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HUGH MACCOLL AND THE TRADITION OF LOGIC

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Part II. Philosophy, Theology, and Literature.

INTRODUCTION

Throughout the short history of modern logic, Hugh MacColl's (1837–1909) pioneering achievements have scarcely been received with adequate esteem. In 1877, two years before the *Begriffsschrift*, he published the first purely symbolical presentation of a variant version of propositional logic. J. M. Bocheński, one of the few scholars familiar with his early system, judged it to be "the climax of mathematical logic before Frege". Towards the end of the century MacColl developed the first modal system in the history of modern logic. Twenty years before Lewis, he defined the concept of strict implication, and earlier than Peano he accounted for inclusion by means of implication.

In spite of some benevolent acknowledgements, neither MacColl's contributions to the development of logic nor the philosophical context in which they came forth have been investigated with appropriate attention. Only during the second half of the 20th century have MacColl's major domains of research, i.e. modal and non-classical logic, gradually turned into acclaimed fields of interest. By then, however, their exploration could rely on methodological standards that for MacColl were not within reach. In addition to such general reasons, specific obstacles have hindered serious efforts to investigate his account of logic. Most of MacColl's articles and books are not readily available. A reliable and comprehensive bibliography has not been published. Had MacColl benefited from an ordinary academic career his works would most likely have come down to us in a more accessible form.

The colloquium on *Hugh MacColl and the Tradition of Logic*, held at the Ernst-Moritz-Arndt University of Greifswald from 29 March to 1 April 1998, was designed with the intention of initiating serious research on this author whose predominant intent was "to bridge the gulf between Symbolic Logic and the Traditional". We are pleased to publish here the proceedings of this colloquium.

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Theories of logic are naturally supposed to strive for utmost generality. Nevertheless, their invention depends on specific historical conditions, even beyond the realm of the formal sciences. MacColl's contributions to logic draw on progress in disciplines such as algebra, analysis and probability theory. Likewise, they depend on some of the author's philosophical assumptions and comply with his metaphysical and religious beliefs. Finally, MacColl's literary works allow us a comprehensive grasp of his views on the human condition. The articles in this volume touch upon these different subjects and perspectives. In this way they evidence the truly interdisciplinary intention with which the colloquium was designed: MacColl's various contributions to the history of symbolic logic should be presented and discussed not just with reference to the field's internal development. Without concern for their broader, cultural preconditions, the occurrence, relevance and meaning of his achievements cannot receive an appropriate understanding.

Due to this methodological stance, the colloquium has also led to a detailed inquiry into MacColl's biography and the conditions of his personal life. The results of this research will be published separately.[1] However, a résumé of our findings is appropriate here.

MacColl was the youngest child of six born to a tenant-farmer in Argyllshire in the Highlands of Scotland in January 1837. His father John died when Hugh was only a year old. His elder brother Malcolm, ordained in the Episcopal Church of Scotland and later in the Anglican Church, tried to support Hugh's education. Hugh became a schoolmaster in England in the late 1850s, eventually moving to France in 1865, settling with his growing family in Boulogne-sur-Mer, where he taught Mathematics (and other subjects) for some forty or more years. He took an External London degree in Mathematics in 1876.

MacColl's literary production falls within four main periods. There is an initial period from around 1870 to the early 1880s in which, arising from the solution of problems in such magazines as the *Educational Times*, he developed logical methods in what is now known as the Boole-Schröder class calculus—featuring prominently in Schröder's huge survey *Lectures on the Algebra of Logic*. From the early 1880s, around the time of his first wife's death, until around 1896 he devoted himself to literary and philosophical studies, publishing two novels. From 1896 he returned to logic, publishing a series of articles which culminated in his book *Symbolic Logic and its Applications* of 1906. In his final years he returned to philosophy, publishing a book on *Man's Origin, Destiny and Duty* in 1909, the year of his death.

[1] M. Astroh, I. Grattan-Guinness, S. Read, "A Biographical Survey on Hugh MacColl (1837–1909)", forthcoming in *History and Philosophy of Logic*.

The colloquium at Greifswald was organised and designed as a component of a larger research project on *The Writings of Hugh MacColl on Logic, Philosophy and Mathematics*. It is being carried out in cooperation with A. J. I. Jones and J. W. Klüwer of the University of Oslo. Meeting the difficulty of getting hold of MacColl's scattered publications, the project will in due course lead to a comprehensive and critical edition of his various publications and correspondence, with a matching bibliography (a pre-print of the bibliography is available at the NJPL web site).

Several initiatives in investigating MacColl's theories on logic have preceded the present one and thus contributed to the setting up of this project. Hence they should be mentioned here. During the nineteen-eighties C. Thiel and V. Peckhaus at the Friedrich-Alexander University of Erlangen carried out a wide-ranging research project on the social history of logic. In the course of their investigations considerable attention was paid to Hugh MacColl. The research reports of 1986 and 1989 by A. Christie, one of their collaborators, contain valuable information on MacColl's biography and the range of his writings. Unfortunately, Christie could not continue his research work on MacColl, so for some years the subject was not pursued any further.

In 1990 and 1991 research seminars at the Friedrich-Schiller University of Jena and the University of the Saarland, organised by M. Astroh, G. Heinzmann and W. Stelzner, focused on MacColl's contributions to logic. Results of this research were presented at the *Frege-Kolloquium*, held at Jena in 1991. Moreover, these joint efforts led to a research project by M. Astroh and K. Lorenz on *Der Begriff des Integrals in den frühen Schriften MacColls*. It was carried out by S. Rahman. His Habilitationsschrift, *Die Logik der zusammenhängenden Sätze im frühen Werk von Hugh MacColl*, partially resulted from these investigations.

With the kind permission of Professor Thiel, it was possible to use Christie's initial research in order to outline the research project on MacColl's writings from which the preparation of this volume ensues. Thus far, the project has led to a colloquium on a pioneer of modern logic whose work contributed to the historical continuity of logical research. In turn the edition of his writings will profit considerably from the efforts of those who contributed to these proceedings.

This volume is divided into two major parts, "History of Logic" and "Philosophy, Theology and Literature". Under the former heading, some contributors concentrate on major issues of MacColl's mature system of modal logic, developed in the years 1895–1905. Others investigate the reception of MacColl's work by eminent authors of his time:

1) I. Grattan-Guinness (Middlesex) looks at the reception of MacColl's ideas on logic in the period immediately around the publication of *Symbolic Logic and its applications* in 1906. MacColl's main critic was Bertrand Russell. MacColl's advocacy of logical pluralism was silenced by Russell's authority.

2) V. Peckhaus (Erlangen) looks back at the much more favourable reception of MacColl's earlier logical work in Germany in the 1880s. But this meant that MacColl's work was eclipsed and forgotten in later years as the conception of logic changed with the turn of the century.

3) S. Rahman (Saarbrücken) sets out from MacColl's notion of symbolic existence, over which he tangled with Russell at the time Russell was trying to free himself from his own not dissimilar views. He uses it to motivate developments of the dialogical logic of Lorenzen and Lorenz. The dialogical free logic which results is even able to contain paraconsistency.

4) S. Read (St. Andrews) gives a systematic development of MacColl's modal algebra, and shows that the basic modal algebra developed by MacColl was Feys' system T. Adding MacColl's conditional operator brings with it the paradoxes of strict implication, a consequence which can be avoided in a more subtle modal construction.

5) P. Simons (Leeds) shows that MacColl's logic is not many-valued, since it is not value-functional. It was intended, and is interpreted better, as a modal probability logic.

6) W. Stelzner (Jena) shows how MacColl's notion of a proposition is context-relative. Accordingly, 'A is true' is interpreted as reporting the value of A in a fixed *de re* situation.

7) G. Sundholm (Leiden) considers MacColl's account of logical consequence. MacColl, like many other logicians of his time, was insensitive to the "Frege-point", that the logical connectives do not connect assertions or judgments but contents of those assertions. The same content can occur both asserted and unasserted.

8) J. Woleński (Cracow) demonstrates the full subtlety of MacColl's treatment of the modalities, in particular, MacColl's notion of a variable—a statement which is possible but uncertain.

Another three articles focus on the wider intellectual context in which MacColl developed his scientific work:

1) M. Astroh (Greifswald) considers MacColl's reaction to Darwin and the doctrine of evolution. MacColl's account of logical form is firmly set in a linguistic theory much influenced by Max Müller's account of the historical development of language.

2) S. E. Cuypers (Leuven) turns to MacColl's last work, on the meaning of life, developing a divine-law conception of ethics. In this,

he opposed Haeckel's explanation using evolutionary biology, wishing to separate the physical sphere where evolution has its place from the human sphere of life, reproduction and mind.

3) S. H. Olsen (Hong Kong) broadens the canvas yet further to set MacColl in his cultural background, mid-century Victorian Britain, a world which changed around him while he remained true to his early upbringing, not least his mother's Puritan and Presbyterian inheritance. Nonetheless, MacColl's novels show his yen to participate in the advancement of science and extension of education in the late nineteenth century.

Both the *Deutsche Forschungsgemeinschaft* and the *Kultusministerium des Landes Mecklenburg–Vorpommern* have supported the colloquium and the publication of its proceedings by substantial grants. We wish to thank both institutions for their generous funding.

Michael Astroh and Stephen Read
Greifswald, Oslo, St. Andrews, Autumn 1999

Ivor Grattan-Guinness

ARE OTHER LOGICS POSSIBLE? MACCOLL'S LOGIC AND SOME ENGLISH REACTIONS, 1905–1912

By the mid-1900s Hugh MacColl had published enough in the way of papers and notes on logic that a more connected presentation could be made. Accordingly, he reworked several items into a book published in 1906 and entitled *Symbolic Logic and its Applications*. In this paper I comment on some main features of the book and note reactions to it at the time, especially by Russell. Consequences of his ignorance of contemporary literature are also considered. The account begins with his little-known book on algebra.

1. Prologue: MacColl's Algebraic Background

In his professional life MacColl worked as a school-teacher in Boulogne-sur-Mer in France. This work led to his first book, 100 pages of *Algebraic Exercises and Problems, With Elliptical Solutions* by "Hugh McColl, *Late Mathematical Master at the Collège Communal*" in the town (MacColl 1870). The costs were seemingly borne by the author; the publisher was the London house of Longmans, Green, who gave it publicity in the 31 August 1870 issue of their trade journal *Notes on books*, with a short text which presumably he wrote himself (Appendix A).[1] A ponderous sub-title explained that the exercises were "Framed so as to combine constant practice in the simple reasoning usually required in the solution of problems, with constant practice in the elementary rules … and the mechanical operations of algebra generally". The word "elliptical" in his title did not signal a return to geometry, but described his model answers for several questions, which the boys were supposed to copy out and then fill in the ellipsis dots with suitable intermediate calculations. Of the exercises themselves,

[1] A copy of *Notes on books*, and MacColl's correspondence, is held in the University of Reading Library Archives, Longmans and Chatto and Windus Archives (hereafter cited as "RULC").

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around 250, many were applied, though often notionally so; for example, given that someone is now $ax/(a-x)$ years old, how old was he $2ax^2/(a^2-x^2)$ years ago? An appendix contained a few simple methods, on one uniform plan, for "resolving algebraical equations" (more sub-title) by methods of indeterminate coefficients.

Given MacColl's later concern with logic, which may have already been active, the link of algebra with algorithm, and thus with logic via "reasoning", is noteworthy, especially as he published at the time when *geometry* was in a state of educational ferment in his home country, with the foundation of the Association for the Improvement of Geometrical Teaching in 1871 (Price 1994, ch. 2). But his book was well enough received for a "new edition" to appear in 1877, with the same text but a much shorter sub-title; the preface was also slightly changed. The mastership was no longer mentioned; instead, MacColl recorded his "B.A. London University".[2] The pages were apparently made up from quires stored after the original printing in 1870, because the publisher's ledger records sale under that form only: 1,000 copies had then been printed, and in 1881, when records cease, 488 copies remained. By 1886 there were still 457 unbound copies left, and it was proposed that MacColl should take back all but 12 of them; he agreed on 29 August, though grudgingly, and presumably the book then disappeared.[3]

By the time of its second edition, MacColl was in his 40th year, in a leisurely and unambitious academic career; but some main ideas in logic had come to him, as described elsewhere in this volume. I turn now to his second phase, during the 1900s.

2. On MacColl's Kinds of Proposition

Like his *Exercises*, MacColl's *Symbolic logic and its applications* was published on commission for the author by Longmans, Green, in a print-run of 1,000 copies (MacColl 1906); in the first two years, fewer

[2]This self-description on the new title page was not correct. His degree was awarded by the University of London (UoL); "London University" was the name of the private company launched in the mid-1820s, and renamed "University College London" in 1836 when the UoL was established as a degree-examining and -awarding body. The distinction between internal and external students came with the Parliamentary UoL Act of 1898; it came up in 1901 in MacColl's correspondence with Russell, when he misunderstood another aspect of the reforms (MacColl 1901a, p. 4; 1901c, pp. 13–14).

[3]Information taken from RULC, Longmans Commission Ledgers 15, 16 and 19 (ledger 19 containing MacColl's letter).

than 200 copies were sold.[4] *Notes on books* again brought publicity, in March (see Appendix B); as stated there and in the Preface (dated August 1905), the text was an assemblage of many of his articles over the decade and even a few bits from earlier ones, all shaped into a coherent text. In the first of its two parts, MacColl laid out (in 105 pages) the principles of his logic, followed by a catalogue of logical operations. Then he treated various standard topics, such as the valid syllogistic modes (where he had some cavils with traditional readings), enthymemes and inference; he also solved some unidentified "recent examination questions". The last chapter began the more mathematical concerns, such as mathematical induction and definitions of infinitude. Some aspects of this part are considered here and in the next two sections; the last one will also take the second part of the book, in which he handled a "Calculus of limits".

The account of the system was both prosodic and algebraic, and the chosen symbolism hinders understanding of the words. Notable from his first phase of the late 1870s for putting forward the proposition rather than the class or the terms as the basic logical unit, he symbolised the form of subject A and predicate B as "A^B"; but he used the same symbol structure to notate the kind of proposition and the corresponding 'attribute', such as 'certainty' with "ϵ" and 'is *certain*' with "$^\epsilon$". He wrote not only "C^ϵ" as 'proposition C is certain', but also "ϵ^ϵ" for 'a certainty is certain'. He is often regarded as using "ϵ" ambiguously; but I see *a clear distinction*, though somewhat spoilt by using the same letter in two different symbols. It seems to be based upon the unstated principle $C^\sharp = (C = \sharp)$, where "$\sharp$" and "$^\sharp$" each runs through its quintet of cases and (his) symbol "=" denotes the equivalence between two propositions. One signal consequence was that he could define each logical connective for propositions (pp. 7–9). But he could have presented the distinction more clearly; the attributes for the main five kinds were given on pp. 6–7, but only three of the kinds themselves were given (on p. 9), after he had stated a string of symbolised propositions.

More doubt surrounds the kinds 'certainty' and its opposite, 'impossibility'. They are associated with the respective probability values 1 and 0 for propositions of these kinds (p. 7). However, and ironically in a logician who had developed his theory from a basis in probability theory, only implication holds here: if C is certain or impossible, then its probability value is 1 or 0, but not necessarily vice versa.

[4]RULC, Longmans Commission Ledger 22. Appendix B comes also from this source.

The kind 'variables', symbolised "θ", is also conceptually unclear. It corresponds naturally to the attribute "is *variable*", occupying the middle ground between certainty and impossibility, as the kind 'possible but uncertain', with some associated probability value within $(0, 1)$ (pp. 6–7).[5] As always with his propositions, the kind has to be understood in terms of the form of words rather than of its reference; for example (his own), 'Mrs Brown is not at home' is a variable according to her status at the (implicit) time of consideration: "To say that [it] is a *different proposition* when it is *false* from what it is when it is *true*, is like saying that Mrs. Brown is a *different person* when she is in, from what she is when she is *out*" (p. 19). He did not consider the case of a proposition which would be orthodoxly true or false, but maybe of unknown value (for example, "Mrs Brown was born in this house"). Otherwise, his position is quite clear; but it clashed with the prevailing philosophy of propositions.

In his review of the book in *Mind*, which in several respects was quite positive, Russell (1907) picked on this feature, regarding the kind as specifying a propositional function; for him a proposition was always true or false, although we may not know which one. In October 1905 he gave a lecture to the Oxford Philosophical Club on "necessity and possibility" (Russell 1905); it is possible that his recent correspondence with MacColl, quite intense that year, had motivated the study. At all events he considered various senses of modalities, including appraisal, as logical or epistemological. He appraised uses in various current figures: Meinong (psychological), Bradley (confusing the necessity of a proposition from that of implication), Bosanquet (confusing hypothesis with disjunction), Moore (failed effort to establish logical priority among propositions) and MacColl (confusing proposition with propositional function). Ironically, his conclusion was illogical:

> I conclude that, so far as it appears, there is not one fundamental logical notion of necessity, nor consequently of possibility. If this conclusion is valid, the subject of modality ought to be banished from logic, since propositions are simply true or false, and there is no such comparative and superlative of truth as is implied by the notions of contingency and necessity.

However, just because there is no definitive version of the theory, why should one abandon all of them? At that time the status of the axioms of choice was similarly unsettled, but nobody wished to abandon the entire concern. Again, to announce bivalency categorically begged

[5] Sadly, MacColl stipulated that these probability values could only be rational numbers. This restricted the scope of this theory in ways of which he may not have been aware.

the question which MacColl wished to address. Finally, the association of certainty and impossibility with comparative and superlative truth-values does not characterise MacColl's position. In a final irony, from the late 1910s Russell was to adopt MacColl's sense of necessity and possibility, though explicitly attributed to propositional functions (Russell 1919, p. 163). However, it never became a main part of either his logic or of his epistemology.[6]

3. MacColl on Realities and "Null Classes"

Since MacColl considered predicates, he had to handle collections of objects which satisfied them, and he made a distinction between individuals with "a *real existence*" and those without it. Construing classes in the usual part-whole sense, he considered "pure" classes composed of individuals of one or another type, and "mixed" ones containing both. No general criterion of distinction was given, but by way of example he assigned to the first kind "*horse, town, triangle, virtue, vice*" (MacColl 1906, p. 42). However, these examples were of concepts, not objects at all. Further, while the first three candidates seem unexceptionable, the latter two are surprising; the reason given was that "the statement 'Virtue exists' or 'Vice exists' really asserts that virtuous persons, or vicious persons, exist; a statement which every one would accept as true". What, however, are the grounds for this reduction? Are philosophies which wish to reify virtue and vice as concepts (or individuals) in their own right categorically rejected, and on what grounds?

In addition, individuals of the second kind were contained in "the class 0" as "unrealities" to which "necessarily belong such as *centaur, mermaid, round square, flat sphere*" (pp. 42–43; curiously, this novelist did not propose the example of fictional characters). He stressed that his "null class" differed from the usual definition as containing no members, and "*contained in every class*, real or unreal; whereas I consider it to be *excluded from every class*" (p. 77). Apart from the unclear distinction, it is most unfortunate that he gave classes of this kind a name which was already widely used in quite different senses in set theory, part-whole theory, algebraic and mathematical logic. In his review Russell did not respond well to this proposal. However, his own use of "existence" is confusingly multiple. In his writings I find the following senses, for individuals x (I) and for classes u (C):

[6]Dejnožka (1990) claims that Russell developed theories of modality; however, while the texts cited are instructive and more numerous than one might guess, the claim is well tempered by Magnell (1991).

Case	Notation(s)	Sense of existence
I1	$\exists x$ ($\exists x$)	As in existential quantification
I2	$E!(\imath \mathrm{x})(\phi x)$	Of a referent of a denoting phrase
C1	$\exists u$ ($\exists u$)	As in existential quantification
C2	Eu	Abstractable from a propositional function
C3	$\exists`u$ $\exists u$ $\exists!u$	Non-emptiness of a class

These senses can conflict; for example, the empty class exists (or may do so) à la C1 and C2, but not C3, while conversely the class which generates Russell's paradox exists only in C3. His own criticism of MacColl was based on restricting existence to the Peanesque sense C3, which is contentious.

4. MacColl on Infinitudes

In a footnote to his calculus of limits MacColl indicated that "the symbol 0, representing *zero*, denotes ... that particular non-existence through which a variable passes when it changes from a positive infinitesimal to a negative infinitesimal" (1906, p. 106). He was referring to another eccentricity, in which he stated that "a *negative infinitesimal* denotes any positive quantity or ratio *too small numerically in any recognised notation*" (p. 104), adding similar "definitions" for the negative counterpart, and also for the positive and negative infinite with the property of "too large" instead of "too small" (pp. 108–110). Thus, just when Russell (and Gottlob Frege before him) had sorted out the tri-distinction between zero, the empty set and nothing, MacColl now specified zero as a limiting value of sequences of infinitesimals. He also distinguished his infinities from "non-existences" such as 1/0 and 3/0, which were unrealities belonging to the (or a) null class 0.

These formulations again show MacColl's view of a proposition as a form of words, and with the word "recognised" the same reduction to personal or social assessment that we saw above over virtue and vice. Thus his position was consistently maintained, but it hardly brings conviction. In his last letter to Russell, written on 18 December 1909 just before his death, he fell into impredicativity as well as error when claiming that, M^M, where M is a million, "would be inexpressible, *and therefore infinite*", in Roman numerals! (MacColl 1909a, p. 9). His philosophy of propositions risks conflating words with their references; one may define (or specify) infinitude finitely, as Georg Cantor and Richard Dedekind, among others, had long done, in literature of which he was unaware.

Finally, links with theology need to be noted. MacColl was a Christian believer; he wrote on religion in the *The Hibbert Journal*, a "quarterly review of religion, theology and philosophy" founded in 1902 with the funds established 50 years earlier by the Victorian philanthropist Robert Hibbert (1770–1849), attacking arguments of chance against divine design (1907a). But he did not make theological links to mathematics via his infinitude, a view which had gained some currency at the time, stemming in part from Cantor himself (Dauben 1979, esp. ch. 6). When the American mathematician and believer C. J. Keyser (1862–1947) wrote a rambling piece in the journal concerning "the message of modern mathematics to theology" (1909), MacColl (1909b) expressed general sympathy in his reply, but rehearsed his own theory of infinitude and "unreal ratios" such as 2/0, with no appeal to the Maker. There is a curious parallel with Russell, who in 1904 had opposed Keyser's claim in the journal, argued partly out of theology, that an axiom of infinity was needed in set theory; in the end Russell also adopted one (Grattan-Guinness 1977, pp. 24, 127).

Some of MacColl's remarks on limits occurred in the second part of the book, in which he presented in 35 pages a "Calculus of limits" in the differential and integral calculus as the principal of his applications. The basis was an analogy between truth-values in compound propositions and sign laws in algebra; for example, True and False is False in logic, Positive times Negative is Negative in algebra.[7] He used superscripts again, with "^{P}" and "^{N}" to define appropriate propositions, such as

$$(x-3)^P = (x>3), \text{ and } (x-3)^N = (x<3) \tag{1}$$

(p. 107, with "=" serving as equality by definition—in addition to its use as arithmetical equality and equivalence between propositions). The ensuing theory advanced to an algebraic method of expressing the change of limits in multiple integrals, where inequalities such as in (1) were used to state that a variable lay between given values. The procedures were quite ingenious, but hardly an application to which mathematicians would rush; some of the cases, such as finding the roots of a quadratic equation (pp. 112–113), can be effected rather more quickly by the usual means.

[7] Indeed, truth tables were introduced with the symbols + and − in the doctoral dissertation of 1920 of Emil Post (a student of Keyser, incidentally).

5. Appraisals by Shearman and Jourdain

Although Russell was the most important and penetrating of commentators on MacColl, he was not the only one; here we note two other compatriots. The first was A. T. Shearman (1866–1937), also a graduate of the University of London (in 1888, when studying at University College Aberystwyth in Wales), and later lecturer in philosophy at University College London.[8] He addressed MacColl's work in a lecture on current symbolic logics to the Aristotelian Society in London (Shearman 1905), and especially in a book of his own published in 1906. This was a pleasant though not profound historico-philosophical survey of these logics, to which I largely confine this summary.

Shearman's knowledge of MacColl's work was based on the recent papers. The main discussion came in a section of 23 pages; in contrast to his praise of MacColl's "very ingenious system" in his 1905 paper, here he focused upon aspects "wherein I think he falls into error" (Shearman 1906a, p. 149). They included MacColl's (apparent) conflation of propositions with propositional functions; the dependence of modalities upon thinkers, for example, with an attendant confusion of "events with statements"; and the theory of unrealities, with attached issues of existence, where he took Russell's side. MacColl commented on this book, and on a short criticism (Shearman 1906b) in *Mind* of his views on existence, in his reply (1907b) to Russell's review. In addition to (rightly) defending his use of symbols, he firmly upheld the independence of modalities from thinkers, giving as an example of a certainty the mathematical theorem that $3 \cdot 141 < \pi < 3 \cdot 142$. A short non-discussion between the two logicians followed in later volumes of *Mind*. Both Shearman and MacColl were reviewed anonymously, and coolly, in *Nature* (Anonymous 1906).

The second commentator was Philip Jourdain (1879–1919), a former student of Russell at Cambridge who devoted much of his career to set theory (not very impressive) and to its history and that of mathematical and algebraic logics (much better). In particular, he wrote a suite of articles on the work of six logicians, publishing them as a three-piece paper in a mathematical journal. The second contained 18 pages on MacColl (Jourdain 1912, pp. 219–236); typically dense with references to many of MacColl's publications from the start, it gave a fair summary of MacColl's theory and its applications. He stressed the role of probability theory (without describing it in detail), and MacColl's priority for asserting the primacy of the proposition.

[8]By a bequest to University College London a trio of "Shearman lectures" on logic and philosophy is held from time to time, of (in my experience) variable merit and relevance to the intended topics.

However, Jourdain's criticisms were quite strong, especially on the non-discussion of correspondence and the non-admittance of propositional functions; he mainly followed Russell's views and quoted some of them. Upon receiving Jourdain's manuscript of the article in September 1909 Russell was "glad to find you so much in agreement with me as regards the points about which MacColl and I have differed". Three years earlier he had judged it "amazing how MacColl's reply [to his review] ignores all the points that I have raised" (Grattan-Guinness 1977, pp. 119, 101). MacColl himself received the manuscript, apparently in March 1909,[9] and Jourdain quoted him in the published version on three points: a detail on symbolism (Jourdain 1912, p. 221); an explanation of real and unreal classes (p. 232); and a general note at the end, where with his usual courtesy he thanked Jourdain for the attention but regretted that he had "sided with the symbolists" on many issues, and stated some of the exercises in logic and uses of the calculus of limits where he felt his own approach to be currently the best (p. 236).

6. MacColl's Historical Place

As the Jourdain article showed, MacColl was working in an environment largely unsympathetic to or uninterested in his main concerns. But MacColl's concessions on some quite elementary features did not help to fight disinterest or resistance. In particular, in his book he distinguished between a statement as any kind of utterance, including "a shake of the head, the sound of a signal gun", and so on, and a proposition as the special case of declarative sentence in subject-predicate form (p. 2); however, in his reply (1907b) to Russell's review he asserted that "a proposition is simply a conventional arrangement of words or symbols employed to convey information or express a judgement", which is surely the previous construal of "statement". Some technical terms were poorly chosen; for example, to use the phrase "null class" in such a non-standard way is surpassed in misfortune only by his ridiculous "definitions" of infinitude, both put forward at a time when Georg Cantor's theory of sets and transfinite numbers had become so important in foundational studies in both mathematics and logics. These cases lay in the applications of the logic rather than at its centre, but they must have discouraged enquiry into the logic itself.

[9] No correspondence with MacColl is held in the surviving fragments of Jourdain's *Nachlass* in the Institut Mittag-Leffler, Djursholm, Sweden. There is also none with John Venn at Gonville and Caius College, Cambridge.

On notations, apart from the ambiguity over the Greek letters on the line and in superscript, MacColl's are among the most forgettable of my acquaintance. His use of normal symbols such as "0" and "=" followed the tradition of multiple interpretation in algebraic logic which stemmed from Boole; but ambiguities arise, especially in the work of Ernst Schröder, and MacColl also manifests them. The tradition of mathematical logic from Frege and Peano onwards had tried to avoid such oversell, and this avoidance marks one of the significant differences between mathematical and algebraic logics (Grattan-Guinness 1975). Another useful influence from algebraic logic was duality; while less marked than in Schröder, it was evident in his five kinds of propositions, with impossibility and certainty as poles on either side of truth and falsehood, and variability in the (unclear) middle.

Perhaps because of his self-education, MacColl comes across as rather half trained in mathematics; the applications of his system there are not exciting. Further, in his last letter to Russell, summarising an argument about limits in a paper published posthumously in *Mind* (MacColl 1910, art. 5), he wondered why the notation "dy/dx" could not be read as a ratio of infinitesimals (MacColl 1909a, p. 5), apparently unaware that this was *exactly* how Leibniz introduced it (with his own sense of infinitesimal) and his successors used it (Bos 1974). Among other cases, in the general journal *The Athenaeum* (MacColl 1904)[10] he held out against non-Euclidean geometries due to their supposed assumption that parallel lines meet, and so placed them among his unrealities; Russell's reply (1904) reads somewhat like a tutorial note.

In addition, MacColl's failure to learn German or Italian, mentioned in his letters to Russell (MacColl (1901b, p. 2), MacColl (1909a, p. 2) and MacColl (1901c, pp. 10–11)), prevented him from knowing some major sources and thus from contributing effectively. His inability to read *technical* Italian when he lived so long in France seems pathetic. He did not even draw on pertinent writings in French, such as papers by Peano, Schröder and others in the proceedings of the 1900 Congress of Philosophy in Paris, to which he himself contributed his longest paper (MacColl 1901d) summarising many features of both logic and his applications. That paper was the best contact he made with the mathematical community this century; it was enhanced by an accurate summary published in an American review article of mathematical lectures given at the Congress (Lovett 1901, pp. 166–168). However, he knew

[10] From 1871 to 1900 *The Athenaeum* had been edited by one Norman MacColl (1843–1904), otherwise a Spanish scholar; although also Scottish-born, he seems to have been no relation to Hugh. His successor as editor was Vernon Randall.

little of the Anglo-Saxon representatives of algebraic logic; he used C. S. Peirce's symbol for implication in a passage of his book where he also noted Schröder's (1906, pp. 78–80), but he did not discuss or use their systems. He also ignored both their logic of relations and an alternative version produced by Russell in 1901 for mathematical logic, and did not develop one of his own, although it was recognised as a major component of both traditions in logic.

For these reasons MacColl got the good idea of modal logic off to a bad start, and has never been given his due in the history of the subject. Thus it is not surprising that the bibliographical sleuth Giuseppe Peano omitted him from the lists of recent writings in his compendium *Formulario mathematico*, even in its final edition of 1908 (where, curiously, Frege is also missing). When C. I. Lewis came to restart modal logic in the early 1910s, especially in various papers in *Journal of Philosophy* and *Mind*, MacColl was almost entirely ignored. In a book of 1918 he found MacColl's systems only "suggest somewhat" his own (1918, p. 108), and omitted them even from the long opening historical chapter. His main inspiration, and negative one, had come from *Principia mathematica*. In a later discussion of Lewis's logic, W. T. Parry went to great pains to contrast it with MacColl's (Parry 1968, pp. 21–24).

7. The Fight for Logical Pluralism

In any case, the philosophical climate was hostile. For example, Russell prepared the manuscript of his lecture (1905) on necessity and possibility to the extent of furnishing most of the needed references; but he never bothered to finish it off for publication. Again, in a review of MacColl's book in *The Philosophical Review*, the American philosopher J. G. Hibben dismissed the three extra kinds of proposition as "a needless complication", and found the section on the calculus of limits to be the most original part (Hibben 1907). Although he writes in a conciliatory way, MacColl may have been arguing for his system as *the* correct logic while Russell and others were opposing him with the traditional bivalency; in this section I shall take MacColl as a source for logical pluralism, in which the traditional form is not the only possibility (as it were).

The struggle for logical pluralism has been long and hard; in this regard I end with an anecdote. In 1972 McMaster University held a conference to celebrate the centenary of Russell's birth, and also the establishment of the Russell Archives. One of the speakers was Nicholas Rescher, who gave a talk on "neglected aspects of Russell's work on logic"; from my memory of the occasion, much of the content appeared also in a later paper on "Russell and modal logic", published as Rescher 1974 in a volume on modality and reprinted five years later in a tributary book on Russell. Not much in the way of tribute was provided, however; apparently, Russell had found MacColl to be "so much old-fashioned fairy tale nonsense", thereby exercising "a baneful influence" on the acceptance of modal logics.

This historical appraisal seems to be rather implausible; if MacColl was old-fashioned, then who were his predecessors? Rescher presented Russell's position on modalities only from his book of 1900 on Leibniz, written before his discovery of Peano or acquaintance with mathematical logic, and thus a quite different figure from the one who confronted MacColl from 1904 onwards.[11] When the lecture finished I began the discussion period with (what I hoped was seen as) an *historical* appraisal of Russell's situation when he came across MacColl, as follows. Thanks to Peano he had been able to envision a comprehensive foundation for (much) mathematics in the propositional and predicate calculi with quantification, in which Cantor's set theory played a central role. Thus, when he found MacColl's alternative approach to which he not only felt unsympathetic but which in any case was fraught with unclarities, weak definitions and unwelcome uses of technical terms, his reaction was understandably cool.

Rescher began a reply, but was interrupted from the audience by Max Black, who denounced categorically non-classical logics of all kinds (that is, without argument). His advocacy of logical monism was immediately acclaimed by several other participants, so that the historical discussion which I had tried to launch degenerated into a philosophical attack on logical pluralism. However, I maintain it, and give credit MacColl for pioneering it in modern times; there is indeed more than contextual difference between the truths of "$7 > 4$" and of the residential properties of Mrs. Brown.[12] But this reaction in the 1970s shows how hard and durable was the resistance.

At some time, perhaps when his manuscripts were being organised for sale in the late 1960s, Russell annotated some of his collections of letters with remarks on the correspondent, often quite warm. For MacColl he merely put "a mathematical logician with whom I disagreed". However, he kept all of the letters that he had received from

[11] For criticisms of the paper, see Dejnožka 1990, pp. 406–412.

[12] At this time Henri Poincaré (1905, p. 827) also read propositional functions this way, as sometimes true and sometimes false; but I take him as obtusely making a (pathetic) criticism of the mathematical logic which he despised but did not trouble properly to learn.

MacColl, unlike other collections of which he kept little (for example, and more typically, about 15% of the letters from Jourdain). Presumably he sensed some merit in MacColl's proposals for others to find. At last we can realise his archival investment and give credit to a pioneer modal logician and maybe logical pluralist whose ideas have their historical place and perhaps still some future of their own.

Acknowledgements

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Appendix A

Notes on Books, August 31 1870:

Algebraic Exercises and Problems, with Elliptical solutions; framed so as to combine Constant Practice in the Simple Reasoning usually required in the Solution of Problems with Constant Practice in the Elementary Rules, the Simplification of Fractions and other expressions, and in the Mechanical Operations of Algebra generally. With an Appendix containing a few Simple Methods, on one uniform plan, for Resolving Algebraical Expressions into their Elementary Factors. By HUGH MCCOLL, late Mathematical Master at the Collège Communal, Boulogne-sur-Mer. 12 mo. pp. 108, price 3*s*.6*d*. cloth. [*June* 16, 1870.

THIS little book, intended for beginners in Algebra, is not meant to supersede any of the ordinary systematic treatises on the subject.

Its main object is to train beginners in the application of the elementary operations of algebra to the solutions of problems. The elliptical solutions at the end of the exercises have been written with this view. The pupil is supposed to copy these on paper, filling up the blanks as he goes along. Each link in a chain of reasoning is thus brought before him in logical succession; but he is expected to join the links and complete the chain himself. If he does all this successfully and without the assistance of his teacher, the latter may feel satisfied that his pupil has understood both the problem proposed and its solution; while, on the other hand, if the pupil meets with any difficulty the teacher can at once see where the difficulty arises, and remove it. This secures economy of labour on the part both of teacher and pupil. The latter, instead of wasting time and energy in attempting to solve by his own unaided powers those problems which upon a fair trial he finds too difficult for him, is enabled to concentrate his attention successively upon single points, and so conquer his difficulties in detail; while the former is spared the trouble of explaining difficulties which his pupil might not perhaps find very formidable if left to himself, and may reserve his assistance for those cases which really require it.

Many of the examples are so framed as to make hardly any demand upon the reasoning powers, while at the same time they afford good practice in the four elementary rules, the simplification of fractions and other expressions, and in the mechanical operations of algebra generally. The pupil will find the answers to these at the end of the exercises, but, generally speaking, no solutions.

Appendix B

Notes on Books, March 1906:

Symbolic Logic and its Applications. By Hugh MacColl, B.A. (London). 8vo. pp. xii + 142, price 4*s.* 6*d.* net.

The system of symbolic logic explained in this volume is founded on a series of papers contributed by the author to various magazines, English and French, including *Mind*, the *Athenæum*, the *Educational Times*, and the *Proceedings of the London Mathematical Society.* The author hopes that in this, its final form, his system will be found so simple that schoolboys of ordinary intelligence can master its fundamental principles and apply them to the solution of problems both in logic and in algebra. Solutions of questions set at recent university examinations in logic are given as illustrations. The last two chapters treat of probability and the limits of multiple integrals.

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HUGH MACCOLL AND THE GERMAN ALGEBRA OF LOGIC

In this paper the early reception of Hugh MacColl's logical system up to the 1890s, by the German algebraist of logic, Ernst Schröder, is investigated. In his monumental *Vorlesungen über die Algebra der Logik*, Schröder refers to MacColl as one of his most important precursors. It will be shown that MacColl was a respected member of the logical community of his time, taking his position in the competition for the best (most effective) logical system. Schröder's comparison of the procedures for solving logical problems provided by different logical systems, in particular the different ways of solving Boole's famous "Example 5", is discussed. This discussion will demonstrate the importance of the organon aspect of symbolic logic. Finally some conclusions are drawn concerning later neglect of MacColl's logic.

There is no doubt that Hugh MacColl (1837–1909) was incapable of pioneering a specific tradition in logic. Today, however, some of the particulars of his "Calculus of Equivalent Statements" are regarded as ingenious anticipations of innovations which were only much later introduced to logic. Thus, for a considerable time he was not counted among the important pioneers of symbolic logic. It is, however, hasty to infer from the lack of tradition that MacColl's logical work was ignored by his contemporaries. This can be shown by an investigation of the reception of this work in the German algebra of logic, which is represented (almost exclusively) by Ernst Schröder (1841–1902). His monumental *Vorlesungen über die Algebra der Logik*[1] shows that MacColl was a respected member of the international community of logicians in the late 19th and the early 20th century. His logic took part in the competition of logical systems. In this competition the conceptions of

[1] This work was published in four parts, of which one appeared posthumously: Schröder 1890, 1891, 1895, and 1905, these parts reprinted as Schröder 1966.

Nordic Journal of Philosophical Logic, Vol. 3, No. 1, pp. 17–34.

the German mathematicians Ernst Schröder and Gottlob Frege (1848–1925), the American logicians Charles S. Peirce (1839–1914) and Christine Ladd-Franklin (1847–1930), the British, William Stanley Jevons (1835–1882) and John Venn (1834–1923), the Polish Russian Platon Sergeevich Poretskii (1846–1907), and others, concurred. The main task was to solve logical problems, i.e. to schematize them with the help of logical formulas and dissolve them to the unknowns. How the solution was found within the different systems facilitated comparative evaluation of the calculi with respect to their efficiency, elegance and economy. It is obvious that the results of such a comparison could affect the further development of a calculus.

In this paper I will deal with this competition concerning the best logical system from the perspective of the German algebra of logic. How MacColl's logic was received may reveal clues as to why his system never found the recognition it deserved, although it was highly, but critically, esteemed by his contemporaries.[2]

1. Friendly Contests

The competition of logical systems concerned not their theoretical perfection, but practicability. In accordance with the organon conception of rationalistic logic, logical systems were regarded as devices for solving logical and mathematical problems. Logical systems had to be easily applicable. MacColl wrote, for example, in a comment on John Venn's diagrammatic method:

> Where is the formidable array of 6×2^6 (or 384) letters which Mr. Venn, unless I misunderstood his words, supposes the logician obliged to face as a necessary preliminary to all inference in every problem requiring six letters? Whether Dr. Boole's or Prof. Jevons's method can fairly be charged with imposing this heavy labour I am not prepared to say; but my method certainly does not impose it. (MacColl 1880, p. 171)

MacColl's criterion of comparison was the degree of complexity of logical problems which could be mastered by the logical system. The competitors agreed that solving problems was not a simple task. Therefore they quickly looked for help from mechanical devices. In the second

[2] In writing this paper I was able to benefit from preparatory work done by Anthony Christie in the Erlangen research project "Case Studies towards a Social History of Formal Logic", not brought to publication. In particular, it was Christie's idea to interpret the reception of MacColl's writings with respect to the late 19th century competition of logical systems. Cf. the unpublished typescript, Christie 1986. For the research project see Peckhaus 1986, Padilla-Gálvez 1991, and Thiel 1996.

half of the 19th century not only programmable calculating automata, but also logical machines, were developed.[3] With the help of his logical piano, for example, William Stanley Jevons was able to solve mechanically inferences with four terms.[4]

MacColl advocated public competition. In his review of William Stanley Jevons's *Studies in Deductive Logic* (1880), he challenged the author outright:

> Friendly contests are at present being waged in the "Educational Times" among the supporters of rival logical methods; I hope Prof. Jevons will not take it amiss if I venture to invite him to enter the lists with me and there make good the charge of "anti-Boolean confusion" which he brings against my method. (MacColl 1881, p. 43)

MacColl referred to a discussion taking place in the "Mathematical Questions" column of the *Educational Times*, the journal of the British College of Preceptors (cf. Grattan-Guinness 1992). It is astonishing that logical problems found their way into a journal of this type, aimed at a broader audience. But the *Educational Times* was not a singular case. The unusual interest in the new logic in Great Britain after Boole's death led to a great number of contributions, reviews and letters on formal logic in science journals like *Nature* (cf. Christie 1990) and in other national and regional journals.[5]

2. The Reception by Ernst Schröder

From the very beginning the learned competition concerning the best system of logic had an international flavor. The knowledge of the new logical systems in Great Britain was carried into the world primarily through the writings of William Stanley Jevons. His *Principles of Science* (1874) in particular worked as a catalyst. Furthermore, the effect of Alexander Bain's (1818–1903) *Logic* (1870) should not be underestimated. It was devoted to John Stuart Mill's inductive logic, but contained a section on Boole's symbolic logic which stimulated the emergence of research on symbolic logic in Poland.[6] It is safe to assume that even MacColl got to know Boole's algebra of logic via Bain's

[3] The Analytical Engine of Charles Babbage (1791–1871) is an example, although Babbage was not able to bring it to full operation (cf. Hyman 1982).

[4] Cf. Jevons 1870 and Jevons 1874, pp. 107–114.

[5] Samuel Neil (1825–1901), for example, published in the years 1864 and 1865 a series of articles on "Modern Logicians" in his journal *The British Controversialist and Literary Magazine*, among them comprehensive biographies and reviews on John Stuart Mill (1806–1873, cf. Neil 1864) and George Boole (1815–1864, cf. Neil 1865).

[6] In 1878 Bain's *Logic* was translated into Polish. Cf. Batóg and Murawski 1996.

logic. In the second paper on "The Calculus of Equivalent Statements" (1877/78b), he quoted Boole according to Bain's presentation.[7]

I have shown elsewhere (cf. Peckhaus 1997) that the roots of Schröder's algebra of logic are to be found in the German abstract algebraical conceptions of his time, but not in Boole's algebra of logic. Schröder relied on the general doctrines of forms of Hermann Günther Graßmann (1809–1877, cf. Graßmann 1844) and Hermann Hankel (1839–1873, cf. Hankel 1867), and on Robert Graßmann's (1815–1901) symbolic logic (Graßmann 1872). However, he knew of Boole's calculus and acknowledged its priority from 1874/75 on. Schröder's principal works were the three volumes of his *Vorlesungen über die Algebra der Logik* published between 1890 and 1905. In the first volume (1890), he primarily treated the class calculus after having founded a more general calculus of domains. The second volume, of which a second part appeared posthumously (1891, 1905), was devoted to the propositional calculus. In these two volumes MacColl was, after Charles S. Peirce, the most frequently mentioned author. Peirce's dominance is understandable, insofar as Schröder, in forming his calculi, followed Peirce's model very closely as it was developed in the two papers "On the Algebra of Logic" (1880, 1884). But Schröder always compared Peirce's considerations with the early parts of MacColl's series of papers "The Calculus of Equivalent Statements" published between 1877 and 1880, and he granted MacColl priority when he had anticipated Peirce's results. As far as Schröder was concerned, MacColl's calculus held the status of a preliminary stage of Peirce's algebra of logic.

There are three aspects in Schröder's reception of MacColl's logic which seem to be especially interesting and therefore deserve closer examination:

1. The different efficiencies of the calculi in eliminating terms and resolving logical formulas could provide a measure for the quality of these calculi. Schröder compared his own procedure, derived from Boole's, with Peirce's, the latter being regarded as a natural way of proceeding. He discussed MacColl's method extensively as a preliminary stage.

2. Schröder explicitly accepted MacColl's priority in formulating a propositional logic. In most cases he spoke of a "MacColl-Peircean propositional logic." Nevertheless, he criticized both authors' at-

[7]Elsewhere (MacColl 1878/79, p. 27), he mentions "the kindness of the Rev. Robert Harley for the loan of Boole's 'Laws of Thought'." Referring to Boole's *An Investigation of the Laws of Thought* (1854), he declares that he found many differences, but also numerous points of contact.

tempts to found logic on propositional logic. This conception, he argued, is less general than his own, because he founded the calculus of propositions on the calculi of domains and classes.

3. Schröder stressed the MacColl-Peircean priority of defining material implication, and adopted this definition in his own propositional logic.

2.1. Solution of logical problems

Schröder treated the solution of logical problems in the last two paragraphs of the first volume of his *Vorlesungen über die Algebra der Logik* (1890, §§ 25, 26). He characterized the type of problems discussed in the beginning of § 26 as follows:

> The preceding discussion only concerned problems whose data can be expressed by subsumptions (or equations [...]) between such classes or functions of such in the identical [i.e. Boolean] calculus, and whose solution can also be expressed by propositions of this form. It was important to eliminate certain classes from the data of the problem, to calculate others from these data [...], i.e. to find their subjects and predicates which can be described with the help of the remaining classes. (Schröder 1890, p. 559)

Here Schröder refers to the thirty problems which he solved in the preceding paragraph with the help of his class calculus as far as it was developed at that stage. He then went on to compare his results with solutions provided by alternative calculi. He mentioned Peirce, who had listed in his paper "On the Algebra of Logic" five logical methods in chronological order, by Boole, Jevons, Schröder, MacColl and himself (Peirce 1880, p. 37). Schröder expressed the opinion that these five methods could be reduced to three, because his own was a modified version of Boole's, which thus became obsolete (Schröder 1890, p. 559). Furthermore, the methods of MacColl and Peirce could be combined, because MacColl had paved the way for Peirce's (p. 589).

Schröder chose a problem first published by Boole as a tool for testing the performance of the calculi. It held some prominence among the mathematical and philosophical logicians of the time because of its complexity.[8] Boole's formulation of the problem is quoted in full, but the different solutions are only sketched:[9]

[8] "Example 5" in Boole 1854, pp. 146–149.

[9] Boole 1854, p. 146, cited by Schröder in translation with some revisions (1890, p. 522). This problem was also treated by Hermann Lotze in his "Anmerkung über logischen Calcül" (Lotze 1880, pp. 265–267). Lotze criticized Boole's claim that his solution of the problem shows the advantage of his calculus over syllogistics. Lotze agreed with Boole that it was senseless to try to solve this problem syllo-

Ex. 5. Let the observation of a class of natural productions be supposed to have led to the following general results.

1st, That in whichsoever of these productions the properties A and C are missing, the property E is found, together with one of the properties B and D, but not with both.

2nd, That wherever the properties A and D are found while E is missing, the properties B and C will either both be found or both be missing.

3rd, That wherever the property A is found in conjunction with either B or E, or both of them, there either the property C or the property D will be found, but not both of them. And conversely, wherever the property C or D is found singly, there the property A will be found in conjunction with either B or E, or both of them.

Let it then be required to ascertain, first, what in any particular instance may be concluded from the ascertained presence of the property A, with reference to the properties B, C, and D; also whether any relations exist independently among the properties B, C, and D. Secondly, what may be concluded in like manner respecting the property B, and the properties A, C, and D. (Boole 1854, p. 146)

In his translation Schröder labelled the data α, β, and γ, and he split the two questions into four:

Let it be required to ascertain,

first, what in any particular instance may be concluded from the ascertained presence of the property A, with reference to the properties B, C, and D,

secondly, also to decide whether any relations exist independently from the presence or absence of the other properties among the presence or absence of the properties B, C, and D (and, if yes, which?),

thirdly, what may be concluded in like manner from the existence of the property B with respect to the properties A, C, and D (and vice versa, when the existence or absence of the property B can be inferred from that of the properties of the latter group),

fourthly, to state what follows for the properties A, C, D as such. (Schröder 1890, p. 522)

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gistically, but did not regard the calculatory procedure as obvious. He preferred a combinatorial way which he obviously adopted from Jevons. This combinatorial way "presents itself automatically as the more appropriate" (Lotze 1880, p. 266). Jevons's combinatorial procedure was a subject of correspondence between Lotze and Schröder. Schröder reported on this correspondence, criticizing Lotze's devaluation of the calculatory method (cf. Schröder 1890, pp. 566–568). Gottlob Frege, like Lotze, criticized the artificiality of this problem in his comparison of the *Begriffsschrift* with the Boolean calculus (Frege 1983, p. 52). Nevertheless, he also tried to solve the problem. Frege's pathbreaking solution is thoroughly discussed by Peter Schroeder-Heister (1997). A favorable treatment of this problem can be found in Wilhelm Wundt's logic (1880, p. 357). Gottfried Gabriel suggested in 1989 the examination of the different solutions in order to obtain criteria for a comparison of different systems of logic, i.e. traditional logic, algebra of logic, and Frege's *Begriffsschrift*(Gabriel 1989, p. XXIII).

Schröder's notations will be used to sketch out his solution. Lowercase Latin letters stand for classes; their properties are marked with respective capitals. Logical addition (adjunction) is marked by $+$, logical multiplication (conjunction) by juxtaposition. Negation is symbolized by an appended negation stroke, e.g. $a_{|}$ stands for the negation of a. The subsumption symbol $\Subset$ indicates class inclusion in the class calculus. The class symbols 0 and 1 stand for the empty class and the universal class, respectively. The functional symbol $f(x)$ represents in the identical (i.e. Boolean) calculus a complex expression containing x (or $x_{|}$) and other symbols connected with the help of basic logical operations, identical multiplication, addition and negation (cf. Schröder 1890, p. 401). Schröder's solution will be outlined in as far as it is necessary to compare it with the alternative solutions discussed.

In an initial step Schröder presented the data, i.e. the conditions α–γ, as subsumptions or equations. In Schröder's calculus, equality is derived from subsumption: $a = b$ stands for a subsumption relation between the terms a ("subject") and b ("predicate") which is valid in both directions at the same time. $a = b$ is thus defined as $(a \Subset b)(b \Subset a)$. In Schröder's symbolism the data α–γ can be formalized as follows:

$$\begin{array}{rrcl} \alpha: & a_{|}c_{|} & \Subset & (bd_{|} + b_{|}d)e \\ \beta: & ade_{|} & \Subset & bc + b_{|}c_{|} \\ \gamma: & a(b+e) & = & cd_{|} + c_{|}d\,. \end{array} \tag{1}$$

These formulas contain the class symbol e as related to the property E, which then has to be eliminated because it does not affect the solutions of the questions. In order to eliminate this class symbol, Schröder put the equations (1) to 0 on the right hand side, and finally combined these three equations by conjunction into one. For this purpose he could use two theorems proven earlier:[10]

$$38_{\times} \qquad (a \Subset b) = (ab_{|} = 0)$$

and

$$39_{\times} \qquad (a = b) = (ab_{|} + a_{|}b = 0).$$

After combining the modified premises the following equation results:

$$\begin{aligned} &a_{|}c_{|}(bd + b_{|}d_{|} + e_{|}) + ade_{|}(bc_{|} + b_{|}c) + \\ &\quad + a(b+e)(cd + c_{|}d_{|}) + (a_{|} + b_{|}e_{|})(cd_{|} + c_{|}d) = 0\,. \end{aligned}$$

[10] Schröder erroneously mentions Theorem 39_{+} instead of $39_{\times}$ which allows, however, an equation to be brought to 1.

Several steps of calculation are required for the elimination of e. They result in the formula

$$a(cd + bc_{\shortmid}d_{\shortmid}) + a_{\shortmid}(cd_{\shortmid} + c_{\shortmid}d + b_{\shortmid}c_{\shortmid}d_{\shortmid}) = 0\,.$$

This formula is the starting point for further eliminations and resolutions of certain class symbols, finally leading to an answer for the questions.

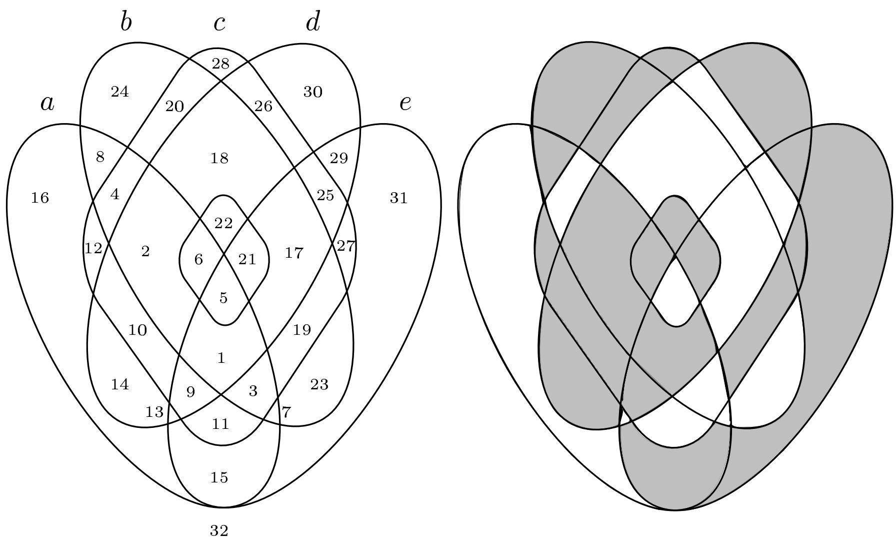
Schröder stressed the similarity of his method with that of Boole, which he considered, however, to be "definitely settled" because of his modifications. As far as Schröder was concerned, Boole's method was therefore only of historical interest. Its disadvantages resulted from the lack of a sign for negation—Boole had to write $1 - x$ for $x_{\shortmid}$—and from the interpretation of the logical "or" as an exclusive "or". The inadequacy of Boole's language led to logically uninterpretable expressions in the course of calculating logical equations according to the model of arithmetic.

Schröder started his discussion of alternatives with Jevons's method which he called "without art" ("kunstlos"), although it was the "nearest at hand or most unsophisticated". Jevons proposed this "Crossing-off procedure" ("Ausmusterungsverfahren") in his *Pure Logic* (1864). According to Schröder, it consisted in

> writing down for all classes mentioned in the formulation of the problem all the possible cases which can be thought of with respect to the presence or absence of one in relation to another, then crossing off all cases which are excluded from the thinkable combinations by the data of the problem as inadmissable, and trying to pick out the answers to the questions posed by the problem from the remaining ones. (Schröder 1890, p. 560)

Schröder applied Jevons's method to Boole's problem (1890, pp. 562–566). It contained five class symbols; therefore $2^5 = 32$ combinations had to be considered, of which eleven are valid. Schröder criticized the complexity of the combinatorial method, which grows with the square of the number of class symbols occurring. He furthermore claimed that the procedure is not really calculatory, but that it is based instead on a "mental comparison" of combinations and premises (pp. 567–568).

Schröder also discussed the graphical extension of this method presented by John Venn in his *Symbolic Logic* of 1881 (Schröder 1890, pp. 569–573). Venn symbolized the relations between the extensions of classes if two or three class symbols are involved by circles, by ellipses with four class symbols, and by ellipses together with a ring in the form of a rhombus with five class symbols. The procedure is similar to Jevons's crossing-off method, because the fields not present according to the data of the problem are erased by hatching them. With respect to the complexity of the problems which can be treated, Venn's procedure was even more restricted than Jevons's, because the schemes for symbols of more than five classes become rather intricate.[11] Schröder admitted that Venn's method had the advantage that every logical problem which could be presented in an intuitive form could be solved as soon as it was symbolized with the help of the graphical scheme. The scheme proposed by Venn for five class-symbols is shown on the left hand side of the figure below. On the right hand side the solution of the Boolean problem is given.[12] The exterior field 32 should be hatched as well. This has not been done for the sake of descriptiveness.[13]



[11]Schröder 1890, p. 569. Today, however, graphical procedures have been developed with which greater complexities can be handled.

[12]Schröder used the following algorithm for numbering the fields of the Venn diagram.

	a	b	c	d	e
1	1	1	1	1	1
2	1	1	1	1	0
3	1	1	1	0	1
4	1	1	1	0	0
5	1	1	0	1	1
6	1	1	0	1	0
7	1	1	0	0	1
8	1	1	0	0	0
			⋮		
32	0	0	0	0	0

I thank Peter Bernhard (Erlangen) for sharing this information.

While Schröder was critical of the methods of Jevons and Venn, he praised those of MacColl and Peirce for being equal to his own in their efficiency (1890, p. 560). He illustrated the relation between the different methods with the following metaphor: While he himself ties the different skeins of premises into a single bale (i.e. the united equation) and then hacks his way through it, Peirce separates each skein into thin threads and cuts them individually or binds them together again if necessary. Jevons, on the other hand, would make chaff and banter of the whole thing.[14] If one modifies Peirce's procedure so that the clues are separated only as far as is needed to isolate the symbols for eliminations and the unknowns, it will come close to MacColl's. Schröder acknowledged that the variants of MacColl and Peirce were natural and simple, but criticized their lengthiness (1890, p. 573).

Schröder reconstructed Peirce's method as a sequence of six steps ("Prozesse") (1890, pp. 574–584). He followed Peirce's own presentation (cf. 1880, pp. 37–42).

1. In an initial step the premises are expressed as subsumptions.

2. Then every subject (the term on the left hand side of a subsumption) is developed as a sum, every predicate (the term on the right hand side) as a product, using the schemes

$$\begin{array}{llcll} 44_{+} & f(x) & = & f(1)x + f(0)x_{\mathsf{l}} & \text{and} \\ 44_{\times} & f(x) & = & \{f(0)+x\}\{f(1)+x_{\mathsf{l}}\}\,. & \end{array}$$

3. In the third step all complex subsumptions are reduced, e.g.

$$s + s' + s'' + \ldots \Subset pp'p'' \ldots$$

into the subsumptions

$$\begin{array}{l} s \Subset p, s \Subset p', s \Subset p'', \ldots \quad, \\ s' \Subset p, s' \Subset p', s' \Subset p'', \ldots \quad, \\ s'' \Subset p, s'' \Subset p', s'' \Subset p'', \ldots \quad, \\ \vdots \end{array}$$

[13]Schröder corrects the solution given by Venn (Venn 1881, p. 281) by hatching field 24. Venn acknowledged this correction in the second edition of his *Symbolic Logic* (1894, p. 352, n. 1).

[14]"Bei dieser wurden die verschiedenen Knäuel der Prämissen oder Data des Problems erst fest zu einem einzigen Knoten geschürzt (der vereinigten Gleichung) und dieser dann durchhauen (bei der Elimination).

Beim Peirce'schen Verfahren aber werden jene Knäuel in ihre dünnsten Fäden auseinandergeleft und die erforderlichen einzeln zerschnitten (oder auch neu nach Bedarf verknüpft)—wogegen die Jevons'sche Methode sogleich ein Häcksel aus dem Ganzen machte!" (Schröder 1890, p. 573)

4. The fourth step is devoted to the necessary eliminations.

5. In the fifth step all the terms, where the unknown x can be found at subject or predicate position, are picked up, and finally ...

6. ... united in the last step. With the help of the resulting formula the unknown can be calculated.

Schröder saw the advantages of Peirce's method over Boole's in the fact that it operates with subsumptions and not with equations, and that it preserves the subject-predicate structure, which "thoroughly matches the judging functions of ordinary reasoning" (1890, p. 584)—but these are advantages in Schröder's method as well. Peirce's method has the further advantage that it is not necessary to bring the equations to zero on the right hand side and then to unite them to one single equation.

Schröder closed his considerations on the class calculus (1890, pp. 589–592) with a discussion of MacColl's method. He stated that MacColl invented this method independently, but nevertheless rather belatedly, in order to solve the problems of the Boolean calculus. It differed, however, not as much from the modified Boolean method (i.e., Schröder's method) as MacColl himself thought. Schröder stressed that he agreed with Venn, who had written in his assessment (Venn 1881, p. 37; 1894, p. 492) that MacColl's symbolical method is "practically identical with those of Peirce and Schröder."

This assessment is accurate if one regards MacColl's own evaluation of the differences between his system and those of Boole and Jevons. In the third part of the series of papers on "The Calculus of Equivalent Statements", he gave a list of the points of difference:

1. With me every single letter, as well as every combination of letters, always denotes a *statement*.
2. I use a symbol (the symbol :) to denote that the statement following it is true provided the statement preceding it be true.
3. I use a special symbol—namely, an accent—to express denial; and this accent, like the minus sign in ordinary algebra, may be made to affect a multinomial statement of any complexity. (MacColl 1878/79, p. 27)

Relating implication and subsumption, the latter being the class logical equivalent of the former, MacColl presented important modifications to the calculi of Boole and Jevons which were later also introduced by Schröder. It is noteworthy that MacColl did not mention his use of the inclusive "or". The reason may be that MacColl, in the context of the quoted passage, paralleled the calculi of Boole and Jevons, using

Jevons's *Pure Logic* (1864), where the exclusive "or" had already been replaced by the inclusive "or".

It is a matter of course that Schröder recognized that MacColl's formulas emerged from the propositional calculus, the "calculus of equivalent statements", in which the symbols 0 and 1 were not class symbols but interpreted as truth values. Schröder discussed this method in his more general class logic because MacColl also treated the Boolean class logical example 5 (cf. MacColl 1878/79, pp. 23–25).

According to Schröder's analysis, MacColl's solution was based on the two equations named "rule 22" (cf. MacColl 1878/79, p. 19):

$$xf(x) = xf(1);\ x'f(x) = x'f(0)\,.$$

Schröder had dicussed these equations as "theorems of MacColl's" at an earlier stage in his *Vorlesungen* (Schröder 1890, § 19, p. 420).[15]

Schröder regarded it as an advantage of MacColl's procedure that the premises were not united. In this point, he stated, MacColl was a precursor of Peirce. But he denied further advantages over his or Peirce's method, for example with respect to printing economy, a better survey, or more comfort in working (1890, pp. 591–592).

It is curious that Schröder returned to MacColl's method in the second volume of his *Vorlesungen*, which is devoted to the calculus of propositions. There he retracted his assessment that MacColl's solution was not really original (1891, p. 391). Schröder then stated that MacColl's method, as presented above, was only a scheme. In fact, Schröder said, MacColl had used another method, which was indeed original and advantageous. Schröder sketched it as follows (1891, pp. 304–305):

In a given product of propositions $F(x, y)$, y is to be eliminated and x is to be calculated. The following four implications are used

$$\begin{array}{lcl|lcl} xy & \mathrel{\underline{\in}} & F(1,1) & x_{|}y & \mathrel{\underline{\in}} & F(0,1) \\ xy_{|} & \mathrel{\underline{\in}} & F(1,0) & x_{|}y_{|} & \mathrel{\underline{\in}} & F(0,0) \end{array}$$

By addition and using the theorems $x = xy + xy_{|}$ and $x = x_{|}y + x_{|}y_{|}$, y can be eliminated, resulting in

$$x \mathrel{\underline{\in}} F(1,1) + F(1,0) \;\Big|\; x_{|} \mathrel{\underline{\in}} F(0,1) + F(0,0)\,.$$

By contraposition the solutions can be given

$$F_{|}(1,1)F_{|}(1,0) \mathrel{\underline{\in}} x_{|} \;\Big|\; F_{|}(0,1)F_{|}(0,0) \mathrel{\underline{\in}} x\,.$$

[15] In MacColl's notation the apostrophe denotes negation. MacColl himself said that he used the implications $xf(x) : f(1)$, $x'f(x) : f(0)$ named as rule 23.

2.2. Calculus of propositions

The fact is often overlooked that Schröder, in the second volume of his *Vorlesungen*, presented a highly elaborate propositional and predicate logic, which was not motivated by Frege. He adopted the quantifiers of Peirce and Oscar Howard Mitchell (1851–1889), using a sum and product notation. In the hierarchy of Schröderian logic, propositional logic was only in third position. Schröder started with a calculus of domains, consisting of manifolds of elements. He spoke of a class calculus only if these elements could be individuated and combined to form classes. It was a further specialization if the calculus concerned propositions (judgements or "statements"). With this architecture in mind, it is reasonable for Schröder to reproach MacColl and Peirce for putting the cart before the horse in founding the logical calculus on the calculus of propositions. His hierarchy, Schröder stressed, had the advantage of being more general, and it was also better in didactical respects, because it was not necessary to have the complete syllogistical apparatus at hand from the beginning. He compared the difficulties of reading Peirce's papers on logic (his argument was, of course, also valid for MacColl's papers) with the difficulties of a student, who "should learn a language which is unknown for him from a grammar which is written in the same language" (1891, p. 276).

2.3. Material implication

In the algebraic view, formulas and symbols in logic are interpreted in different ways, depending on the different contexts of application. The subsumption, which is interpreted in the class calculus as an "integrative operation" ("Einordnung") leading to the equality or subordination of classes, becomes an implication in the propositional calculus. In the identical calculus Schröder defined subsumption as material implication

$$(A \preccurlyeq B) = (A_{|} + B)\,,$$

and interpreted the formula as follows: "The 'validity class' of the subsumption $A \preccurlyeq B$ is the class of occasions, during which A is not valid or B is valid" (Schröder 1891, p. 69). But he differed from Peirce before him, in that he obtained the equation by identical transformations:

$$(A \preccurlyeq B) = (AB_{|} = 0) = \{(AB_{|})_{|} = \dot{1}\} = (A_{|} + B = \dot{1}) = A_{|} + B\,.$$

The dotted 1 stands for the truth value "true". Schröder used the term $A = \dot{1}$ more exactly to express that the proposition A is always, at any time, and on all occasions, valid. Marking the 1 with a dot is necessary to distinguish it from the arithmetical 1, which occurs in quantifications over a numerical index.

Schröder granted the priority of material implication to MacColl and Peirce. It is astonishing that MacColl gave a far more modern interpretation of the formulas, ten years before Schröder. In his interpretation, $A = 1$ means that the statement A is true; $A = 0$ that it is false (1877/78a, p. 9). The expression $A : B$ means that statement A implies statement B, i.e. in any case if A is true, then B is true (1877/78b, p. 177).

I would finally like to suggest that MacColl didn't grasp the algebraic approach of Schröder and Peirce. This became obvious when he criticized the opinion that Peirce's operation of illation and Schröder's subsumption were equivalent to his implication, whereas, according to his opinion, these operations only denote class inclusion (MacColl 1906, § 74). He did not realize that in the Peirce-Schröderian algebra of logic, symbols for operations and for relations between variables were only denoted in a schematic way. The meaning of the symbols thus depended on the meaning of the variables.

3. Conclusion

As far as Schröder was concerned, MacColl appeared to be a successor of Boole's in his algebra of logic who, unlike Jevons, kept a logical notation closely analogous to mathematics, but who, like Jevons, was also able to avoid the limitations of Boole's logic. In combining Boolean Algebra (which was founded by Jevons) with mathematical symbolism, he became a precursor of Peirce. Schröder appears to have heard of MacColl via Peirce. But throughout his work he acknowledged MacColl's priority. His comments sometimes read as if MacColl's calculus is a purely historical precursor of Peirce's logic. Schröder was quite attracted to Peirce's ideas, possibly because his interests in logic changed while he was still writing the first two volumes of his *Vorlesungen*. His new interest was an "Algebra and Logic of Relatives", the first volume appearing in 1895, which he elaborated according to Peirce's model. Therefore Schröder's presentation of MacColl's system had no crucial effect on the reception of MacColl's ideas.

This state of affairs was not changed by the fact that MacColl cut a good figure in the competition of logical systems during the heyday of the algebra of logic between 1864 and 1890, at least according to Schröder. The modal operations, characteristic for MacColl' s *Symbolic Logic* of 1906, were not formulated in the early parts of the series of papers "The Calculus of Equivalent Statements" to which Schröder referred. Nevertheless, it is possible to draw some conclusions from the early reception, compared to the acceptance of MacColl's later work. The logic discussion of the time shows the importance of the organon aspect of logic. The logical calculus was regarded as a tool for the solution of logical problems, but also for problems from other areas which could be translated into logical language. These areas included mathematics, the philosophy of science, jurisprudence, but also genealogy. Questions of symbolism played a predominant role in evaluating the usability of the calculi. The bulky symbolism of MacColl's later work could only prevent broader acceptance. Thus, MacColl shared the same fate as Frege.

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WAYS OF UNDERSTANDING HUGH MACCOLL'S CONCEPT OF *SYMBOLIC EXISTENCE*

Hugh MacColl (1837–1909) proposed, in several papers, a non-standard way of understanding the ontology underlying what we today call quantified propositions. His ideas, mixed with reflections about the use of arbitrary objects, were not greatly successful and were ruthlessly criticised by Bertrand Russell especially. The aim of this paper is to show that a thorough reading of MacColl's general understanding of *symbolic existence*, a concept which is connected with his view of traditional *hypotheticals*, elucidates his proposals on the role of ontology in logics. The interpretation of MacColl's concept of *symbolic existence* put forward in this paper and embedded in a dialogical system of free logic can be expressed in a nutshell: in an argumentation, it sometimes makes sense to restrict the use and introduction of singular terms in the context of quantification to a formal use of those terms. That is, the Proponent is allowed to use a constant iff this constant has been explicitly conceded by the Opponent. The paper also offers a second way of reconstructing MacColl's ideas on contradictory objects by means of combining the concept of formal use of constants in free logics and that of the formal use of elementary negations in paraconsistent logics.

1. Introduction

1.1. Aims of the paper

Hugh MacColl (1837–1909) proposed, in several papers, a non-standard way of understanding the ontology underlying what we today call quantified propositions. His ideas, mixed with reflections about the use of arbitrary objects, were not greatly successful and were ruthlessly criticised by Bertrand Russell especially.

The aim of this paper is to show that a thorough reading of MacColl's general understanding of *symbolic existence*, a concept which is strongly connected with his view of *hypotheticals*, elucidates

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MacColl's proposals on the role of ontology in logics. I will make an abstraction of MacColl's use of arbitrary objects by replacing them with quantifiers and will also make brief comments on the connections he establishes between symbolic existence and the formulation of a strong conditional. This move centres the discussion on the main idea, although I concede it may also bend MacColl's own argumentation style in some way.

1.2. Symbolic reasoning and the problematic modality of hypotheticals

MacColl's formulation of traditional syllogistic is part of a general framework where rules of logic are considered as rules for *hypotheticals*. According to MacColl, Boole's logical equations for hypotheticals should be replaced by a system of equivalent propositions including disjunctions and conditionals, which reflects the natural semantics of traditional hypothetical forms. This natural semantics is best described by saying that a hypothetical form expresses (1) a necessary connection between the two parts of the hypothetical for the conditional and (2) some doubt on the part of the user of a given hypothetical as to the actual truth, in a given instance, of the pair of statements which compose this connection. The formal translation of the necessary connection between the two parts of the hypothetical in conditional form led MacColl to formulate a strong concept of implication, which in his early work was defined as relevant and connexive and in his later work as a strict implication. The translation of the problematic modality of hypotheticals was achieved through the distinction between formal and non-formal (or material) truth, which MacColl misses in Boole's use of the symbol '1': According to MacColl a complex formula expresses a hypothetical proposition if the subformulae occurring in it are stated hypothetically. The subformulae are said to be stated hypothetically if the truth-value of the complex formula is independent of the actual truth of these subformulae:

> The premisses $\mathrm{A} : \mathrm{B}$ and $\mathrm{B} : \mathrm{C}$ [i.e. $\mathrm{A} \rightarrow \mathrm{B}$ and $\mathrm{B} \rightarrow \mathrm{C}$—S.R.] of the latter [implication: $\{(\mathrm{A} \rightarrow \mathrm{B}) \wedge (\mathrm{B} \rightarrow \mathrm{C})\} \rightarrow (\mathrm{A} \rightarrow \mathrm{C})$—S.R.] are hypothetical concepts of the mind—concepts which may be true or false (as may also the conclusion), without in the least invalidating the formula (MacColl 1902, p. 368).

Actually, there is some ambiguity in MacColl's use of the word 'hypothetical'. Instead of describing hypotheticals as having subformulae with a problematic modality, he speaks, as already mentioned, of suformulae *stated hypothetically*. In general, we can say that when MacColl wishes to stress the problematic modality of propositions he calls them *statements*:

Def. 6.—Statements represented by letters or any other arbitrary symbols, to which we attach only a *temporary* meaning, are usually statements whose truth or falsehood may be considered an open question, like the statements of witnesses in a court of justice (MacColl 1880, p. 47).

MacColl's use of the word 'statement' is unfortunately not always consistent, but as I have argued elsewhere the concept of truth-determining propositional variables can be fruitfully applied instead (see Rahman 1998). This concept follows the lines of MacColl's main purposes and provides a good basis for reconstructing his reflections on hypotheticals. Thus I will say that a set of (occurrences of) propositional variables is *truth-determining* for a proposition A iff the truth value of A may be determined as true or false on all assignments of true or false to the set. I will say further that a propositional variable occurring in A is *redundant* iff there is a truth-determining set for A that does not contain this propositional variable. Thus, clearly, the set $\{p\}$ is not truth-determining for $p \to q$, but the set $\{p_1, p_2\}$ is truth-determining for $p \to p$. In this way we can reconstruct MacColl's use of elementary statements, that is, elementary propositions with a problematic mood, as truth-determining propositional variables, regarded as truth-determining independently of their actual truth-value—only their possible truth-values should be considered. By these means it is even possible to link the problematic modality of hypotheticals with the strong connection required for them, namely: A proposition A is *strongly connected* iff no redundant propositional variable (nor any of the occurrences of a propositional variable) occurs in A.[1]

Now all this allows the ideas behind MacColl's general framework to be expressed in the following way: The elementary expressions of the logical language are truth-determining propositional variables. Propositions in which the actual truth-values of their propositional variables are known are called categoricals. Hypotheticals in which the actual truth-value of their propositional variables cannot modify the truth of the propositions in which they occur are formally valid. Symbolic reasoning is to reason under two conditions, namely: 1. Only those propositions are allowed in which no non-truth-determining propositional variables occur; 2. Propositional variables are used independently of their actual truth. The first condition yields a system with a strong conditional; the second condition, which reflects the problematic modality of traditional hypotheticals, commits itself to a formal use of propositional variables. In other words, to reason symbolically means to reason with hypotheticals.

[1]See details in Rahman 1997a, 1998 and Rahman and Rückert 1998, 1999a.

But what does it mean to use propositional variables formally? The dialogical approach to logic, which will be introduced in detail in the next section, has a very appealing answer to this question and offers a consistent way for understanding MacColl's reflections on the problematic modality of hypotheticals: Let an argumentation be given in which someone, the Proponent, states a thesis, and someone else, the Opponent, rejects it. In the course of the argumentation the use of the propositional variables is said to be used formally iff 1. Propositional variables cannot be attacked; 2. The Proponent may use a propositional variable in a move iff the Opponent has already stated the same statement before—that is, instead of committing himself to the empirical defence of a given atomic proposition a, the Proponent chooses the following way of justifying his use of a: "If you (the Opponent) concede that a holds, so do I" (observe that, because of the difference between game and strategy levels in dialogical logic, the winning of a dialogue with help of the formal rule does not necessarily yield the validity of the formula involved—see 2.2).

Hugh MacColl tried to build a system for quantified propositions which should inherit this general framework (see Rahman 1997a, II). MacColl thought that this implied not only creating a system of first-order logic with a strong conditional but also the formulation of a system where the use of propositions stating facts about the elements of the corresponding universe of discourse commits one only to a symbolic existence of the objects introduced by these propositions. That is, MacColl tried to formulate a logic where even the use of constants assumes a problematic modality:

Let e_1, e_2, e_3, etc. (up to any number of individuals mentioned in our argument or investigation) denote our universe of *real existences*. Let $0_1, 0_2, 0_3$, etc., denote our universe of *non-existences*, that is to say, of unrealities, such as *centaurs*, *nectar*, *ambrosia*, *fairies*, with self-contradictions, such as *round squares*, *square circles*, *flat spheres*, etc., including, I fear, the non-Euclidean geometry of four dimensions and other hyperspatial geometries. Finally, let $\mathrm{S}_1, \mathrm{S}_2, \mathrm{S}_3$, etc., denote our Symbolic Universe, or "Universe of Discourse," composed of all things real or unreal that are named or expressed by words or other symbols in our argument or investigation ...

When a class A belongs wholly to the universe e, or wholly to the class 0, we may call it a pure class ...

We may sum up briefly as follows: Firstly, when any symbol A denotes an *individual*; then, any intelligible statement $\phi(\mathrm{A})$, containing the symbol A, implies that the individual represented by A has a *symbolic* existence; but whether the statement $\phi(\mathrm{A})$ implies that the individual represented by A has a *real* existence depends upon the context. Secondly, when any symbol A denotes a *class*, then, any intelligible statement $\phi(\mathrm{A})$ containing the symbol A implies that the whole class A has a *symbolic* existence; but whether the statement ϕ(A) implies that the class A is *wholly real*, or *wholly unreal*, or *partly real and partly unreal*, depends upon the context. (MacColl 1905b, pp. 74–77; see also MacColl 1905a,b and MacColl 1906, pp. 76–77).

But how can i) a symbolic use of constants, ii) classifications in such a symbolic universe of discourse, and iii) propositions about flat spheres and round squares be introduced in formal logic?

All these questions can be answered in the context of the dialogical approach to free logic developed recently (Rahman et al. 1999) in a way which is congenial with MacColl's proposal of formulating a first-order logic which reflects the problematic modality of propositional logic. In a nutshell: in an argumentation, it sometimes makes sense to restrict the use and introduction of singular terms in the context of quantification to a formal (or *symbolic*) use of those terms. That is, the Proponent is allowed to use a constant iff this constant has been explicitly conceded by the Opponent. The symbolic use of constants amounts to allowing the use of these constants under the sole restriction that they name an individual: their ontological characterisation besides individuality does not play any role in logics. This paper also offers a second way of reconstructing MacColl's ideas on contradictory objects by means of combining the concept of formal use of constants in free logics and that of the formal use of elementary negations in paraconsistent logics.

In the next section I do not go into the details of free logics based on reference. Instead, I show how the dialogical approach to free logic can capture the ideas behind MacColl's concept of symbolic existence.

2. Symbolic Existence and the Dialogical Approach to Free Logic

2.1. The dialogical approach to free logic

Dialogical logic, suggested by Paul Lorenzen in 1958 and developed by Kuno Lorenz in several papers from 1961 onwards,[2] was introduced as a pragmatical semantics for both classical and intuitionistic logic.

The dialogical approach studies logic as an inherently pragmatic notion with the help of an overtly externalised argumentation formulated as a *dialogue* between two parties taking up the roles of an *Opponent* (O in the following) and a *Proponent* (P) of the issue at stake, called the principal *thesis* of the dialogue. P has to try to defend the thesis against all possible allowed criticism (*attacks*) by O, thereby being allowed to use statements that O may have made at the outset of the dialogue. The thesis A is logically valid if and only if P can succeed in

[2]Lorenzen and Lorenz 1978. Further work has been done by Rahman (1993).

defending A against all possible allowed criticism by the Opponent. In the jargon of game theory: P has a *winning strategy* for A.

The philosophical point of dialogical logic is that this approach does not understand semantics as mapping names and relationships into the real world to obtain an abstract counterpart of it, but as acting upon them in a particular way.

I will now describe an intuitionistic and a classical version of a very basic system called DFL (dialogical free logic) introduced in Rahman et al. 1999. Since the principal aim of the paper is the elucidation of MacColl's concept of symbolic existence, I will not introduce a system which contemplates strongly connected propositions. For such a system, cf. Rahman 1997a, 1998, Rahman and Rückert 1998, 1999a.

Suppose the elements of first-order language are given with small letters $(a, b, c, \dots)$ for elementary formulae, capital italic letters for formulae that might be complex $(A, B, C, \dots)$, capital sans serif letters $(\mathsf{A}, \mathsf{B}, \mathsf{C}, \dots)$ for predicators and constants τ_i. A dialogue is a sequence of labelled formulae of this first-order language that are stated by either P or O.[3] The label of a formula describes its role in the dialogue, whether it is an aggressive or a defensive act. An *attack* is labelled with $?_{n/\dots}$, while $!_{n/\dots}$ tags a defence. (n is the number of the formula the attack or defence reacts to, and the dots are sometimes completed with more information. The use of indices on labels will be made clear in the following). In dialogical logic the meaning in use of the logical particles is given by two types of rules which determine their *local* (*particle rules*) and their *global* (*structural rules*) meaning. The particle rules specify for each particle a pair of moves consisting of an attack and (if possible) the corresponding defence. Each such pair is called a *round*. An attack *opens* a round, which in turn is *closed* by a defence if possible. Before presenting a dialogical system DFL for free logics, we need the following definition.

DEFINITION 1. *A constant τ is said to be* introduced by X *if (1)* X *states a formula $A[\tau/x]$ to defend $\bigvee_x A$ or (2)* X *attacks a formula $\bigwedge_x A$ with $?_{n/\tau}$, and τ has not been used in the same way before. Moreover, an atomic formula is said to be introduced by* X *if it is stated by* X *and has not been stated before.*

DFL is closely related to Lorenz's standard dialogues for both intuitionistic and classical logic. The particle rules are identical, and the sets of structural rules differ in only one point, namely when determining the way constants are dealt with. Before presenting the formal definition of DFL, we should take a look at a simple propositional dialogue as an example of notational conventions:

[3] Sometimes I use X and Y to denote P and O with $X \neq Y$.

Opponent		Proponent	
		$a \wedge b \rightarrow a$	(0)
(1)	$?_0 a \wedge b$	$!_1 a$	(4)
(3)	$!_2 a$	$?_{1/\text{left}}$	(2)
		The Proponent wins	

Formulae are labelled in (chronological) order of appearance. They are not listed in the order of utterance, but in such a way that every defence appears on the same level as the corresponding attack. Informally, the argument goes like this:

P: "If a and b, then a."
O: "Given a and b, show me that a holds."
P: "If you assume a and b, you should be able to show me that both hold. Thus show me that the left part holds."
O: "OK, a."
P: "If you can say that a holds, so can I."
O runs out of arguments, P wins.

DEFINITION 2. PARTICLE RULES.

	Attack	Defence
$\neg A$	$?_n A$	$\otimes$ *(The symbol '$\otimes$' indicates that no defence, but only counterattack is allowed)*
$A \wedge B$	$\dfrac{?_{n/\text{left}}}{?_{n/\text{right}}}$ *(The attacker chooses)*	$\dfrac{!_m A}{!_m B}$
$A \vee B$	$?_n$	$\dfrac{!_m A}{!_m B}$ *(The defender chooses)*
$A \rightarrow B$	$?_n A$	$!_m B$
$\bigwedge_x A$	$?_{n/\tau}$ *(The attacker chooses)*	$!_m A[\tau/x]$
$\bigvee_x A$	$?_n$	$!_m A[\tau/x]$ *(The defender chooses)*

The first row contains the form of the formula in question, the second row possible attacks against this formula, and the last one possible defences against those attacks. (The symbol "$\otimes$" indicates that no defence is possible.) Note that $?_{n/\ldots}$ is a move—more precisely it is an attack—and not a formula. Thus if one partner in the dialogue states a conjunction, the other may initiate the attack by asking either for the left side of the conjunction ("show me that the left side of the conjunction holds", or $?_{n/\text{left}}$ for short) or the right one ("show me that the right side of the conjunction holds", or $?_{n/\text{right}}$). If one partner in the dialogue states a disjunction, the other may initiate the attack by asking to be shown *any* side of the disjunction ($?_n$). As already mentioned, the number in the index denotes the formula the attack refers to. The notation of defences is used in analogy to that of attacks. Rules for quantifiers work similarly.

Next, we fix the way formulae are sequenced to form dialogues with a set of structural rules (orig. *Rahmenregeln*).

(DFL0). *Formulae are alternately uttered by* P *and* O. *The* initial formula *is uttered by* P. *It does not have a label, but provides the topic of argument. Every formula below the initial formula is either an attack or a defence against an earlier formula of the other player.*

(DFL1). *Both* P *and* O *may only make moves that change the situation.*[4]

(DFL2). FORMAL RULE FOR ATOMIC FORMULAE. P *may not introduce atomic formulae: any atomic formula must be stated by* O *first.*

(DFL3). FORMAL RULE FOR CONSTANTS. *Only* O *may introduce constants.*

(DFL4). WINNING RULE. X *wins iff it is* Y*'s turn but he cannot move (either attack or defend).*

($\text{DFL}_\text{I}5$). INTUITIONISTIC RULE. *In any move, each player may attack a (complex) formula asserted by his partner or he may defend himself against* the last not already defended *attack. Only the latest open attack may be answered. If it is* X*'s turn at position n and there are two open attacks m, l such that $m < l < n$, then* X *may not defend against m.*

DFL is an intuitionistic as well as a classical semantics. To obtain the classical version simply replace ($\text{DFL}_\text{I}5$) by the following rule:

($\text{DFL}_\text{C}5$). CLASSICAL RULE. *In any move, each player may attack a (complex) formula asserted by his partner or he may defend himself against* any attack *(including those which have already been defended).*

[4]This rule replaces Lorenz's *Angriffsschranken*, but this point still remains to be made clear on a formal basis. The idea is that a new situation is defined when O provides an atomic formula which can be used by the Proponent.

If we need to specify (explicitly) which system is meant, we write $\mathrm{DFL_I}$ or $\mathrm{DFL_C}$ instead of DFL.

The crucial rule that makes DFL behave like a free logic is (DFL3).

To see the difference between standard and free dialogues (those with and those without (DFL3)), consider another example. Without (DFL3), we would obtain the following dialogue proving that if nothing is a vampire, Nosferatu is no vampire:

	Opponent	Proponent	
		$\bigwedge_x \neg\mathsf{A}_x \to \neg\mathsf{A}_\tau$	(0)
(1)	$?_0 \bigwedge_x \neg\mathsf{A}_x$	$!_1\neg\mathsf{A}_\tau$	(2)
(3)	$?_2\mathsf{A}_\tau$	$\otimes$	
(5)	$!_4\neg\mathsf{A}_\tau$	$?_{1/\tau}$	(4)
	$\otimes$	$?_5\mathsf{A}_\tau$	(6)
		The Proponent wins	

If we play the same dialogue again in DFL, things look different:

	Opponent	Proponent	
		$\bigwedge_x \neg\mathsf{A}_x \to \neg\mathsf{A}_\tau$	(0)
(1)	$?_0 \bigwedge_x \neg\mathsf{A}_x$	$!_1\neg\mathsf{A}_\tau$	(2)
(3)	$?_2\mathsf{A}_\tau$	$\otimes$	
	The Opponent wins		

We observe that P runs out of arguments. He cannot attack (1) any more, because not a single constant has been introduced so far, and he may not introduce one on its own. Neither can he defend himself against the atomic formula in (3) due to the particle rule for negation.

It is obvious that the (Proponent's) thesis $A_\tau \to \bigvee_x A_x$ cannot be won. This shows that the Opponent may state a proposition about a fictive entity without committing himself to its existence. MacColl's reflections on non-existence amount to this analysis of $A_\tau \to \bigvee_x A_x$.[5]

2.2. Winning strategies and dialogical tableaux for DFL

As already mentioned, validity is defined in dialogical logic via winning strategies of P, i.e. the thesis A is logically valid iff P can succeed in defending A against all possible allowed criticism by O. In this case, P has a *winning strategy* for A. It should be clear that the formal rule

[5]M. Astroh's thorough discussion of MacColl's conception of existence (Astroh 1996, pp. 1399–1401) amounts to the failure of this thesis in DFL.

which elucidates MacColl's understanding of the problematic modality of hypotheticals does not necessarily imply that winning a dialogue with the help of this rule yields the validity of the formula involved: The Proponent may win a dialogue, even formally, because the Opponent did not play the best moves. Validity, on the other hand, forces the consideration of all possibilities available. A systematic description of the winning strategies available can be obtained from the following considerations:

If P *shall win against any choice of* O*, we will have to consider two main different situations, namely the dialogical situations in which* O *has stated a complex formula and those in which* P *has stated a complex formula. We call these main situations the* O*-cases and the* P*-cases, respectively.*

In both of these situations another distinction has to be examined:

1. P wins by *choosing* an attack in the O-cases or a defence in the P-cases, iff he can win *at least one* of the dialogues he has chosen.

2. When O can *choose* a defence in the O-cases or an attack in the P-cases, P can win iff he can win *all of the* dialogues O can choose.

The closing rules for dialogical tableaux are the usual ones: a branch is closed iff it contains two copies of the same formula, one stated by O and the other one by P. A tree is closed iff each branch is closed.

For the intuitionistic tableaux, the structural rule about the restriction on defences has to be considered. The idea is quite simple: the tableaux system allows all the possible defences (even the atomic ones) to be written down, but as soon as determinate formulae (negations, conditionals, universal quantifiers) of P are attacked all others will be deleted. Those formulae which compel the rest of P's formulae to be deleted will be indicated with the expression "$\mathrm{O}_{[\mathrm{O}]}$" (or "$\mathrm{P}_{[\mathrm{O}]}$"), which reads *save* O*'s formulae and delete all of* P*'s formulae stated before.*

To obtain a tableaux system for DFL from those described above, add the following restriction to the closing rules and recall the rule (DFL3) for constants:

DFL-RESTRICTION. *Check that for every step in which* P *chooses a constant (i.e. for every* P*-attack on a universally quantified* O*-formula and for every* P*-defence of an existentially quantified* P*-formula) this constant has been already introduced by* O *(by means of an* O*-attack on a universally quantified* P*-formula or a defence of an existentially quantified* O*-formula).*

This restriction can be technically implemented by a device which provides a label (namely a star) for each constant introduced by O. Thus, the DFL-restriction can be simplified in the following way:

DFL-RESTRICTION WITH LABELS. *Check that for every step in which* P *chooses a constant this constant has already been there, labelled with a star.*

All these considerations can be expressed by means of the tableaux systems for classical and intuitionistic DFL.[6]

CLASSICAL TABLEAUX FOR DFL.

(O)-Cases	(P)-Cases
$\dfrac{(\mathrm{O})A \wedge B}{\langle(\mathrm{P})_{?}\rangle(\mathrm{O})A \mid \langle(\mathrm{P})_{?}\rangle(\mathrm{O})B}$	$\dfrac{(\mathrm{P})A \wedge B}{\begin{array}{c}\langle(\mathrm{O})_{?}\rangle(\mathrm{P})A\\(\langle(\mathrm{O})_{?}\rangle(\mathrm{P})B)\end{array}}$
$\dfrac{(\mathrm{O})A \wedge B}{\begin{array}{c}\langle(\mathrm{P})_{?\mathrm{left}}\rangle(\mathrm{O})A\\(\langle(\mathrm{P})_{?\mathrm{right}}\rangle(\mathrm{O})B)\end{array}}$	$\dfrac{(\mathrm{P})A \wedge B}{\langle(\mathrm{O})_{?\mathrm{left}}\rangle(\mathrm{P})A \mid \langle(\mathrm{O})_{?\mathrm{right}}\rangle(\mathrm{P})B}$
$\dfrac{(\mathrm{O})A \to B}{(\mathrm{P})A\ldots \mid \langle(\mathrm{P})A\rangle(\mathrm{O})B}$	$\dfrac{(\mathrm{P})A \to B}{\begin{array}{c}(\mathrm{O})A\\(\mathrm{P})B\end{array}}$
$\dfrac{(\mathrm{O})\neg A}{(\mathrm{P})A, \otimes}$	$\dfrac{(\mathrm{P})\neg A}{(\mathrm{O})A, \otimes}$
$\dfrac{(\mathrm{O})\bigwedge_x A}{\langle(\mathrm{P})_{?\tau}\rangle(\mathrm{O})A_{[\tau^*/x]}}$ *τ has been labelled with a star before*	$\dfrac{(\mathrm{P})\bigwedge_x A}{\langle(\mathrm{O})_{?\tau^*}\rangle(\mathrm{P})A_{[\tau^*/x]\tau}}$ *τ is new*
$\dfrac{(\mathrm{O})\bigvee_x A}{\langle(\mathrm{P})_{?}\rangle(\mathrm{O})A_{[\tau^*/x]}}$ *τ is new*	$\dfrac{(\mathrm{P})\bigvee_x A}{\langle(\mathrm{O})_{?}\rangle(\mathrm{P})A_{[\tau/x]}}$ *τ has been labelled with a star before*

[6]See details on how to build tableaux systems from dialogues in Rahman 1993 and Rahman and Rückert 1997. The use of these tableaux systems follows the very well-known analytic trees of Raymund Smullyan (1968).

The closing rules are the usual ones. Observe that the formulae below the line represent pairs of attack-defence moves. In other words, they represent rounds.

Note that the expressions between the symbols '$\langle$' and '$\rangle$', such as $\langle(\mathrm{P})_?\rangle$, $\langle(\mathrm{O})_?\rangle$ or $\langle(\mathrm{P})A\rangle$ are moves—more precisely they are attacks—but not statements.

Intuitionistic Tableaux for DFL.

(O)-Cases | (P)-Cases

$$\frac{(\mathrm{O})A \wedge B}{\langle(\mathrm{P})_?\rangle(\mathrm{O})A \mid \langle(\mathrm{P})_?\rangle(\mathrm{O})B} \qquad \frac{(\mathrm{P})A \wedge B}{\begin{array}{c}\langle(\mathrm{O})_?\rangle(\mathrm{P})A\\ (\langle(\mathrm{O})_?\rangle(\mathrm{P})B)\end{array}}$$

$$\frac{(\mathrm{O})A \wedge B}{\begin{array}{c}\langle(\mathrm{P})_{?\mathrm{left}}\rangle(\mathrm{O})A\\ (\langle(\mathrm{P})_{?\mathrm{right}}\rangle(\mathrm{O})B)\end{array}} \qquad \frac{(\mathrm{P})A \wedge B}{\langle(\mathrm{O})_{?\mathrm{left}}\rangle(\mathrm{P})A \mid \langle(\mathrm{O})_{?\mathrm{right}}\rangle(\mathrm{P})B}$$

$$\frac{(\mathrm{O})A \to B}{(\mathrm{P})A\ldots \mid \langle(\mathrm{P})A\rangle(\mathrm{O})B} \qquad \frac{(\mathrm{P})A \to B}{\begin{array}{c}(\mathrm{O})_{[\mathrm{O}]}A\\ (\mathrm{P})B\end{array}}$$

$$\frac{(\mathrm{O})\neg A}{(\mathrm{P})A, \otimes} \qquad \frac{(\mathrm{P})\neg A}{(\mathrm{O})_{[\mathrm{O}]}A, \otimes}$$

$$\frac{(\mathrm{O})\bigwedge_x A}{\langle(\mathrm{P})_{?\tau}\rangle(\mathrm{O})A_{[\tau^*/x]}} \qquad \frac{(\mathrm{P})\bigwedge_x A}{\langle(\mathrm{O})_{?\tau^*}\rangle(\mathrm{P})_{[\mathrm{O}]}A_{[\tau^*/x]\tau}}$$

τ *has been labelled with a star before* | τ *is new*

$$\frac{(\mathrm{O})\bigvee_x A}{\langle(\mathrm{P})_?\rangle(\mathrm{O})A_{[\tau^*/x]}} \qquad \frac{(\mathrm{P})\bigvee_x A}{\langle(\mathrm{O})_?\rangle(\mathrm{P})A_{[\tau/x]}}$$

τ *is new* | τ *has been labelled with a star before*

Let us look at two examples, namely one for classical DFL and one for intuitionistic DFL:

(P)	$\bigwedge_x \neg \mathsf{A}_x \to \neg \mathsf{A}_\tau$
(O)	$\bigwedge_x \neg \mathsf{A}_x$
(P)	$\neg \mathsf{A}_t$
(O)	A_t

The tableau remains open as P cannot choose τ to attack the universal quantifier of O. The following intuitionistic tableau is slightly more complex:

	$\cancel{(\mathrm{P})}$	$\cancel{\bigwedge_x \mathsf{A}_x \to \neg \bigvee_x \neg \mathsf{A}_x}$
	$(\mathrm{O})_{[\mathrm{O}]}$	$\bigwedge_x \mathsf{A}_x$
	$\cancel{(\mathrm{P})}$	$\cancel{\neg \bigvee_x \neg \mathsf{A}_x}$
	$(\mathrm{O})_{[\mathrm{O}]}$	$\bigvee_x \neg \mathsf{A}_x$
$\langle (\mathrm{P})_? \rangle$	(O)	$\neg \mathsf{A}_{t^*}$
$\langle (\mathrm{P})_{?\tau} \rangle$	(O)	A_t
	(P)	A_t

The tree closes.

2.3. Many quantifiers and sorts of objects in the symbolic universe: The systems DFL^n and DFL$^{\langle n \rangle}$

Consider the situation expressed by the following proposition:

> *The novel contains a passage in which Sherlock Holmes dreams that he shot Dr. Watson.*

There is an underlying reality that the novel is part of, the outer reality of the story told in the novel and an even further outer reality of the dream of the protagonist. To distinguish between the reality of Conan Doyle writing stories, Holmes's reality and the reality of Holmes' dream, we need three pairs of quantifiers expressing the three sorts of reality (or fiction), for which in a first step we do not need to assume that they introduce an order of levels of fiction (or reality). Actually MacColl, as can be read in the text quoted in 1.2, formulated a system in which different sorts of elements of the symbolic universe are considered. Now, if the motivation of introducing a formal use of constants (or in MacColl's words a symbolic universe) is, as already mentioned, an ontologically neutral treatment of these constants, it is not very clear why levels of reality should be considered at all. Narahari Rao, for example, thinks that such a graduation is incompatible with the very idea of a symbolic universe (see Rao 1999). Formally, the introduction of sorts of elements is very simple: Think of the pair of quantifiers of

DFL as having the upper index 0 and add new pairs of quantifiers with higher indices, as many as we need to express every sort of reality (or fiction) that could possibly appear. We call the dialogical logic thus derived $\mathrm{DFL}^{\mathrm{n}}$. The new particle rules to be added to DFL are:

	Attack	Defence
$\bigwedge^i_x A$	$?_{n/\tau}$ (The attacker chooses)	$!_m A[\tau/x]$
$\bigvee^i_x A$	$?_n$	$!_{m/\tau} A[\tau/x]$ (The defender chooses)

The extended set of quantifiers requires a new notion of introduction.

Definition 3. *A constant τ is said to be* introduced as belonging to the sort *i iff it is used to attack a universal quantifier of sort i or to defend an existential quantifier of sort i and has not been used in the same way before.*

I adapt (DFL3) to $\mathrm{DFL}^{\mathrm{n}}$:

($\mathrm{DFL}^{\mathrm{n}}$3). First extended formal rule for constants. *For each sort of quantification the following rule holds: constants may only be introduced by* O.

These formulations yield a logic containing an arbitrary number of disjoint pairs of quantifiers dealing with different sorts of reality and fiction—this logic contains also the standard (non-free quantifiers) $\exists$ and $\forall$ for which neither DFL3 nor $\mathrm{DFL}^{\mathrm{n}}$3 hold.

In some contexts, it might be useful to have a logic where these different realities are ordered in a hierarchy. We call the system that establishes this ordering $\mathrm{DFL}^{\langle\mathrm{n}\rangle}$; it results from modifying (DFL3) again:

($\mathrm{DFL}^{\langle\mathrm{n}\rangle}$3). Second extended formal rule for constants. P *may introduce a constant τ on a level m iff* O *has introduced τ on some level n with $n < m$ before.*

I leave as an exercise for the reader two examples. The first states that in $\mathrm{DFL}^{\mathrm{n}}$, whenever A has an instance in the scope of one or another $\vee$-quantifier, it has an instance in the scope of $\exists$; the second makes use of the ordering in $\mathrm{DFL}^{\langle\mathrm{n}\rangle}$:

1. $(\bigvee^1_x \mathsf{A}_x \vee \bigvee^2_x \mathsf{A}_x) \rightarrow \exists_x \mathsf{A}_x$ (to be solved with DFL^{n})
2. $(\bigvee^1_x \mathsf{A}_x \wedge \bigwedge^2_x(\mathsf{A}_x \rightarrow \mathsf{B}_x)) \bigvee^1_x \mathsf{B}_x$ (to be solved with $\mathrm{DFL}^{\langle n\rangle}$)

3. Symbolic Universe and Inconsistent Objects

3.1. Paraconsistency

MacColl's system contains inconsistent objects like round squares, flat spheres and so on. The logic described above can deal with these objects as elements of the symbolic universe. Another way of dealing with this situation is to understand arguments containing propositions about inconsistent objects as arguments in which inconsistent elementary propositions about given elements of the universe of discourse are allowed. That is, instead of allowing the use of constants which name inconsistent objects, you have arguments in which two contradictory elementary propositions are allowed—this way of thinking about inconsistent objects was proposed by Richard Routley (see Routley 1979) in his interpretation of Felix Meinong (for a brief exposition of the main ideas of Richard Routley's monumental work, see Manuel Bremer 1998). This requires a logic in which such contradictions are allowed. Such a logic was the aim of the founders of paraconsistent logic, namely the Polish logician Stanisl aw Jaśkowski (see Jaśkowski 1948) and the Brazilian logician Newton C. A. da Costa (see da Costa 1974).

The work of da Costa takes the assumption that contradictions can appear in a logical system without making this system trivial. Actually this leads to the standard definition of paraconsistent logics:

Definition 4. Paraconsistency. *Let us consider a theory $\mathcal{T}$ as a triple $\langle \mathcal{L}, \mathcal{A}, \mathcal{G} \rangle$, where $\mathcal{L}$ is a language, $\mathcal{A}$ is a set of propositions (closed formulae) of $\mathcal{L}$, called the axioms of $\mathcal{T}$, and $\mathcal{G}$ is the underlying logic of $\mathcal{T}$. We suppose that $\mathcal{L}$ has a negation symbol, and that, as usual, the theorems of $\mathcal{T}$ are derived from $\mathcal{A}$ by the rules of $\mathcal{G}$ (cf. da Costa et al. 1998, p. 46).*

In such a context, $\mathcal{T}$ is said to be inconsistent *if it has two theorems A and $\neg A$, where A is a formula of $\mathcal{L}$. $\mathcal{T}$ is called* trivial *if any formula of $\mathcal{L}$ is a theorem of $\mathcal{T}$. $\mathcal{T}$ is called* paraconsistent *if it can be inconsistent without being trivial. Equivalently $\mathcal{T}$ is paraconsistent if it is not the case that when A and $\neg A$ hold in $\mathcal{T}$, any B (from $\mathcal{L}$) also holds in $\mathcal{T}$.*

Thus, if $\mathcal{T}$ is a paraconsistent theory it is not the case that every formula of $\mathcal{L}$ and its negation are theorems of $\mathcal{T}$. Typically, in a paraconsistent theory $\mathcal{T}$, there are theorems whose negations are not theorems of $\mathcal{T}$. Nonetheless, there are formulae which are theorems of $\mathcal{T}$ and whose negations are also theorems (da Costa et al. 1998, p. 46).

Actually there are two main interpretations possible. The one, which I will call the *compelling interpretation*, based on a naive correspondence theory, stresses that paraconsistent theories are ontologically committed to inconsistent objects. The other, which I call the *permissive interpretation*, does not assume this ontological commitment of paraconsistent theories. The usual referential semantics for paraconsistent logics is not really compatible with the idea of a permissive interpretation of paraconsistency—the permissive interpretation of inconsistencies can be seen as another way of stating their symbolic existence. Rahman and Carnielli (1998) developed a dialogical approach to paraconsistency which yields several systems called *literal dialogues* (shorter: L-D) and takes its permissive non-referential interpretation seriously. I will adapt L-D to the purposes of the present paper.

3.2. The dialogical approach to paraconsistent logic

As already mentioned, MacColl's symbolic universe contains non-existent objects and (formally) existent ones. Non-existent objects are in my reconstruction those objects which are named by constants that have been *used*—i.e. which occur in a formula stated in a dialogue—without having been introduced (in the sense of DFL3) before. Now, contradictory objects are in MacColl's view to be included in the sub-universe of non-existent objects, and this is quite in the sense of a permissive interpretation of paraconsistency. Thus, I will provide the system(s) of free-logic DFL with a rule introducing paraconsistency—I call this rule the *negative literal rule* (DFL4)—but with the following caveat: *The* negative literal rule *applies only for formulae in which constants occur that have not been introduced in the sense of DFL3.*

(DFL4). Negative literal rule. *The Proponent is allowed to attack the negation of an atomic (propositional) statement (the so-called* negative literal*) if and only if the Opponent has attacked the* same *statement before.*

This structural rule can be considered analogous with the formal rule for positive literals. The idea behind this rule can be connected with MacColl's concept of symbolic existence in the following way: A contradiction between literals, say a and $\neg a$, expresses that one proposition ascribes a predicator to a given object while the other denies that a predicator applies to this object. Now, if the Opponent is the one who introduces such a contradiction between literals, this contradiction can be seen as having a pure problematic modality, i.e. as being stated symbolically. This means that the Proponent—who has proposed $(a \wedge \neg a) \rightarrow \neg a$, for example—is also allowed to use the conceded symbolical contradiction $a \wedge \neg a$ (of the Opponent), stating himself for example $\neg a$. Expressed intuitively: "If you (the Opponent) concede that a flat sphere is not flat, so can I (the Proponent)". Now, suppose that the Opponent attacks $\neg a$ with a. This allows the Proponent to attack the corresponding negation (and no other) of the Opponent (i.e., "If you (the Opponent) attack my proposition that a flat sphere is not flat, so can I (the Proponent)").

When I want to distinguish between the intuitionistic and the classical version I write L-D$^{\mathrm{i}}$ (for the intuitionistic version) and L-D$^{\mathrm{c}}$ (for the classical version). To be precise, we should call these logical systems literal dialogues with classical structural rule and literal dialogues with intuitionistic structural rule, respectively. Actually, *strictu sensu* they are neither classical nor intuitionistic because neither in L-D$^{\mathrm{i}}$ nor in L-D$^{\mathrm{c}}$ are *ex falso sequitur quodlibet*, $(a \to b) \to ((a \to \neg b) \to \neg a)$, or $a \to \neg\neg a$ winnable.

In L-D the (from a paraconsistent point of view) dangerous formulae $(a \wedge \neg a) \to b$, $a \to (\neg a \to b)$ and $(a \to b) \to ((a \to \neg b) \to \neg a)$ are not valid. Let us see the corresponding literal dialogues in L-D$^{\mathrm{c}}$ for the first and the last one:

	Opponent	Proponent	
		$(a \wedge \neg a) \to b$	(0)
(1)	$?_0 a \wedge \neg a$		
(3)	$!_2 a$	$?_{1/\text{left}}$	(2)
(5)	$!_2 \neg a$	$?_{1/\text{right}}$	(4)
	The Opponent wins		

The Proponent loses because he is not allowed to attack the move (5) (see negative literal rule). In other words, the Opponent has stated the contradiction $a \wedge \neg a$ about an object, but this contradiction, being conceded as part of the symbolic reasoning in the argument, cannot be attacked by the Proponent until the Opponent starts an attack on the negative literal $\neg a$—an attack which in this case will not take place.

Similar considerations hold for $(a \to b) \to ((a \to \neg b) \to \neg a)$:

	Opponent	Proponent	
		$(a \to b) \to ((a \to \neg b) \to \neg a)$	(0)
(1)	$?_0 a \to b$	$!_1 (a \to \neg b) \to \neg a$	(2)
(3)	$?_2 a \to \neg b$	$!_3 \neg a$	(4)
(5)	$?_4 a$	$\otimes$	
(7)	$!_6 b$	$?_1 a$	(6)
(9)	$!_8 \neg b$	$?_3 a$	(8)
	The Opponent wins		

The Proponent loses here because he cannot attack $\neg b$.

All classically valid formulae without negation are also valid in L-D^{c}. All intuitionistically valid formulae without negation are also valid in L-D^{i}. As in da Costa's system C_1 none of the following are valid in L-D^{c}:

$(a \wedge \neg a) \to b$	$(a \to (b \vee c)) \to ((a \wedge \neg b) \to c)$
$(a \wedge \neg a) \to \neg b$	$((a \to \neg a) \wedge (\neg a \to a)) \to \neg b$
$\neg(a \wedge \neg a)$	$((a \wedge b) \to c) \to ((a \wedge \neg c) \to \neg b)$
$a \to \neg\neg a$	$(a \to b) \vee (\neg a \to b)$
$(a \to b) \to ((a \to \neg b) \to \neg a)$	$((a \vee b) \wedge \neg a) \to b$
$((a \to b) \wedge (a \to \neg b)) \to \neg a$	$(a \vee b) \to (\neg a \to b)$
$((\neg a \to b) \wedge (\neg a \to \neg b)) \to a$	$(a \to b) \to (\neg b \to \neg a)$
$\neg a \to (a \to b)$	$(\neg a \vee \neg b) \to \neg(a \wedge b)$
$\neg a \to (a \to \neg b)$	$(\neg a \wedge \neg b) \to \neg(a \vee b)$
$a \to (\neg a \to b)$	$(\neg a \vee b) \to (a \to b)$
$a \to (\neg a \to \neg b)$	$(a \to b) \to \neg(a \wedge \neg b)$
$((a \to \neg a) \wedge (\neg a \to a)) \to b$	$\neg a \to ((a \vee b) \to b)$

In L-D^{i} all the intuitionistically non-valid formulae have to be added to the list, for example:

$\neg\neg A \to A$	$A \vee (A \to B)$
$A \vee \neg A$	$A \vee ((A \vee B) \to B)$
$((A \to B) \to A) \to A$	$\neg(A \to B) \to A$

The extension of literal dialogues for propositional logic to first-order quantifiers is straightforward. To build *Quantified Literal Dialogues*, we have only to extend the structural negative literal rule to elementary statements of first-order logic. The way to do that is to generalise the rule for elementary statements:

DEFINITION 5. (GENERAL) NEGATIVE LITERAL RULE. *The Proponent is allowed to attack the negation of an elementary statement (i.e., the* negative literal*) if and only if the Opponent has attacked the* same *statement before.*

Let us look at an example:

	Opponent	Proponent	
		$\bigwedge_x((\mathsf{A}_x \wedge \neg\mathsf{A}_x) \to \mathsf{B}_x)$	(0)
(1)	$?_{0/\tau}$	$!_1(\mathsf{A}_\tau \wedge \neg\mathsf{A}_\tau) \to \mathsf{B}_\tau$	(2)
(3)	$?_2\mathsf{A}_\tau \wedge \neg\mathsf{A}_\tau$		
(5)	$!_4\mathsf{A}_\tau$	$?_{3/\text{left}}$	(4)
(7)	$!_6\neg\mathsf{A}_\tau$	$?_{3/\text{right}}$	(6)

The Opponent wins

The Proponent loses here because (according to the general negative literal rule) he is not allowed to attack move (7) using the Opponent's move (5).

Similarly, the literal rule blocks validity of $\bigwedge_x(\mathsf{A}_x \to (\neg\mathsf{A}_x \to \mathsf{B}_x))$ and the quantified forms of other non-paraconsistent formulae. Here again it is possible to define quantified literal dialogues for intuitionistic and classical logic.

Let us consider an example of a thesis which is not intuitionistically but classically winnable: A quantified literal dialogue in L-D$^{\mathrm{i}}$ for $\bigwedge_x \neg\neg\mathsf{A}_x \to \neg\neg \bigwedge_x \mathsf{A}_x$ runs as follows:

	Opponent	Proponent	
		$\bigwedge_x \neg\neg\mathsf{A}_x \to \neg\neg \bigwedge_x \mathsf{A}_x$	(0)
(1)	$?_0 \bigwedge_x \neg\neg\mathsf{A}_x$	$!_1\neg\neg \bigwedge_x \mathsf{A}_x$	(2)
(3)	$?_2\neg \bigwedge_x \mathsf{A}_x$	$\otimes$	
	$\otimes$	$?_3 \bigwedge_x \mathsf{A}_x$	(4)
(5)	$?_{4/\tau}$		
(7)	$!_6\neg\neg\mathsf{A}_n$	$?_{1/\tau}$	(6)
	$\otimes$	$?_7\neg\mathsf{A}_\tau$	(8)
(9)	$?_8\mathsf{A}_\tau$	$\otimes$	

The Opponent wins

The Proponent loses in L-D$^{\mathrm{i}}$ because he is not allowed to defend himself against the attack of the Opponent in move (5)—the last Opponent's attack not already defended by the Proponent was stated in move (9).

The Proponent wins in L-D$^{\mathrm{c}}$ because the restriction mentioned above does not hold. Thus the Proponent can answer the attack of move (5) with move (10) in the following dialogue in (quantified) L-D$^{\mathrm{c}}$ and win:

	Opponent	Proponent	
		$\bigwedge_x \neg\neg\mathsf{A}_x \to \neg\neg \bigwedge_x \mathsf{A}_x$	(0)
(1)	$?_0 \bigwedge_x \neg\neg\mathsf{A}_x$	$!_1\neg\neg \bigwedge_x \mathsf{A}_x$	(2)
(3)	$?_2\neg \bigwedge_x \mathsf{A}_x$	$\otimes$	
	$\otimes$	$?_3 \bigwedge_x \mathsf{A}_x$	(4)
(5)	$?_{4/\tau}$	$!_5\mathsf{A}_\tau$	(10)
(7)	$!_6\neg\neg\mathsf{A}_\tau$	$?_{1/\tau}$	(6)
	$\otimes$	$?_7\neg\mathsf{A}_\tau$	(8)
(9)	$?_8\mathsf{A}_\tau$	$\otimes$	

The Proponent wins

It is possible to define tableaux for the winning strategies which correspond to these dialogue systems (see Rahman and Carnielli 1998). To obtain paraconsistent tableaux systems considered as an extension of those for DFL add the following restriction to the closing rules:[7]

Definition 6. Paraconsistent Restriction. *Check after finishing the tableau and before closing branches that for every elementary* P*-statement which follows from the application of an* O*-rule to the corresponding negative* O*-literal (i.e. for every attack on a negative* O*-literal) there is an application of a* P*-rule to a negative* P*-literal which yields an* O*-positive literal with the same atomic formula as the above-mentioned attack of the Proponent. Those elementary* P*-attacks on the corresponding negative* O*-literals which do not meet this condition cannot be used for closing branches and can thus be deleted.*

This approach to paraconsistency blocks triviality for the literal case only, that is, a thesis of the form $((a \wedge b) \wedge \neg(a \wedge b)) \rightarrow c$ is still valid. One way to see the literal rule is to think of it as distinguishing between the internal or copulative negation from the external or sentential negation.[8] That is, in the standard approaches to logic, the elementary proposition A_n has the internal logical form: $n \,\epsilon\, \mathsf{A}$ (where ϵ stands for the copula: n *is* A) and the negation of it the form: $n \,\epsilon'\, \mathsf{A}$ (n *is not* A). Now in this standard interpretation the negative copula is equivalent to the expression $\neg A$, where A can also be complex. This equivalence ignores the distinction between the internal (copulative) form and the external or sentential form of elementary propositions. The literal approach to paraconsistency takes this distinction seriously, with the result that contradictions which cannot be carried on at the literal level should be freed of paraconsistent restrictions.

4. Conclusions

This article is one of a series based on the seminar “Erweiterungen der Dialogischen Logik” (“extensions to dialogical logic”) held in Saarbrücken in the summer of 1998 by Shahid Rahman and Helge

[7]Although you can produce intuitionistic and classical free-paraconsistent systems the intuitionistic version seems more appropriate. Such a system is defended in Rahman 1999, where, following Read, (see Read 1994, p. 137) I support the idea that although classical logic might have some plausibility for existents it loses this plausibility for non-existents. In the paper mentioned I connect this argument with the theory of *privatio* of the Spanish philosopher Francisco Suárez (1548–1617) (see Suárez 1966, pp. 434–440).

[8]The difference between internal and external negation has been worked out for other purposes by A. A. Sinowjew (1970) and Wessels/Sinowjew (1975).

Rückert. The same seminar has motivated the publication of *The Dialogical Approach to Paraconsistency* by Rahman and Carnielli (1998), *On Dialogues and Ontology. The Dialogical Approach to Free Logic* by Rahman, Fischmann, and Rückert (1999), *Dialogische Modallogik für T, B, S4 und S5* (Rahman and Rückert 1999b) and *Dialogische Logik und Relevanz* (Rahman and Rückert 1998) and *Die Logik der zusammenhängenden Aussagen: ein dialogischer Ansatz zur konnexen Logik* (1999a) by Rahman and Rückert. One important aim of these articles (and the present paper) is to show how to build a common semantic language for different non-standard logics in such a way that 1. the semantic intuitions behind these logics can be made transparent, 2. combinations of these logics can be easily achieved, 3. a common basis is proposed for discussion of the philosophical consequences of these logics—the philosophical point here is to undertake the task of discussing the semantics of non-classical logics from a pragmatical point of view, one which commits itself neither to a correspondence theory of truth nor to a possible-world semantics.

One of the consequences of the dialogical approach is that two of the above-mentioned logics can be seen as extending the formal rule for elementary propositions, namely free and paraconsistent logics. This offers a perspective on these logics which seems to be close to Hugh MacColl's reflections on symbolic existence and demands a new concept of logical form. This new concept of logical form should allow valid and invalid forms to be differentiated without going back to a mere syntactic notion—but this is another interesting story.

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HUGH MACCOLL AND THE ALGEBRA OF STRICT IMPLICATION

C. I. Lewis repeatedly exempts MacColl from criticisms of his predecessors in their accounts of implication. They had all taken a true implication, or conditional, to be one with false antecedent or true consequent. MacColl uniquely, and correctly in Lewis' view, rejected this account, identifying a true implication with the impossibility of true antecedent and false consequent. Lewis' development of the calculus of strict implication arises directly and explicitly out of MacColl's work.

A close analysis of MacColl's calculatory methods, and summaries of his main theses, serve to show that MacColl's modal logic is in fact the logic T introduced by Feys and von Wright many decades later, the smallest normal epistemic modal logic.

1. MacColl and Lewis

The received wisdom is that strict implication was invented and developed by the American logician C. I. Lewis. Careful reading of Lewis' papers, and of his book, *A Survey of Symbolic Logic* (1918), reveals that he repeatedly exempts MacColl from criticisms of his predecessors in their accounts of implication. From this fact, it is clear that Lewis knew MacColl's work; he is not fully candid, however, in acknowledging his debt to MacColl. Indeed, in later life Lewis seemed to take great pains to obscure the origins of his modal calculi as they are presented in his joint work with Langford (1932). Thus, for the 1960 Dover reprint of Lewis 1918, a whole third of the book (chapters 5 and 6), containing Lewis' first treatment of modal logic in book form, was completely omitted at Lewis' instigation (the ground being that what was said there had been superseded by the later treatment—and indeed, the system of the *Survey* was flawed, in containing too powerful a form of the Consistency Postulate). Furthermore, in collecting his articles for their 1970 reprinting (Lewis 1970), Lewis omitted all

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but one of the papers published prior to 1918 on the notion of strict implication. The result was a wholesale removal of many of what brief acknowledgements of MacColl there were in Lewis' writings.

MacColl suffered, however, from an even greater eclipsing of his logical contribution than simply from being excluded from Lewis' revisionist history. In the second half of the nineteenth century, the dominant programme in logic was the Boole-Schröder algebraic system, construing the logical constants as operators in a class algebra. The first two volumes of Schröder's famous *Lectures on the Algebra of Logic* (1890–1905) are studded with (complimentary) references to MacColl. As we will see, MacColl recognized that one had to supplement the existing extensional algebras (extensional in that they admitted a class interpretation) with a further (intensional) operator in order to have any prospect of properly capturing the logic of implication. Within months of MacColl's death in December 1909 came the publication of Whitehead and Russell's *Principia Mathematica* (1910–1912). The result was the rapid replacement of the Boole-Schröder algebraic paradigm by the logistic methods developed by Frege, Peano and others. Only a few years after MacColl's death, the method of systematic proof from axioms of a logistic formulation had replaced the algebraic methods of calculation of the nineteenth century. Lewis caught the spirit of the times, recasting MacColl's modal calculus in logistic terms. I do not want to deny that Lewis provided deep logical insights in his presentation of strict implication in the fashionable new guise. But again, the presentational novelty served to obscure MacColl's contribution from all but the keenest observers.

MacColl's logic was developed in several series of papers whose driving methodology is the calculatory and applied paradigm of the late nineteenth century. The preferred argument for a logical method was its success in application to specific problems. MacColl repeatedly claims superiority of his calculus over that of the "Boolian logicians" as he calls them (Jevons, Schröder et al.) on the grounds of its greater success in solving specific problems from the *Educational Times* or the *Proceedings of the London Mathematical Society*.[1] The development of the background theory, and in particular, the systematic specification of that theory in axiomatic and deductive terms, was a requirement that only came later, following the development of the logistic method.

MacColl saw his calculus as differing in two main regards from the "Boolian" calculus. First, he emphasized its propositional interpretation as at least as important as the customary class interpretation.

[1] See, e.g., MacColl 1903b, cited in MacColl 1998, 15 May 1905.

In fact, one of MacColl's repeated themes is the preference for multiplicity of interpretation. MacColl wrote (1902, p. 362):

> Perhaps the most important principle underlying my system of notation is the principle that we may vary the meaning of any symbol or arrangement of symbols, provided, firstly, we accompany the change of signification by a new explanatory definition; and provided, secondly, the nature of our argument be such that we run no risk of confounding the old meaning with the new. Of course this variation of sense should not be resorted to wantonly and without cause; but the cases are numerous in which it leads both to clearness of expression and to an enormous economy in symbolic operations.

Thus his calculus admits of a class interpretation, a propositional interpretation and an interpretation in probability theory. But the propositional interpretation was the novelty for such an algebraic system, and the one MacColl emphasized as distinctive of his approach.

Secondly, he claimed that the two alethic modalities, 'true' and 'false', symbolized by '1' and '0' in the "Boolian" system, were inadequate to deal with all problems arising in mathematics. Three further modalities he introduced were 'certain' (or 'necessary'), 'impossible' and 'contingent'. One motivation for this was the probabilistic interpretation. A true proposition can have any probability value greater than 0, not necessarily 1. Again, a false proposition can have any value less than 1, not necessarily 0. When we discover that the probability of a proposition is 1, we know it is certain, not just that it is true; when we find its probability is 0, we know it is impossible, not just false. Of course, these values are relative to certain evidence, so we know that the proposition is certain, impossible or neither, relative to certain data. MacColl first introduced ϵ (for certainty), η (for impossibility) and θ (for variability, or contingency) relative to the data. Subsequently, he generalized them to stand also for certainty *tout court*, that is, necessity, for impossibility and for contingency. Thus a^ϵ reads 'a is certain (or necessary)', a^η reads 'a is impossible' and a^θ reads 'a is variable (or contingent)'.[2]

My aim in this paper is to give a systematic presentation of MacColl's modal algebra. It is based on Boole's algebra, extending it by the alethic modalities and strict implication as further operators. Although, as we will see, only one of these need be taken as primitive, the rest being definable, I will take both possibility and strict implication as primitive. The situation is not unlike that in Boolean algebras, which take meet, join and complement as primitive, though meet and join can each be defined in terms of the other and complement.

[2]See, e.g., MacColl 1901, § 3, p. 138. He also wrote a^τ for 'a is true' and a^ι for 'a is false'.

I will start with a reminder of the formal theory of Boolean algebras, presented in a more systematic way than was common in MacColl's time. This presentation will also serve to exhibit the terminology and methodology of proof and demonstration.

2. Boolean Algebras

An algebra consists of a set of elements closed under one or more operations satisfying certain conditions. The idea of algebraic logic is to define a class of algebras which characterize logical validity (or more generally, logical consequence). For example, Boolean algebras characterize classical (propositional) logic, pseudo-Boolean (sometimes called Heyting) algebras characterize intuitionistic logic, and cylindric algebras characterize full (first-order) classical logic. In the Russell/Whitehead paradigm, a logic is taken to be a class of wffs and a subset of validities (or better, a consequence relation on those wffs). The well-formed formulae of the logic are mapped to elements of the algebra via a homomorphism[3] defined on the atomic wffs which identifies equivalent wffs. Much of algebraic logic then consists of the identification of the algebra of equivalent wffs (the Lindenbaum algebra), showing that it is free[4] in a certain class of algebras, that the set of validities is a filter[5] (commonly, as in the cases cited above, the trivial filter consisting of the maximum element alone), and studying homomorphisms from the Lindenbaum algebra to more manageable, in particular, finite algebras in the class.

In the Boole-Schröder paradigm, the representation of situations goes directly to the elements of the algebra, omitting the syntactic intermediary of a language of wffs. The emphasis is on exploration of the algebraic structure and solution of mathematical (and other) problems, rather than the metalogical analysis and representation theory more common today. Logical algebra was seen as a further mathematical tool which proved itself by its utility. The algebras were studied as individuals of a type whose properties were developed piecemeal, rather than systematically as a class. The whole approach was—at least when compared with that which replaced it—calculatory and unsystematic.

[3] A homomorphism is a mapping between algebras of the same type which preserves the operations, e.g., if $\circ$ is a two-place operation on A matching a similar operation on B, and h is a homomorphism from A to B, then $h(a \circ b) = ha \circ hb$.

[4] An informal account of when an algebra is free in a class is if it satisfies no further conditions than those on the class as a whole.

[5] A filter F in a lattice (see below) is a subset such that $a \cap b \in F$ iff $a \in F$ and $b \in F$.

The insight which the two paradigms share, however, is recognition that logical structure responds productively to the application of algebraic techniques.

A Boolean algebra is a complemented distributive lattice. Some authors take lattices to be a particular type of poset (partially ordered set), one in which every two-element subset has a *sup* (supremum) and *inf* (infimum). However, I want to present MacColl's ideas purely algebraically, so I will take lattices to be algebras, even though, as we will see, they can also be viewed as relational structures of a certain sort. I will by and large follow MacColl's notation, in particular, taking . (or concatenation) and + for meet and join respectively, and $'$ for complement, and writing ϵ and η for the maximum and minimum elements of the algebra.

DEFINITION 2.1. *A* lattice L *consists of a set of elements closed under the operations of meet (.) and join (+): $\langle A, ., + \rangle$, subject to the following constraints:*

$$
\begin{array}{rll}
a.(b.c) = (a.b).c & \textit{for all } a,b,c \in A. & \textit{(Associativity for meet)}\\
a+(b+c) = (a+b)+c & \textit{for all } a,b,c \in A. & \textit{(Associativity for join)}\\
a.b = b.a,\ a+b = b+a & \textit{for all } a,b \in A. & \textit{(Commutativity)}\\
a.(a+b) = a + a.b = a & \textit{for all } a,b \in A. & \textit{(Absorption)}
\end{array}
$$

L *is* non-trivial *if there are $a, b \in L$ such that $a \neq b$.*

LEMMA 2.1. *In any lattice, $a.a = a = a + a$.*[6]

Proof. $a.a = a.(a + a.a) = a = a + a.(a + a) = a + a$. □

DEFINITION 2.2. *A* preordering *on a set X is a relation $\leq$ such that*

$$
\begin{array}{rll}
a \leq a & \textit{for all } a \in X. & \textit{(Reflexivity)}\\
\textit{if } a \leq b \textit{ and } b \leq c \textit{ then } a \leq c & \textit{for all } a,b,c \in X. & \textit{(Transitivity)}
\end{array}
$$

DEFINITION 2.3. *X is* partially ordered *by $\leq$ if $\leq$ is a preordering on X such that*

$$
\textit{if } a \leq b \textit{ and } b \leq a \textit{ then } a = b \quad \textit{for all } a,b \in X \qquad \textit{(Antisymmetry)}
$$

DEFINITION 2.4. *A* poset *is a set X with a partial ordering $\leq$: $\langle X, \leq \rangle$.*

DEFINITION 2.5. *1. a is an* upper bound *of $A \subseteq X$ if for all $b \in A$, $b \leq a$.*

2. a is a lower bound *of $A \subseteq X$ if for all $b \in A$, $a \leq b$.*

[6] MacColl 1896, p. 178 (21).

Definition 2.6. *1. a is the* supremum *(or* least upper bound*) of $A \subseteq X$ if a is an upper bound of A and for all $b \in X$, if b is an upper bound of A then $a \leq b$. We write $a = sup(A)$.*

2. a is the infimum *(or* greatest lower bound*) of $A \subseteq X$ if a is a lower bound of A and for all $b \in X$, if b is a lower bound of A then $b \leq a$. We write $a = inf(A)$.*

Lemma 2.2. *$a.b = a$ iff $a + b = b$.*

Proof. Suppose $a.b = a$. Then

$$\begin{aligned} b &= b + (a.b) && \text{Absorption} \\ &= b + a \\ &= a + b && \text{Commutativity} \end{aligned}$$

Converse: similar. □

Definition 2.7. *Given a lattice L, define $\leq$ on L by: $a \leq b$ iff $a.b = a$.*

Lemma 2.3. *In any lattice, if $a \leq b$ then $a.c \leq b.c$ and $a+c \leq b+c$.*

Proof. Suppose $a \leq b$. Then $a.b = a$ and by Lemma 2.2, $a + b = b$. So $(a.c).(b.c) = (a.b).(c.c) = a.c$. So $a.c \leq b.c$.

Similarly, $(a + c) + (b + c) = (a + b) + (c + c) = b + c$. So again by Lemma 2.2, $a + c \leq b + c$. □

Theorem 2.1. *Each lattice $\langle L, ., +\rangle$ induces a poset $\langle L, \leq\rangle$ in which every pair of elements has a sup and an inf, and vice versa.*

Proof. First, we show that $\leq$ is a p.o. on L.

1. $\leq$ is reflexive, by Lemma 2.1.

2. Suppose $a \leq b$ and $b \leq c$, i.e., $a.b = a$ and $b.c = b$.
   Then $a.c = (a.b).c = a.(b.c) = a.b = a$
   i.e., $a \leq c$. So $\leq$ is transitive.

3. Suppose $a \leq b$ and $b \leq a$, i.e., $a.b = a$ and $b.a = b$.
   Then $a = a.b = b.a$ Commutativity
   $= b$. So $\leq$ is anti-symmetric.

Next we show that $a + b = sup\{a, b\}$. Note that $a = a.(a + b)$, so $a \leq a + b$; and that $b = b.(b + a)$
$= b.(a + b)$, so $b \leq a + b$.
Now, suppose $a \leq c$ and $b \leq c$. Then $a.c = a$ and $b.c = b$. So $a + c = c$ and $b + c = c$, by Lemma 2.2, whence $(a + b) + c = a + (b + c) = a + c = c$, and so $(a + b).c = a + b$, by Lemma 2.2, i.e., $a + b \leq c$. So $a + b = sup\{a, b\}$.

Similarly, $a.b = inf\{a, b\}$.

Conversely, given a poset P in which every pair of elements has a *sup* and an *inf*, define $a.b$ as $inf\{a, b\}$ and $a + b$ as $sup\{a, b\}$. Clearly, on this definition, meet and join are associative and commutative. For absorption, since $a \leq sup\{a, b\}$, $a = inf\{a, sup\{a, b\}\} = a.(a + b)$, and similarly, $a = sup\{a, inf\{a, b\}\} = a + (a.b)$. □

LEMMA 2.4. *Take any lattice, L. If $z \leq x$ iff $z \leq y$ for all $z \in L$, then $x = y$.*

Proof. Since $x \leq x$, $x \leq y$ and since $y \leq y$, $y \leq x$. So as in Theorem 2.1, $x = y$. □

LEMMA 2.5. *In any lattice,*

$$(a.b) + (a.c) \leq a.(b + c)$$

and

$$a + (b.c) \leq (a + b).(a + c).$$

Proof. $a.b \leq a.(b + c)$ and $a.c \leq a.(b + c)$, Lemma 2.3
so $(a.b) + (a.c) \leq a.(b + c)$. by Theorem 2.1
Moreover, $a \leq a + b$ and $a \leq a + c$, by Theorem 2.1
so $a \leq (a + b).(a + c)$. Theorem 2.1
Furthermore, $b \leq a + b$ and $c \leq a + c$,
so $b.c \leq (a + b).(a + c)$, by Lemma 2.3
so $a + (b.c) \leq (a + b).(a + c)$. by Theorem 2.1. □

DEFINITION 2.8. *A lattice is* distributive *if*

$$a.(b + c) = (a.b) + (a.c).\text{[7]} \qquad \textit{(Distributive law of meet over join)}$$

LEMMA 2.6. *In a distributive lattice,*

$$a + (b.c) = (a + b).(a + c). \qquad \textit{(Distributive law of join over meet)}$$

Proof. $(a + b).(a + c) = (a + b).a + (a + b).c = a + c.(a + b)$
$= a + (c.a) + c.b = a + b.c.$ by Lemma 2.3
So by Lemma 2.5, $a + b.c = (a + b).(a + c)$. □

In fact, either distributive law may be derived from the other.

DEFINITION 2.9. *A lattice L has a* maximum *(ϵ) if for all $a \in L$, $a \leq \epsilon$, and it has a* minimum *(η) if for all $a \in L$, $\eta \leq a$.*

DEFINITION 2.10. *A lattice L is* complemented *if for all $a \in L$ there is $b \in L$ such that $a + b = \epsilon$ and $a.b = \eta$.*

LEMMA 2.7. *In a distributive lattice, complements are unique.*

Proof. Suppose a has complements b and c, i.e.,

$$a + b = \epsilon = a + c$$

and

[7] MacColl 1901, p. 141.

$$a.b = \eta = a.c.$$

Then $b = b.(b+a) = b.(a+b) = b.(a+c) = ba+bc = ab+bc = ac+bc = (a+b).c = (a+c).c = c.(c+a) = c.$ □

THEOREM 2.2.

1. $a.\epsilon = a$;[8]
2. $a.\eta = \eta$;[9]
3. $\epsilon + a = \epsilon$;[10]
4. $\eta + a = a$.[11]

Proof.

1. Since $a \leq \epsilon$, $a.\epsilon = a$.
2. Since $\eta \leq a$, $a.\eta = \eta$.
3. Since $a \leq \epsilon$, $\epsilon + a = \epsilon$ by Lemma 2.2.
4. Since $\eta \leq a$, $\eta + a = a$ by Lemma 2.2. □

DEFINITION 2.11. *A* Boolean algebra *consists of a set of elements closed under meet, join and complement (′): $\langle A, ., +, ', \eta, \epsilon\rangle$, such that $\langle A, ., +\rangle$ is a distributive lattice with maximum and minimum, and*

$$a.a' = \eta \text{ and } a + a' = \epsilon \text{ for all } a \in A.$$

THEOREM 2.3. *In a Boolean algebra,*

1. $a.b' = \eta$ *iff* $a \leq b$
2. $a + b' = \epsilon$ *iff* $b \leq a$
3. $a'' = a$.

Proof.

1. Suppose $a.b' = \eta$. Then
$$\begin{aligned} a &= a.\epsilon \\ &= a.(b+b') \\ &= (a.b) + (a.b') \\ &= a.b. \text{ So } a \leq b. \end{aligned}$$
Conversely, suppose $a \leq b$. Then $a.b' \leq b.b'$ by Lemma 2.3 $= \eta$. So $a.b' = \eta$.

[8] MacColl 1906, p. 8 (22).
[9] MacColl 1906, p. 8 (23).
[10] MacColl 1901, p. 143 (5).
[11] MacColl 1901, p. 143 (6).

2. The dual case is similar.[12]
3. Immediate from Lemma 2.7. □

THEOREM 2.4.

1. $(a.b)' = a' + b'$;[13]
2. $(a + b)' = a'.b'$;[14]
3. $a \leq b$ *iff* $b' \leq a'$;
4. $(a + b'c)' = a'b + a'c'$.[15]

Proof.

1. $ab(a' + b') = aba' + abb' = \eta + \eta = \eta$ and
   $ab + (a' + b') = (a + a' + b')(b + a' + b') = \epsilon.\epsilon = \epsilon.$
   So $(ab)' = a' + b'$.
2. $a'b'(a + b) = a'b'a + a'b'b = \eta$ and
   $a'b' + (a + b) = (a' + a + b)(b' + a + b) = \epsilon.\epsilon = \epsilon.$
   So $a'b' = (a + b)'$.
3. $a \leq b$ iff $a = a.b$ iff $a' = (a.b)' = a' + b'$ iff $b' \leq a'$.
4. $$\begin{aligned} (a + b'c)' &= a'(b'c)' && \text{by (2)} \\ &= a'(b + c') && \text{by (1)} \\ &= a'b + a'c' && \text{by Definition 2.8.} \end{aligned}$$ □

3. MODAL ALGEBRAS

MacColl's several attempts at systematic presentation of his logic[16] do not satisfy modern standards of rigour. His various statements make clear what theses his algebra contains; what is harder to ascertain is what it does not contain, that is, precisely how strong it is. Hughes and Cresswell (1996) repeat the question they raised in their original text (1968):

> MacColl does give a list of 'self-evident formulae' and it would be interesting to know which of the more recent modal systems is the weakest in which all these are true. (Hughes and Cresswell 1968, p. 214 n. 177; 1996, p. 206 n. 4)

[12] Let P be any statement about lattices, Boolean algebras, etc. If in P we replace $\leq$ by $\geq$, . by $+$, $+$ by ., and each element a by a' (and a' by a), we obtain the dual (statement) P'. Then P is true iff P' is true.

[13] MacColl 1901, p. 141; MacColl 1906, p. 8 (2).

[14] MacColl 1901, p. 141; MacColl 1906, p. 8 (3).

[15] MacColl 1906, p. 9 (1).

[16] E.g., MacColl 1901, 1906.

My claim is that MacColl's modal algebra is what has later come to be called the normal modal logic T. The algebraic treatment of T along with other weak modal logics was presented in Lemmon 1966, building on work in Lemmon 1960, developing, for the modal logics T, S2, S3 and so on, what he called "extension algebras" generalizing the closure algebras of McKinsey and Tarski 1944. The system T was characterized as that of normal epistemic extension algebras. I will call extension algebras (including closure algebras), modal algebras.

DEFINITION 3.1. *A* modal algebra *consists of a set of elements closed under meet, join, complement and extension (possibility,* $^\pi$*):* $\langle A, ., +, ', ^\pi, \eta, \epsilon\rangle$ *such that*

1. $\langle A, ., +, ', \eta, \epsilon\rangle$ *is a Boolean algebra, and*
2. $(a+b)^\pi = a^\pi + b^\pi$. *(K)*

DEFINITION 3.2. *A modal algebra is* epistemic *if it also satisfies the postulate*

$$a \leq a^\pi. \qquad (T)$$

DEFINITION 3.3. *A modal algebra is* normal *if it also satisfies the postulate*

$$\eta^\pi = \eta. \qquad (N)$$

DEFINITION 3.4. *Let*

$$a^\eta = a^{\pi\prime}, \qquad a^\epsilon = a^{\prime\eta} \qquad \text{and} \qquad a^\theta = (a^\epsilon + a^\eta)'.$$

LEMMA 3.1.

1. $a^\pi = a^{\prime\epsilon\prime}$;
2. $a^\epsilon = a^{\prime\pi\prime}$.

Proof.

1. $a^{\prime\epsilon\prime} = a^{\prime\prime\eta\prime}$ by Definition 3.4 (2)
   $= a^{\eta\prime}$ since $a'' = a$
   $= a^{\pi\prime\prime}$ by Definition 3.4 (1)
   $= a^\pi$.
2. $a^{\prime\pi\prime} = a^{\prime\eta}$ by Definition 3.4 (1)
   $= a^\epsilon$ by Definition 3.4 (2). □

THEOREM 3.1.

1. $(a.b)^\epsilon = a^\epsilon.b^\epsilon$;[17]
2. $(a+b)^\theta = (a'b')^\theta$;[18]
3. *if* $a \leq b$ *then* $a^\pi \leq b^\pi$ *and* $a^\epsilon \leq b^\epsilon$;[19]
4. $a^\epsilon \leq a$.[20]

Proof.

1. $$\begin{aligned}(a.b)^\epsilon = (a.b)'^\eta = (a'+b')^\eta &= (a'+b')^{\pi\prime}\\ &= (a'^\pi + b'^\pi)' \qquad \text{by } (K)\\ &= a'^{\pi\prime}.b'^{\pi\prime} = a^\epsilon.b^\epsilon.\end{aligned}$$
2. $(a+b)^\theta = ((a+b)^\epsilon + (a+b)^\eta)' = (a+b)^{\epsilon\prime}.(a+b)^{\eta\prime} = (a+b)'^\pi.(a+b)^\pi = (a'b')^\pi.(a'b')'^\pi = (a'b')'^\pi.(a'b')^\pi = (a'b')^{\epsilon\prime}.(a'b')^{\eta\prime} = ((a'b')^\epsilon + (a'b')^\eta)' = (a'b')^\theta$.
3. Suppose $a \leq b$. Then $a+b=b$ and $a.b=a$.
   Hence $b^\pi = (a+b)^\pi = a^\pi + b^\pi$ by (K)
   i.e., $a^\pi \leq b^\pi$
   and $a^\epsilon = (a.b)^\epsilon = a^\epsilon.b^\epsilon$ by (1)
   i.e., $a^\epsilon \leq b^\epsilon$.
4. $a' \leq a'^\pi$ by (T)
   so $a'^{\pi\prime} \leq a''$ by Theorem 2.4 (3)
   i.e., $a^\epsilon \leq a$. □

THEOREM 3.2. [21]

1. $\epsilon^\eta = \eta$;
2. $\eta^\epsilon = \eta$;
3. $\eta^\eta = \epsilon$;[22]
4. $\epsilon^\epsilon = \epsilon$;
5. $\epsilon^\theta = \eta$.

---

[17]MacColl 1896, p. 169 ; cf. MacColl 1906, p. 72 (7).

[18]MacColl 1896, p. 169 .

[19]MacColl 1906, p. 9 § 13.

[20]Cf. MacColl 1906, p. 8 (15), which reads $A^\epsilon : A^\tau$, meaning 'If A is certain, then A is true'—see §4 below. MacColl writes (op.cit. § 8, p. 7) that A^ϵ asserts more than A^τ, which "only asserts that A is true in a particular case or instance." A^ϵ asserts "that A is *certain*, that A is *always* true (or true in *every* case)."

[21]MacColl 1901, p. 140.

[22]See also MacColl 1998, 6 Oct 1901.

Proof.

1. By (T), $\epsilon \leq \epsilon^\pi$. So $\epsilon^\eta = \epsilon^{\pi\prime} \leq \epsilon' = \eta$. Hence $\epsilon^\eta = \eta$.
2. $\eta^\epsilon = \eta'^\eta = \epsilon^\eta = \eta$ by (1).
3. $\eta^\eta = \eta^{\pi\prime} = \eta'$ (by N) $= \epsilon$.
4. $\epsilon^\epsilon = \epsilon'^\eta = \eta^\eta = \epsilon$ by (3).
5. $\epsilon^\theta = (\epsilon^\epsilon + \epsilon^\eta)' = (\epsilon + \eta)'$ by (1) and (4) $= \epsilon' = \eta$. □

THEOREM 3.3.

1. $(a + a')^\epsilon = \epsilon$;[23]
2. $(a.a')^\eta = \epsilon$;[24]
3. $(a^\epsilon + a^\eta + a^\theta)^\epsilon = \epsilon$.[25]

Proof.

1. $(a + a')^\epsilon = \epsilon^\epsilon = \epsilon$.
2. $(a.a')^\eta = \eta^\eta = \epsilon$.
3. $(a^\epsilon + a^\eta + a^\theta)^\epsilon = ((a^\epsilon + a^\eta) + (a^\epsilon + a^\eta)')^\epsilon = \epsilon^\epsilon = \epsilon$. □

THEOREM 3.4. *Where* $^\circ$ *is* $^\pi$, $^\epsilon$, $^\eta$, $^\theta$, *let* $a^{-\circ} = a^{\circ\prime}$. *Then*[26]

1. $a^\theta a^{-\theta} = \eta$;
2. $a^{-\theta} = a^\epsilon + a^\eta$;
3. $a^{-\epsilon} = a^\eta + a^\theta$;
4. $(a^\epsilon + b^{-\epsilon}c^\epsilon)' = (a^\eta + a^\theta)(b^\epsilon + c^\eta + c^\theta)$;
5. $(a^{-\theta} + a^\theta b^\theta)' = a^\theta(b^\epsilon + b^\eta)$.

Proof.

1. $a^\theta a^{-\theta} = a^\theta a^{\theta\prime} = \eta$.
2. $a^{-\theta} = a^{\theta\prime} = (a^\epsilon + a^\eta)'' = a^\epsilon + a^\eta$.
3. By Theorem 3.1 (4), $a^\epsilon \leq a$
   and by T, $a \leq a^\pi$, so $a^\epsilon \leq a^\pi$ by the proof of Theorem 2.1,
   i.e., $a^\epsilon = a^\pi.a^\epsilon = a^\pi.a^\epsilon + \eta = a^\pi.a^\epsilon + a^\pi.a^\eta = a^\pi(a^\epsilon + a^\eta) =$
   $(a^\eta + (a^\epsilon + a^\eta)')' = (a^\eta + a^\theta)'$.
   So $a^{-\epsilon} = a^\eta + a^\theta$.

[23] MacColl 1896, p. 177 (2).
[24] MacColl 1906, p. 8 (13).
[25] MacColl 1906, p. 8 (14).
[26] MacColl 1906, p. 9.

4. $(a^{\epsilon} + b^{-\epsilon}c^{\epsilon})' = a^{\epsilon\prime}.(b^{-\epsilon}c^{\epsilon})' = a^{-\epsilon}.(b^{\epsilon\prime\prime} + c^{\epsilon\prime})$
$$= (a^{\eta} + a^{\theta}).(b^{\epsilon} + c^{-\epsilon}) \quad \text{by (3)}$$
$$= (a^{\eta} + a^{\theta}).(b^{\epsilon} + c^{\eta} + c^{\theta}). \quad \text{by (3) again.}$$
5. $(a^{-\theta} + a^{\theta}b^{\theta})' = a^{\theta\prime\prime}.(a^{\theta}b^{\theta})' = a^{\theta}.(a^{-\theta} + b^{-\theta}) = a^{\theta}.a^{-\theta} + a^{\theta}.b^{-\theta} = \eta + a^{\theta}.b^{-\theta} = a^{\theta}(b^{\epsilon} + b^{\eta})$. □

4. MacColl Algebras

A MacColl algebra is, in essence, a normal epistemic modal algebra (or a T-algebra, for short). However, as we have noted, MacColl adds a further operator, a conditional operator, to his algebras. Thus we can best represent his algebra as a T-algebra with a further conditional operator, ':'.

DEFINITION 4.1. *A* MacColl algebra *consists of a normal epistemic modal algebra equipped with a conditional operator, :, i.e., a structure* $\langle A, ., +, ', ^{\pi}, :, \eta, \epsilon\rangle$ *such that*

$$a : b = (ab')^{\eta} \qquad \textit{(SI)}$$

$a : b$ *represents strict implication.*

THEOREM 4.1.

1. $a : b = b' : a'$;[27]
2. $a : b = (a' + b)^{\epsilon}$;
3. $(x : a)(x : b) = x : ab$;[28]
4. $(a + b) : x = (a : x)(b : x)$.[29]

Proof.

1. $a : b = (ab')^{\eta} = (b'a'')^{\eta} = b' : a'$.
2. $a : b = (ab')^{\eta} = (ab')^{\prime\epsilon} = (a' + b)^{\epsilon}$.
3. $(x : a)(x : b) = (x' + a)^{\epsilon}.(x' + b)^{\epsilon}$
$$= [(x' + a).(x' + b)]^{\epsilon} \quad \text{by } (K)$$
$$= (x' + ab)^{\epsilon} \quad \text{by Lemma 2.6}$$
$$= x : ab.$$
4. $(a : x)(b : x) = (a' + x)^{\epsilon}.(b' + x)^{\epsilon} = [(a' + x).(b' + x)]^{\epsilon} = (a'b' + x)^{\epsilon} = ab : x$. □

[27] MacColl 1901, p. 144 (7); MacColl 1906, p. 8 (4).
[28] MacColl 1906, p. 8 (5).
[29] MacColl 1906, p. 8 (6).

LEMMA 4.1.

1. *$a : b = \epsilon$ iff $a \leq b$.*
2. *Let $a :: b =_{df} (a : b)(b : a)$. Then $a :: b = \epsilon$ iff $a = b$.*

Proof.

1. $a \leq b$ iff $ab' = \eta$ by Theorem 2.3. Suppose $ab' = \eta$. Then $a : b = (ab')^\eta = \eta^\eta = \epsilon$.
   Conversely, suppose $a : b = \epsilon$. Then $(ab')^\eta = \epsilon$, so $(ab')^\pi = \eta$.
   But $ab' \leq (ab')^\pi$ by T, so $ab' = \eta$.
2. if $a :: b = \epsilon$ then $a : b = b : a = \epsilon$, so $a \leq b$ and $b \leq a$, whence $a = a.b = b$.
   Conversely, if $a = b$ then $a :: b = a :: a = a : a = (aa')^\eta = \eta^\eta = \epsilon$. □

THEOREM 4.2.

1. *$a^\epsilon = a :: \epsilon$;*[30]
2. *$a : \epsilon = \epsilon$;*[31]
3. *$a^\eta = a :: \eta = a : \eta$.*[32]

Proof.

1. $a :: \epsilon = (a : \epsilon)(\epsilon : a) = (a\epsilon')^\eta(\epsilon a')^\eta = (a\eta)^\eta(\epsilon a')^\eta$
   $= \eta^\eta(\epsilon a')^\eta = \epsilon(\epsilon a')^\eta = (\epsilon a')^\eta = a'^\eta = a^\epsilon$.
2. $a : \epsilon = (a\epsilon')^\eta = (a\eta)^\eta = \eta^\eta = \epsilon$.
3. $a :: \eta = (a : \eta)(\eta : a) = (a\eta')^\eta(\eta a')^\eta = a^\eta.\eta^\eta = a^\eta = (a\eta')^\eta$
   $= a : \eta$. □

LEMMA 4.2. *Let $a \supset b =_{df} a' + b$ ($\supset$ is material implication). Then*

$$a.b \leq c \text{ iff } a \leq b \supset c.$$

Proof. Suppose $a.b \leq c$.

$$\begin{aligned} \text{Then } a = a(b' + b) = ab' + ab &= a(b' + ab) \\ &\leq b' + a.b \leq b' + c \qquad \text{by Lemma 2.3} \\ &= b \supset c. \end{aligned}$$

Conversely, suppose $a \leq b \supset c = b' + c$.

$$\begin{aligned} \text{Then } a.b &\leq (b' + c).b \qquad \text{by Lemma 2.3} \\ &= b'.b + c.b = b.c \leq c. \end{aligned}$$

□

[30] MacColl 1901, p. 144 (10).
[31] MacColl 1998, 6 Oct 1901.
[32] MacColl 1901, p. 144 (11).

LEMMA 4.3. $(a \supset b)^\epsilon \leq a^\epsilon \supset b^\epsilon$.

Proof. Note that $a.(a' + b) = a.a' + a.b = ab \leq b$.

So $a^\epsilon.(a' + b)^\epsilon = (a.(a' + b))^\epsilon$ by Theorem 3.1 (1)

$\leq b^\epsilon$ by Theorem 3.1 (3).

So $(a \supset b)^\epsilon = (a' + b)^\epsilon \leq a^\epsilon \supset b^\epsilon$ by Lemma 4.2. □

THEOREM 4.3. $a^\epsilon.(a : b) \leq b^\epsilon$.[33]

Proof. By Lemma 4.3, $a : b \leq a^\epsilon \supset b^\epsilon$.

So by Lemma 4.2, $a^\epsilon.(a : b) \leq b^\epsilon$. □

We have now shown that MacColl's logic was at least as strong as the modal logic T. The three principles which are crucial to this are:

Theorem 3.1 (1)	$(a.b)^\epsilon = a^\epsilon.b^\epsilon$	i.e., K
Theorem 3.1 (4)	$a^\epsilon \leq a$	i.e., T
Theorem 3.2 (3)	$\eta^\eta = \epsilon$	i.e., N.

Let us show that these results are each equivalent to the principles stated. We can see from the results adduced that each of the principles K, T and N entails the results given. Conversely: first, suppose

$$(a.b)^\epsilon = a^\epsilon.b^\epsilon. \tag{*}$$

Then $(a + b)^\pi = (a'b')'^\pi = (a'b')^{\epsilon\prime} = (a'^\epsilon.b'^\epsilon)'$ by (*)

$= (a^\eta b^\eta)' = a^{\eta\prime} + b^{\eta\prime} = a^\pi + b^\pi$,

i.e., (*) entails K.

Next, suppose

$$a^\epsilon \leq a. \tag{**}$$

We need to derive T, *viz* $a \leq a^\pi$. Substituting a' for a in (**), we have $a'^\epsilon \leq a'$, so by Theorem 2.4 (3), $a'' \leq a'^{\epsilon\prime}$, whence $a \leq a^\pi$ by Theorem 2.3 (3) and Theorem 3.1 (1).

Finally, suppose

$$\eta^\eta = \epsilon. \tag{***}$$

We need to derive N, *viz* $\eta^\pi = \eta$.

From (***), $\eta = \epsilon' = \eta^{\eta\prime} = \eta^{\pi\prime\prime}$ by Definition 3.4 (1)

$= \eta^\pi$.

Might MacColl's calculus be stronger than T? There is good reason to think not. For T is among the strongest systems in which there are infinitely many modalities. Any stronger system would contain reduction laws, such as $a^{\pi\pi} = a^\pi$. But MacColl makes no reference

[33] MacColl 1906, p. 9 § 13; cf. Spencer 1973, p. 57 (10).

to any such reduction.[34] Note that η and ϵ behave differently when used as exponents and as formulae themselves. For $a^{\eta\epsilon}$, for example, means $(a^{\eta})^{\epsilon}$, not $a^{(\eta\epsilon)}$, so the fact that, say, $\eta\epsilon = \eta$ is irrelevant to such possible reductions of exponents. In MacColl 1903a, p. 361, he considers the formula $a^{\theta\theta\epsilon} + a^{\theta\theta\eta} + a^{\theta\theta\theta}$, but makes no suggestion that the complex modalities can be reduced. In fact, in MacColl 1897 he explicitly rejects $a^{\epsilon} : a^{\epsilon\epsilon}$ (the characteristic axiom of S4) and its like:

> when the statement α or β may belong sometimes to one and sometimes to another of the three classes ϵ, η, θ, the formulae $(\alpha : \beta)^{\epsilon} : (\alpha : \beta)$ and $(\alpha : \beta)^{\eta} : (\alpha : \beta)'$ will of course still be valid, but *not* always the converse formulae $(\alpha : \beta) : (\alpha : \beta)^{\epsilon}$ and $(\alpha : \beta)' : (\alpha : \beta)^{\eta}$. Similarly, we may still accept $\alpha^{\epsilon\epsilon} : \alpha^{\epsilon}$, $\alpha^{\epsilon\eta} : \alpha^{\epsilon\iota}$ [i.e., $\alpha^{\epsilon\eta} : \alpha^{\epsilon\prime}$], $\alpha^{\eta\eta} : \alpha^{\eta\iota}$, &c., as valid, but *not* their converses $\alpha^{\epsilon} : \alpha^{\epsilon\epsilon}$, $\alpha^{\epsilon\iota} : \alpha^{\epsilon\eta}$, $\alpha^{\eta\iota} : \alpha^{\eta\eta}$, &c. (MacColl 1897, p. 579)

In none of his calculations does he try to reduce the number of modalities by such laws.

McCall (1967) claims that MacColl's system was "in many respects identical to Lewis' system *S3*" (p. 546). But the characteristic axiom of S3 does not figure in the nine theses McCall attributes to MacColl[35]—indeed, if it did, then since MacColl's logic is normal, as shown above (i.e., $\eta^{\pi} = \eta$), there would ensue reduction theses such as $a^{\pi\pi} = a^{\pi}$, characteristic of S4, since S4 is the union of S3 and T. Since MacColl explicitly endorses normality and denies any reduction laws, his logic is T.

5. The Paradoxes of Implication

MacColl introduced his connective ':' out of dissatisfaction with the material implication $\supset$ of the "Boolian" logicians. So it is important to him that his algebraic analysis reject the following formulae:[36]

$$(a : b) + (b : a) \tag{1}$$

and

$$(ab : c) : ((a : c) + (b : c)). \tag{2}$$

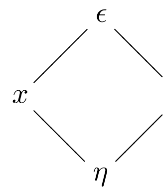
We can show, with a suitable MacColl algebra, and suitable assignments to a, b and c, that we can set (1) and (2) different from ϵ.

[34] See Spencer 1973, pp. 26–27. Spencer infers that *if* MacColl's system is any of Lewis', it will be S1–S3. But as we have seen, it is not.

[35] The nine theses either follow immediately from Definition 3.4 or are proved in or follow from Theorems 3.1, 3.3, 4.1 and 4.2.

[36] Spencer 1973, p. 57 (17) and (18).

Let M be based on the Boolean algebra:



with operations $^\pi$ and : defined by the tables:

$a : b$	ϵ	x	x'	η	a^π
ϵ	ϵ	x	η	η	ϵ
x	ϵ	ϵ	η	η	ϵ
x'	ϵ	x	ϵ	x	x'
η	ϵ	ϵ	ϵ	ϵ	η

(The table for : is of course derivative from that for $^\pi$.) So $x'^\epsilon = \eta$. M is a MacColl algebra. Let $a = x$ and $b = x'$. Then (1) $(a : b) + (b : a) = (x : x') + (x' : x) = x'^\epsilon + x^\epsilon = \eta + x = x \neq \epsilon$. So (1) is invalid in M.

M also serves to invalidate (2). Let $a = x$, $b = x'$ and $c = \eta$. Then $(ab : c) : ((a : c) + (b : c)) = (xx' : \eta) : ((x : \eta) + (x' : \eta)) = (xx'\eta')^\eta : ((x' + \eta)^\epsilon + (x'' + \eta)^\epsilon) = (\eta\epsilon)^\eta : (x'^\epsilon + x^\epsilon) = \eta^\eta : (\eta + x) = \epsilon : x = (\eta + x)^\epsilon = x^\epsilon = x \neq \epsilon$.[37]

Unsurprisingly, therefore, MacColl's theory of implication avoids the so-called paradoxes of material implication. The following are invalid in M:

$$b : (a : b) \tag{3}$$

and

$$a' : (a : b). \tag{4}$$

In the case of (3), let $a = x$ and $b = x'$ in M; then $b : (a : b) = x' : (x : x') = x' : (x' + x')^\epsilon = x' : x'^\epsilon = x' : \eta = (x'' + \eta)^\epsilon = x^\epsilon = x \neq \epsilon$. The same assignment invalidates (4) as well, for then $a' : (a : b) = x' : (x : x') = x \neq \epsilon$.

MacColl gives natural language examples to support this rejection of material implication as the correct account of implication. He suggests letting a = 'He will persist in his extravagance' and b = 'He will be ruined'. Then (3) is rejected because even if he is ruined, we may

[37] MacColl (1906, § 70 pp. 74–5) gives a counterexample to (2). See also MacColl 1903a, p. 362. Cf. Shearman 1906, pp. 29–30.

still hold that he might have persisted in his extravagance and not have been ruined; and (4) is rejected because, even if he does not persist in his extravagance, we may again hold that he might have persisted and still not have been ruined. On a material account of implication these thoughts are simply contradictory, for $(a \supset b)' = ab'$, which contradicts both b and a', leaving no room to distinguish between supposing he might persist and not be ruined and supposing he does persist and is not ruined. The conditional, says MacColl, contains a modal element, revealed by negating it. $(a : b)' = (ab')^{\eta\prime} = (ab')^{\pi}$, that is, it is possible that a (he persists) and b' (he is not ruined).

Nonetheless, this analysis does open MacColl (as it did Lewis[38]) to the so-called paradoxes of strict implication.

Theorem 5.1. [39]

1. $a : \epsilon = \epsilon$;

2. $\eta : a = \epsilon$;

3. $b^{\epsilon} : (a : b) = \epsilon$;

4. $a^{\eta} : (a : b) = \epsilon$.

Proof.

1. $a : \epsilon = (a' + \epsilon)^{\epsilon} = \epsilon^{\epsilon} = \epsilon$.
2. $\eta : a = (\eta a')^{\eta} = \eta^{\eta} = \epsilon$.
3. $b \leq a' + b$, so $b^{\epsilon} \leq (a' + b)^{\epsilon}$ by Theorem 3.1 (3)
   $= a : b$.
   So $b^{\epsilon} : (a : b) = \epsilon$ by Lemma 4.1 (1).
4. $ab' \leq a$, so $a' \leq (ab')'$ by Theorem 2.4 (3)
   whence $a^{\eta} = a'^{\epsilon} \leq (ab')'^{\epsilon}$ by Theorem 3.1 (3)
   $= (ab')^{\eta} = a : b$.
   So $a^{\eta} : (a : b) = \epsilon$ by Lemma 4.1 (1). □

In fact, it is a mistake simply to identify Theorem 5.1 (1) and (2) with the paradoxes of strict implication. For all they say is that there is a maximum (weakest) proposition (ϵ) implied by all others, and a minimum (strongest) proposition (η) which implies all others. Such so-called "Church constants" can in fact be added conservatively to relevance logic, that is, they can be added without necessarily disturbing the relevance features of the implication relation between other propositions.[40] It is (3) and (4) from Theorem 5.1 which exhibit a wider

[38] Lewis 1918, pp. 335 ff.; Lewis and Langford 1932, pp. 175 ff.
[39] MacColl 1901, p. 143 (3); Spencer 1973, p. 57 (12), (13).
[40] Anderson and Belnap 1975, § 27.1.2.

spread of irrelevance, that any necessary proposition (not just ϵ) is implied by any other, and that any impossible proposition (not just η) implies any other.

In a letter to Russell, MacColl says that

> [i]t is true that in ordinary speech the conjunction *if* usually suggests some necessary relation between the two sentences it connects; but the exigencies of logic force us to adhere to our definition, $A : B = (AB')^{\eta}$ and disregard this suggested relation. (MacColl 1998, 19 July 1901)

But this is an overstatement. If we choose to adhere to MacColl's definition, the exigencies of logic do indeed force us to disregard the suggested relation. But we might choose to explore an alternative definition. Elsewhere, MacColl dismisses this as psychologism.[41] But his own example in the letter to Russell shows that it is not a fair charge. He instances three (large) numbers, a, b and c, where $ab \neq c$ but not obviously so. Nonetheless, urges MacColl, Russell should concede, even before calculating the product of a and b, that

(5) $$\text{if } ab = c \text{ then } 2ab = 2c.$$

But he goes on to observe that what makes (5) true is the impossibility of the equation $ab = c$ (since $ab \neq c$). But that undercuts his demand that Russell concede (5) before calculating. Obviously, if $ab = c$ then $2ab = 2c$; whereas it is not clear that, say, if $ab = c$ then $2ab = 7c$. (For if $ab = c$ and $2ab = 7c$, then $c = 0$ and so $a = b = 0$ too.) MacColl starts his example by recognising the relevance of implication, even though he ends by denying it.

To avoid even the irrelevance of the paradoxes of strict implication, one has to take a further step not contemplated by MacColl or Lewis. The source of their failure here lies in the fact that ':' is not *dyadically* intensional. It is the modalization of a truth-function. The truth-function 'and not' is dyadic; but the modal operator 'is impossible' is monadic. Sugihara (1955) produced a matrix to sieve out maximal and minimal formulae in implications; and Meyer (1974)[42] showed that no modalization of a truth-function could capture implication in any logic contained in that characterized by the Sugihara matrix, *viz RM*.[43] Not until implication is introduced by a truly dyadic intensional operator can the paradoxes of strict implication be excluded.

[41] MacColl 1906, §§ 77–8, pp. 81–3.
[42] Cf. Anderson and Belnap 1975, § 29.12.
[43] See also Anderson and Belnap 1975, § 27.1.1.

Definition 5.1. *A* semi-group *consists of a set of elements closed under an associative operation, $\circ$ (fusion): $\langle A, \circ\rangle$ such that*

$$a \circ (b \circ c) = (a \circ b) \circ c \quad \textit{for all } a, b, c \in A. \qquad \textit{(Associativity for } \circ)$$

Definition 5.2. *A* monoid *is a semi-group with an identity, ϵ: $\langle A, \circ, \epsilon\rangle$ such that*

$$a \circ \epsilon = a = \epsilon \circ a. \qquad \textit{(Identity)}$$

Definition 5.3. *A semi-group is* commutative *if*

$$a \circ b = b \circ a. \qquad \textit{(Commutativity for } \circ)$$

Definition 5.4. *A* lattice-ordered semi-group *consists of a set of elements closed under meet, join and fusion: $\langle A, ., +, \circ\rangle$ such that $\langle A, ., +\rangle$ is a lattice, $\langle A, \circ\rangle$ is a semi-group and*

$$a \circ (b + c) = a \circ b + a \circ c \textit{ for all } a, b, c \in A \qquad \textit{(Distribution of } \circ \textit{ over } +)$$

Definition 5.5. *A lattice-ordered semi-group A is* residuated *if $\forall a, b \in A, \exists x, y \in A$ such that*

$$\forall c \in A, c \leq x \textit{ iff } c \circ a \leq b$$

and

$$\forall c \in A, c \leq y \textit{ iff } a \circ c \leq b.$$

We write $x = a \to b$ and $y = b \leftarrow a$.

Lemma 5.1. *If a lattice-ordered semi-group is commutative, $a \to b = b \leftarrow a$.*

Proof. $c \leq a \to b$ iff $c \circ b \leq a$ iff $b \circ c \leq a$ iff $c \leq b \leftarrow a$. So by Lemma 2.4, $a \to b = b \leftarrow a$. □

Definition 5.6. *A lattice-ordered semi-group A is* square-increasing *if $a \leq a \circ a$ for all $a \in A$.*

Definition 5.7.[44] *A* De Morgan monoid $\langle A, ., +, ', \circ, \epsilon\rangle$ *consists of a set of elements closed under meet, join, complement and fusion, such that $\langle A, ., +, \circ\rangle$ is a commutative square-increasing lattice-ordered monoid, the lattice $\langle A, ., +\rangle$ is distributive and for all $a, b \in A$:*

$$\textit{if } a \leq b \textit{ then } b' \leq a' \qquad \textit{(Contraposition)}$$

$$a'' = a \qquad \textit{(Double Negation)}$$

and

$$a \circ b \leq c \textit{ iff } b \circ c' \leq a' \textit{ iff } c' \circ a \leq b' \qquad \textit{(Antilogism)}$$

[44] Anderson and Belnap 1975, § 28.2.

LEMMA 5.2. *In a De Morgan monoid*

$$(a+b)' = a'.b'$$

Proof. Since $a \leq a+b$ and $b \leq a+b$,

$(a+b)' \leq a'$ and $(a+b)' \leq b'$ by Contraposition

so $(a+b)' \leq a'.b'$.

Conversely, since $a'.b' \leq a'$ and $a'.b' \leq b'$,

$a = a'' \leq (a'.b')'$ and $b = b'' \leq (a'.b')'$ by Contraposition

so $a+b \leq (a'.b')'$, whence $a'.b' = (a'.b')'' \leq (a+b)'$.

So $(a+b)' = a'.b'$. □

THEOREM 5.2. *Each De Morgan monoid is residuated.*

Proof. Take $a, b \in A$, the De Morgan monoid. Then $\forall c \in A$,

$c \circ a \leq b$ iff $a \circ b' \leq c'$ by Antilogism

iff $c \leq (a \circ b')'$ by Contraposition

Hence $a \rightarrow b = (a \circ b')'$

$= b \leftarrow a$ by Lemma 5.1, since A is commutative. □

De Morgan monoids give the algebraic structure of the logic of relevant implication, R, where the residual $a \rightarrow b$ expresses relevant implication. The logic $\mathrm{R}^\Box$ adds to R an S4-necessity. In $\mathrm{R}^\Box$, a modal relevant implication (entailment), $a \,\Box\!\!\rightarrow b$, can be defined as $\Box(a \rightarrow b)$, equivalently, $(a \circ b')^\eta$, where $a^\eta =_{df} \Box(a')$, i.e., a'^ϵ. The algebra of $\mathrm{R}^\Box$ adds to De Morgan monoids a closure operation, $^\pi$ (possibility), as in the modal algebras above.[45]

DEFINITION 5.8. *A* modal l-monoid $\langle A, ., +, ', ^\pi, \circ, \epsilon\rangle$ *consists of a set of elements closed under meet, join, complement, possibility and fusion, such that* $\langle A, ., +, ', \circ, \epsilon\rangle$ *is a De Morgan monoid, and*

$$(a+b)^\pi = a^\pi + b^\pi \qquad (K)$$
$$a \leq a^\pi \qquad (T)$$
$$\eta^\pi = \eta \qquad (N)$$
$$a^{\pi\pi} \leq a^\pi \qquad (4)$$

and

$$a^\pi \circ b^\epsilon \leq (a \circ b)^\pi \qquad (MP)$$

where $\eta = \epsilon'$ *and* $a^\epsilon = a'^{\pi\prime}$.

One could of course drop the postulate (4) if one wanted an algebra without reduction theses, more in MacColl's tradition.

[45] Anderson and Belnap 1975, § 28.2.5.

THEOREM 5.3.

1. *Recall the definition of $a \supset b$ as $a' + b$. It follows that*

$$(a \supset b)^\epsilon \leq a^\epsilon \supset b^\epsilon;$$

2. *(MP) entails that*

$$(a \to b)^\epsilon \leq a^\epsilon \to b^\epsilon.$$

Proof.

1. The proof of Lemma 4.2 remains sound.
2. Recall that $a \to b = (a \circ b')'$. From (MP) we have (with b' for a and a for b)

$$b'^\pi \circ a^\epsilon \leq (b' \circ a)^\pi.$$

So $(a \to b)^\epsilon = (a \circ b')'^\epsilon = (a \circ b')^{\pi\prime}$
$\leq (b'^\pi \circ a^\epsilon)' = (a^\epsilon \circ b^{\epsilon\prime})' = a^\epsilon \to b^\epsilon.$ □

We can show, by use of the following modal l-monoid, N, that the paradoxes of strict implication are invalidated. Let N be based on the same Boolean algebra as M, with the operation $^\pi$ as before, but now defining $\circ$ independently:

$a \circ b$	ϵ	x	x'	η	a^π
ϵ	ϵ	ϵ	ϵ	η	ϵ
x	ϵ	x	x'	η	ϵ
x'	ϵ	x'	ϵ	η	x'
η	η	η	η	η	η

Then $\to$ and $^\epsilon$ are given by the tables:

$a \to b$	ϵ	x	x'	η	a^ϵ
ϵ	ϵ	η	η	η	ϵ
x	ϵ	x	x'	η	x
x'	ϵ	η	x	η	η
η	ϵ	ϵ	ϵ	ϵ	η

Consequently, in N, let $a = x'$ and $b = x$. Then $b^\epsilon \to (a \to b)^\epsilon = x^\epsilon \to (x' \to x)^\epsilon = x \to \eta^\epsilon = x \to \eta = \eta$. Similarly, $a^\eta \to (a \to b)^\epsilon = \eta$ for the same assignment to a and b. Thus the paradoxes of strict implication are avoided in $\mathrm{R}^\Box$.

6. Conclusion

In this paper, I have attempted to exhibit some of the richness of MacColl's logic by recasting it in terms of the modal algebras (closure and extension algebras) of McKinsey, Tarski and Lemmon. Within this algebraic framework, I have derived many of MacColl's characteristic theses. Three of his theses, in particular, regarding necessity ($^{\epsilon}$) show that MacColl's logic was in fact the logic now known as T later introduced independently (of MacColl) by Feys (1937–1938) (his system t of § 28) and von Wright (1951) (his system M of Appendix II). It is interesting to speculate whether it was not the fact that MacColl was working within the Boole-Schröder algebraic paradigm which led him to normality ($\eta^{\pi} = \eta$, equivalently, $\epsilon^{\epsilon} = \epsilon$, von Wright's Rule of Tautology) and T, while Lewis' reformulation of modal logic within the proof-theoretic logistic of Frege, Peano and Russell took him away into the dead end of S1, S2 and S3.

I have explored only a small portion of MacColl's logic. There are many further original and fecund ideas remaining for investigation. I hope the framework I have developed here will prove a fruitful one for at least some of this exploration.

It should be noted, however, that I have left certain ideas deliberately unexplored because of an initial resistance to interpretation. Recall that I have used ϵ and η both as elements of the algebra and as operators (exponents). As I have used them, there is a systematic ambiguity. Nothing warranted use of the same symbol other than the two equations from Theorem 4.2:

$$a^{\epsilon} = a :: \epsilon$$

and

$$a^{\eta} = a :: \eta.$$

MacColl proceeds to use the connection between the element $a :: b$ and the equation $a = b$ expressed in Lemma 4.1 (2) to write these as[46]

$$a^{\epsilon} = (a = \epsilon) \tag{†}$$

and

$$a^{\eta} = (a = \eta) \tag{††}$$

and so to read a^{ϵ} not as an element but as expressing the validity of a, i.e., $a = \epsilon$, and similarly for η to express invalidity. (†) and (††) are

[46]MacColl 1901 pp. 143–4 (10) and (11).

ill-formed in my canon. For MacColl they support the identification of ϵ and η as element and exponent.

So far I can follow him, though only by the systematic ambiguity noted. However, MacColl proceeds to introduce θ as an element too, corresponding to the operator, $^{\theta}$. Thus he claims, for example,

$$\theta^{\epsilon} = \theta^{\eta} = \eta,$$ [47]

that is, it is impossible that a variable (contingent) element be either certain (ϵ) or impossible (η). It is an interesting question whether this idea can be so expressed in the language of modal algebras.

A different direction for research on MacColl's ideas would be to take further the suggestions I made in § 5, to develop an implication which is truly dyadic. The theory of modal l-monoids, combining the ideas of modal (i.e., closure and extension) algebras and De Morgan monoids, is largely unexplored.

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[47] MacColl 1901 p. 140.

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MACCOLL AND MANY-VALUED LOGIC: AN EXCLUSIVE CONJUNCTION

1. Rescher's Statement

In his valuable compendium *Many-Valued Logic* Nicholas Rescher states (1969, p. 4) that the founding fathers of many-valued logic prior to Lukasiewicz are Charles Sanders Peirce, Nicolai Vasil'ev and Hugh MacColl. This paper shows very simply, against Rescher, that MacColl's logic cannot reasonably be counted as many-valued.

2. Many-Valued Logic, What

A logic can be given a many-valued semantics and still not be *essentially* many-valued. Here is an example. A propositional logic C2 based on negation and conjunction is given a semantics with the following four-valued truth-tables (conjunction on the right):

	$\neg$	1	2	3	4
$*1$	4	1	2	3	4
2	3	2	2	4	4
3	2	3	4	3	4
4	1	4	4	4	4

The reason C2 is not essentially many-valued is that its tautologies and valid deductions coincide with those of classical two-valued logic C: the matrices of C2 are simply the Cartesian product of the bivalent matrices of classical logic with themselves. By contrast, if one adds a further connective Γ with the matrix

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	Γ
$*1$	2
2	2
3	4
4	4

then the resulting logic is essentially many-valued: it is in this case equivalent to Lukasiewicz's last many-valued and intendedly modal logic L (1953). A similarly essentially many-valued system is Lukasiewicz's first trivalent logic, which is poorer in tautologies than classical logic.

I say that a logical system L is *essentially many-valued* when any semantics with respect to which L is sound and complete is such that:

MV1 the semantic values or statuses of its sentences (closed wffs) include both true (T or 1) and false (F or 0) and at least one other value besides, distinct from T and F.

MV2 having the values T, F and any of the others are *pairwise exclusive* and *jointly exhaustive* (PEJE) of the semantic statuses of any sentence on any given valuation of sentences.

MV3 the connectives of L are *value-functional*, that is, for any connective K and any sentences $S_1, \ldots S_n$, the value of $K(S_1, \ldots , S_n)$ under a given interpretation I, which we write $|K(S_1, \ldots , S_n)|_I$, is a function of $|S_1|_I, \ldots , |S_n|_I$ alone, as determined by the fixed interpretation of K.

In the case where a logic has higher-order operators such as quantifiers the analogous principle to MV3 applies:

MV4 the value of a sentence containing an operator as main symbol is a value-function of the values of its instantiations.

It follows from these conditions that the tautologies and valid inferences of L do not coincide with those of classical logic, for if they did it could be given a bivalent semantics.

3. Why MacColl's Logic is Not Many-Valued

At first sight, the statuses of propositions in MacColl's logic make it look as though one can support the contention that his logic is essentially many-valued. In the definitive statement of his views in 'La logique symbolique' (1901, p. 138), he introduces five semantic values for propositions, giving them these glosses:

$$\tau - \text{true}, \iota - \text{false}, \epsilon - \text{certain}, \eta - \text{impossible}, \theta - \text{variable}$$

and on pp. 140-142 he asserts a number of equations linking them and their associated single-place connectives A^τ A^ι A^ϵ A^θ A^η (A is true, false, certain, variable, impossible):

$$\epsilon^\eta = \epsilon^\theta = \theta^\epsilon = \theta^\eta = \eta^\epsilon = \epsilon^\iota = \eta$$
$$\eta^\eta = \epsilon^\iota = \eta^\iota = \epsilon^\tau = \epsilon$$

All five values together cannot give a 5-valued semantics because τ and ι are PEJE: MacColl asserts that $A^\tau + A^\iota$ (where '+' stands for disjunction) and $(A^\tau A^\iota)^\eta$ (where juxtaposition stands for conjunction). They are clearly simply the two classical values so could not be considered unless added to others, which is ruled out by these principles. The three values ϵ, θ and η likewise but more promisingly form a PEJE set because

$$A^\epsilon + A^\theta + A^\eta, (A^\epsilon A^\eta)^\eta, (A^\epsilon A^\theta)^\eta, (A^\theta A^\eta)^\eta.$$

MacColl defines a strict implication connective: '$A : B$' is read as 'If A then B' and understood as synonymous with 'it is impossible that A and not B' or $(AB')^\eta$ where B' is the negation of B and defined as synonymous with B^ι. He affirms these implications

$$A^\epsilon : A^\tau \qquad A^\eta : A^\iota$$

the first being akin to the modal formula T. Since on p. 144 formula (10) MacColl also affirms that $A^\epsilon = (A = \epsilon)$ presumably also

$$\epsilon^\epsilon = \theta^\theta = \epsilon \text{ and } \theta^\eta = \eta$$

so since $\tau^\tau = \epsilon$ and $\iota^\tau = \eta$ (for as terms and factors τ and ι are said by MacColl to be equivalent respectively to ϵ and η—p. 140 ftn. 3), in general

$$\alpha^\beta = \epsilon \text{ if } \alpha = \beta$$
$$\alpha^\beta = \eta \text{ if } \alpha \neq \beta$$

and the following seemingly value-functional connectives seem to emerge (the last two values for A^θ being as conjectured):

A	A^ϵ	A^θ	A^η	A^τ	A^ι
$*\epsilon$	ϵ	η	η	ϵ	η
θ	η	ϵ	η	θ	θ
η	η	η	ϵ	η	ϵ

The connective A^ϵ looks then like a "strong assertion" functor: it gives the strongly designated value ϵ when its argument has value ϵ and the strongly anti-designated value η otherwise.

But ϵ, θ and η do not sustain a value-functional semantics because the implication connective written ':' has an incomplete matrix with respect to the values as follows

			A	
	$A : B$	ϵ	θ	η
	$*\epsilon$	ϵ	η	η
B	θ	ϵ		η
	η	ϵ	ϵ	ϵ

$|A : B|$ can be anything for $|A| = \theta = |B|$. Suppose A is 'It is raining' then for $B = A, |A : B| = \epsilon, |A : B'| = \eta$, while if B is 'It is Wednesday', independent of $A, |A : B| = \theta$.

Similarly, the matrix for conjunction is incomplete:

			A	
	AB	ϵ	θ	η
	$*\epsilon$	ϵ	θ	η
B	θ	θ		η
	η	η	η	η

Hence neither the three nor all five values on offer provide a value-functional semantics for the important implication and conjunction connectives.

4. MacColl's as a Modal Probability Logic

The intended and stated intepretations of A^ϵ, A^η and A^θ are as probability propositions

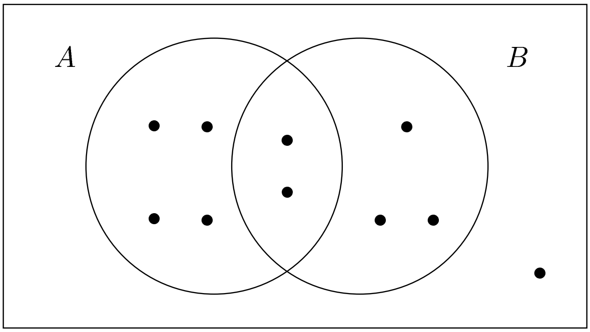
$$\begin{array}{ll} A^\epsilon & P(A) = 1 \\ A^\eta & P(A) = 0 \\ A^\theta & 0 < P(A) < 1 \end{array}$$

where MacColl distinguished between formal and material necessity (certainty), impossibility and variability. The uncertainty attaching to the interpretation of some of MacColl's constants and connectives supports Russell's contention (1906, p. 256 f.) that MacColl fails clearly to distinguish between propositions and propositional functions. Nevertheless, with some charity (which Russell was unwilling to dispense to a proponent of modal logic) MacColl's general intentions are clear enough as outlined above.

If we interpret the three functors as above, then the lack of value-functionality is immediately explained: the probability function P is not value-functional: $P(AB)$ is not a function of $P(A)$ and $P(B)$, but satisfies merely the inequality $0 \leq P(AB) \leq \min\{P(A), P(B)\}$. If $P(A) = 0.4$ then $P(A') = 0.6$ and $P(AA') = 0$. The summation law for probability

$$P(A) + P(B) = P(A + B) + P(AB)$$

was known to MacColl: it follows from the Kolmogorov axioms. If we take A and B as finite sets given by the Venn diagram below, where $P(X)$ gives the probability that a dot chosen at random is within the area X and $P(X')$ gives the probability that such a dot is in the complementary area to X,



then here $P(A) = 0.6$, $P(B) = 0.5$, $P(A+B) = 0.9$, $P(AB) = 0.2$ and $P(A/B) = P(AB)/P(B)$, here 1/3, again a fact known to MacColl (cf. 1901, p. 154).

Because the intended and actual application of MacColl's logic is to probabilities, then despite there being many "values" for propositions it is not value-*functional*, so it is misleading to regard MacColl's logic as essentially many-valued. Rather it is a modal logic of probability, which is not fully value-functional. It is true that not just in MacColl's day but for some time afterwards, logicians such as Lukasiewicz did not clearly distinguish probability logic from many-valued logic. In his *Grundlagen der Wahrscheinlichkeitsrechnung* of 1913, and even after developing many-valued logic as such, Lukasiewicz still tends to run the two together (as in Lukasiewicz 1930, cf. Lukasiewicz 1970, p. 173). Since the point of a many-valued system is to interpret the logical constants in a way analogously with that of bivalent logic, that point is lost if value-functionality goes. Instead one is dealing as here with a modal rather than a many-valued system, even if we use a plurality (greater than two) of other "statuses", as e.g. when talking about truth "at" several different possible worlds in the standard semantics for modal logic.

Ironically, a philosopher whose views on the several values a proposition may have (including others apart from true and false) were also forged in conjunction with a theory of probability was Alexius Meinong, whose work was influential on Lukasiewicz. As I have shown elsewhere (Simons 1989), Meinong's clear affirmation of values for propositions other than the two classical ones makes him, though not himself a logician, a precursor of Lukasiewicz's work and a founding father of many-valued logic with greater title to this status than MacColl.

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CONTEXT-SENSITIVITY AND THE TRUTH-OPERATOR IN HUGH MACCOLL'S MODAL DISTINCTIONS

MacColl finds the logical foundation for his modal distinctions in the assumption that propositions can change their truth-value depending on context. This opens the way for the introduction of a non-redundant truth-operator. Following MacColl, the expressions A^τ ("A is true") and A should be strictly equivalent without being synonymous. In the paper, a semantic treatment of MacColl's truth-operator is developed which fulfils these formal claims and is in accordance with the logical intuition of MacColl concerning his modal distinctions. According to this treatment, the following features of MacColl's truth-operator are stated: A^τ reports the belonging of the value *true* to A in the given situation. Depending on the context of the occurrence of A^τ in an expression B, the value of A in a given situation is transmitted to other situations which should be considered in order to evaluate B. By this context-fixation, the use of A^τ produces a kind of rigid designation (or rigid valuation). Consequently, in the corresponding *de-re*-treatment of the truth-operator there is a difference between being true and being certain, but there is no difference between being true and being certain of being true. This *de-re*-use of the truth-operator is accompanied by a *de-dicto*-treatment in which the strict equivalence between A^τ and A holds.

1. Introduction

One of the main divergences of the logical intuition developed by MacColl in his *Symbolic Logic*, compared to the classical intuition of Frege consists in the fact that MacColl takes for granted that a distinction between *true* and *certain*, *false* and *impossible* is of fundamental logical relevance and that there are *variable* propositions (or statements) which are sometimes true and sometimes false. In these differences MacColl finds the basis for his modal logic of *Symbolic Logic and its Applications* (1906) with the strict implication introduced there.[1]

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MacColl finds room for his modal distinctions in orienting his logical efforts to language in use, rather than to the treatment of language as just a static system of signs. In contrast to the Fregean Aussage (proposition), a statement in MacColl's treatment is a linguistic entity, an *intelligible arrangement of words*, which is physically located in space and time and which can be psychologically perceived. One can say that this linguistic turn is fundamental to MacColl's non-classical achievements. In natural language one is immediately confronted with pragmatic characterizations like ambiguity, context-sensitivity, linguistic acts, epistemic dimensions, soundness claims, justification procedures, etc. With respect to these characterizations (but not in relation to all of them), MacColl justifies the usefulness and the logical relevance of his modal distinctions. His fundamental modal characterizations do not rest on ambiguity (one accusation of Russell's against MacColl's logical enterprise[2]). MacColl finds the main source of his non-classical departure in the context-sensitivity of language and, additionally, in different kinds of epistemic justification.

The context-sensitivity in MacColl consists not only in the presence of different applicationary or justificationary contexts, but also in a syntactic context-sensitivity of the expressions in his symbolic systems. MacColl underlines this aspect in a letter to Russell dated May 15, 1905. He writes: "This enormous superiority of my system is due in great measure to the very principle which you find so defective, namely, the principle of leaving to *context* everything in the reasoning or symbolical operations which it is not absolutely necessary to express." The application of this internal context-sensitivity principle reflects a confidence in the ability of the prospective reader to find the right connections and in his willingness to try this with charity. Nevertheless, for some devices introduced by MacColl it is difficult to catch the right connections and to come up with any sound and adequate explanation in the sense of MacColl. One very resistant place is the logical distinction between an expression A and the expression predicating truth of A, "A is true" (A^τ). Already in the 1960s Storrs McCall

[1] In the course of his intellectual life, the logical views of MacColl, especially those concerning implication and modality, went through important changes. This paper does not focus on those developments, but tries to give an explication of MacColl's views concerning these topics in his main and almost final work (MacColl 1906). The intended explication should be coherent with the views uttered and the formal claims presented there. So I will not focus on the fact that MacColl held different views about modality and implication in different periods of his logical work. Even in 1906 one is confronted with the traces of views formerly favored by MacColl. For detailed information see: Astroh 1993, Rahman 1997, Rahman and Christen 1997.

[2] See Russell's review in *Mind* (1906).

closed his report about MacColl in the *Encyclopedia of Philosophy* with the words: "The many other idiosyncracies in MacColl's system, such as ... his distinction between a and a^{τ}, still await a competent interpreter" (McCall 1967, p. 546). It seems that this statement has not lost its correctness.[3] In the remainder of this paper I try to take some steps in the direction of a sound interpretation of the truth-predicate in the sense of MacColl 1906, i.e., in a sense in which MacColl would have interpreted this predicate, and which delivers more than just the indication of an intuitively possible solution, as I did in my paper of 1993 (Stelzner 1993).

MacColl finds the logical foundation for his modal distinctions in the specialty that the propositions treated in his system can change their truth-value depending on context:

> Some logicians say [Russell, e.g., did this in his review of MacColl's *Symbolic Logic and its Applications* (Russell 1906)] that it is not correct to speak of any statement as "sometimes true and sometimes false"; that if true, it must be true always; and if false, it must be false always. To this I reply ... that when I say "A is sometimes true and sometimes false", or "A is a *variable*," I merely mean that the symbol, word, or collection of words, denoted by A sometimes represents a truth and sometimes an untruth. For example, suppose the symbol A denotes the statement "Mrs. Brown is not at home." This is neither a formal certainty, like $3 > 2$, nor a formal impossibility, like $3 < 2$, so that *when we have no data but the mere arrangement of words*, "Mrs. Brown is not at home," we are justified in calling this *proposition*, that is to say, *this intelligible arrangement of words*, a variable, and in asserting A^{θ} ["A is a variable"]. ... To say that the proposition A is a *different proposition* when it is *false* from what it is when it is *true*, is like saying that Mrs. Brown is a *different person* when she is *in* from what she is when she is *out.* (MacColl 1906, pp. 18 f.)

According to this, we can have the same proposition in different contexts of use. The truth-value for the statements or propositions depends on the context of the application of such statements. A proposition in the Fregean-Russellian sense could be interpreted against the background of MacColl's view as a pair $\langle \text{statement}, \text{context} \rangle$. For such a pair we have a fixed truth-value, i.e. (in Frege) a fixed meaning (Bedeutung) of the proposition (Aussage), and the proposition appears to be the sense (Sinn) of the statement (Aussagesatz). Following Russell,

[3] One witness to this is Shahid Rahman, who writes: "Schließlich müssen viele seiner Ideen noch kritisch erarbeitet werden, wie z.B. sein Argument für die Nicht-Synonymie der Ausdrücke 'A' und 'A ist wahr', die eine entscheidende Rolle in seinem Begriff der pure logic spielt." ["Eventually, many of his ideas should be critically worked out, e.g., his argument for the non-synonymy of the expressions 'A' and 'A is true', which plays a decisive role in his notion of pure logic."] (Rahman 1997, p. 166)

one could conclude from this: If we give MacColl the classically right entities, namely truth-value definite propositions, to be treated in his modal system, then the modal distinctions between *true* and *certain* and between *false* and *impossible* will be superficial and MacColl's modal logic will collapse to classical logic.

Getting those classically right entities could be managed in two ways: Confining MacColl's system by the exclusion of variable statements[4] or extending the system by the introduction of symbolic tools expressing different contexts and pairs between statement and context. Neither way, I think, is in the spirit of MacColl: The general exclusion of variable statements would lead away too much from natural language use. In addition, the expression of contexts seems not to be necessary for MacColl, because he is confident that one should be able to recognize the actual context-relatedness in the context of use of a statement.

Not only I, but MacColl too would have had to agree with the above redundancy claim for both positive and negative modal distinctions if his modal distinctions rested only on context-sensitivity. But the modal distinctions in MacColl's treatment do not vanish, because, firstly, MacColl recognizes the general confinement to truth-definite propositions as unnatural and not logically forced, and, secondly, even confining our considerations to truth-value definite propositions, the modal distinctions do not vanish when we take into account MacColl's second source of this differentiation: namely, the kinds and strength of justification we have for the truth or falsehood of a (maybe truth-value definite) proposition or statement:

> A proposition is called a formal certainty when it follows necessarily from our definitions, or our understood linguistic conventions, without further data ... It is called a material certainty when it follows necessarily from some special data not necessarily contained in our definitions. (MacColl 1906, p. 97)

Contrary to the certainties mentioned here, MacColl's truth-predicate in A^τ does not depend on a kind of justification. A^τ just reports that A is true:

> The symbol A^τ only asserts that A is *true* in a particular case or instance. The symbol A^ε asserts more than this: it asserts that A is *certain*, that A is always true (or true in every case) within the limits of our data and definitions, that its probability is 1. The symbol A^ι only asserts that A is false in a particular case or instance; it says nothing as to the truth or falsehood of A in other

[4] MacColl considers this case, when in chapter IV he writes: "For the rest of this chapter we shall exclude the consideration of variables, so that A, A^τ, A^ε will be considered mutually equivalent, as will also A', A^ι, A^η." (MacColl 1906, p. 21)

instances. Thus A^τ and A^ι are simply assertive; each refers only to one case, and raises no general questions as to data or probability. (MacColl 1906, p. 7)

The explication given in the above quotation is sharpened by two formal demands which should be fulfilled in relation to A^τ and A: In the treatment intended by MacColl (1906), the expressions A^τ and A should be strictly equivalent (this is symbolized by $A^\tau = A$), but, nevertheless, A should not be synonymous with A^τ (i.e., it does not hold logically that A^τ can be replaced salva veritate in every context of its occurrence by A). MacColl demonstrates this non-synonymy of A and A^τ by examples, but these cannot suffice as full-fledged explanation of these predicates.[5]

The use of the truth-predicate in MacColl 1906 has an extensive pre-history, beginning with MacColl's first logical writings in 1877.[6] And this pre-history shows very changing intuitions concerning the content of expressions containing the truth-predicate. Only the words "is true" remained unchanged in this history. It would be highly misleading to assume that the analysis of the early uses of this predicate could explain the use of "τ" as a symbolic form for "is true" in *Symbolic Logic* of 1906. This holds for the unsatisfactory early semantic uses of "$= 1$" as an expression for "is true" before 1906 and for the pragmatic-epistemic use of such expressions in the sense of a report about a finished decision concerning the acknowledgment or confirmation of the truth of a statement or proposition, explained by MacColl. Even if MacColl (1906) explicitly repeats such a pragmatic explanation,[7] we can take this merely as a remark about a possible use of a truth-predicate. But this pragmatic use in no way explains the semantic features of the truth-predicate "τ" used in the 1906 formalization. The pragmatic interpretation of a truth-predicate would be good for explaining the non-synonymy between "A" and "It is true that A", but in the pragmatic interpretation "A" and "It is true that A" are not equivalent to each other, and in the formal system of 1906 MacColl even claims the unrestricted soundness of the strict equivalence $A^\tau = A$.

Taking seriously the above-mentioned remark of MacColl about the reference of A^τ to only one case, we come to the conclusion that A^τ

[5] I discuss these examples later.

[6] Cp. Michael Astroh's and Shahid Rahman's analysis of the changing uses of symbolic forms for the expression "is true" (Astroh 1993, Rahman and Christen 1997)

[7] "The statement A^τ is a revision and *confirmation* of the judgement A ... We suppose two incompatible alternatives, A and A', to be placed before us with fresh data, and we are to decide which is true. If we pronounce in favour of A and write A^τ, we *confirm* the previous judgement A and write A^τ; if we pronounce in favour of A', we reverse the previous judgement A and write A^ι." (MacColl 1906, p. 18)

asserts the belonging of the value *true* to A in the given situation and carries this relatedness from the given context of valuation to other situations.[8] Treated this way concerning the valuation of A in A^τ, MacColl's predicate $^\tau$ has a context-fixing function and brings about a kind of rigid designation or rigid valuation: If the value for A^τ has to be determined in the given context in order to determine the value of an expression H, part of which is A^τ, then the value of every subformula B^τ of H is the value of B in the given situation. So we should have A and A^τ equivalent in every given unique imaginable situation, but we don't: If it is true that A is true in the given situation (i.e., A^τ), then A is true in every situation. But A^τ, that A is true in the given situation, is then true in every other situation related to the given (formerly actual) situation: In every other situation it is true that A is true in the formerly actual situation. Explicating A^τ in this way, we have a difference between being true and being certain, but we do not have any difference between being true and being certain of being true. In this sense, statements of kind A^τ would be classically right entities, i.e., entities directed to which all positive modal distinctions collapse (analogously for negative characterizations).

This informal explanation of the intuition behind the truth-predicate $^\tau$ gives a preliminary sketch of how I will try to give a sound explication of its syntax and semantics in accordance with the logical system of *Symbolic Logic*. If we attempt to give adequate semantic explications for MacColl's modal system, the problems with this intuition will be sharpened.

2. Syntax and Semantics for MacColl's Modal Distinctions

In order to prepare for the explication of the semantic features of the truth-predicate, I define the semantics of its non-modal and modal background in this Section.

2.1. The non-modal basis

One problem with the semantics for MacColl's symbolism is the inner semantic context-sensitivity of the symbolic signs he uses. Depending on the syntactic context in which such a symbolic sign occurs, its semantic features can change.[9] This means that the semantics of

[8] Basically this is the view developed in Stelzner 1993.

[9] This context-sensitivity is characteristic for all phases of the development of MacColl's logical views, beginning with his papers in the late 1870s. Already at the early stage of his work, MacColl sees it not as a defect, but as a tool which he

MacColl's system (like semantics of natural languages) is not a purely compositionally constructed semantics. For MacColl's systems, Frege's statement that the meaning of a word has to be determined in the context of the sentence, not in isolation,[10] holds in a much stronger sense than for those of Frege himself. In this sense, MacColl makes a much more consequential use of the non-traditional dictum shared by him and Frege: "The *complete proposition* is the unit of all reasoning" (MacColl 1906, p. 11).

Depending on their occurrence in a proposition, the signs A, B, C, ... in MacColl's logical language can denote individual subjects or conceptual predicates:

> The symbol A^{B} denotes a proposition of which the individual A is the subject and B the predicate. Thus, if A represents *my aunt*, and B represents *brown-haired*, then A^{B} represents the proposition 'My *aunt* is *brown-haired*.' (MacColl 1906, p. 4)

Besides this, they can be attributes of classes and in this way they can determine subjects:

> When A is a class term, A_{B} denotes the individual (or an individual) of whom or of which the proposition A^{B} is true. For example, let H mean "*the horse*";

uses in order to avoid introducing new symbolic signs and unfamiliar-looking formal explanations: "Strange-looking symbols somehow offend the eye; and we do not take to them kindly, even when they are of simple and easy formation. Provided we can avoid ambiguity, it is generally better to intrust an old symbol with new duties than to employ the services of a perfect stranger. In the case just considered, and in many analogous cases, the context will be quite sufficient to prevent us from confounding one meaning with another, just as in ordinary discourse we run no risk of confounding the meanings of the word *air* in the two statements—'He assumed an air of authority,' and 'He resolved the air into its component gases.'" (MacColl 1882, pp. 229 f.). The context determines not just the meaning of descriptive signs, like "air" in the given example. The syntactic context in some cases determines even to what category a special sign belongs. In most cases the context says enough for a clearly unambiguous reconstruction. But in other cases the flavor of ambiguity cannot be avoided. The danger of ambiguity is increased by the intensive development of MacColl's logical views. So I agree with Michael Astroh, when he (after the examination of MacColl's use of $A = 1$) comes to the conclusion: "Aufgrund der Korrekturen, die MacColl von einem Aufsatz zum nächsten einfließen läßt, ohne die Konsequenzen seiner Interpretationen zu diskutieren, ist es zumindest im Hinblick auf seine frühen Schriften schwierig, ihm von vornherein eine einheitliche Position zu unterstellen, die er nur zunehmend genauer zu artikulieren wüßte." ["Because of the corrections which MacColl introduces from one paper to the next without discussing the consequences of his interpretations, it is at least in respect of his early writings difficult to presuppose for MacColl a homogeneous position which he is able to articulate more and more precisely."] (Astroh 1993, p. 132).

[10] "Nach der Bedeutung der Wörter muß im Satzzusammenhange, nicht in der Vereinzelung gefragt werden." (Frege 1884, p. X)

let w mean "*it won the race*"; and let s mean "*I sold it*," or "*it has been sold by me*." Then $\mathrm{H}_{\mathrm{w}}^{\mathrm{s}}$, which is short for $(\mathrm{H}_{\mathrm{w}})^{\mathrm{s}}$, represents the complex proposition "The *horse* which *won* the race has been *sold* by me," or "I have sold the horse which won the race" ... Thus the suffix w is *adjectival*; the exponent s *predicative* ... The symbol H^{w}, without an adjectival suffix, merely asserts that a horse or the horse, won the race without specifying which horse of the series $\mathrm{H}_1, \mathrm{H}_2$, &c. (MacColl 1906, p. 4 f.)

In the propositional logic (MacColl speaks in this case about *pure* or *abstract* logic) the same symbols $A, B, C, \ldots$ can denote statements or propositions (cf. MacColl 1906, p. 6). So, in defining the (non-modal) syntax of the system, we have just one kind of non-logical terms, predicate-/class-/subject-/individual-/statement-/proposition-terms: $A, B, C, \ldots$ (short: P-terms).

MacColl introduces in his system a denial (or negation) which is directed (again like the Frege-negation) to sentences or propositions, but does not constitute negative terms (even if the syntactic place of its occurrence could suggest so):

A small *minus* before the predicate or exponent, or an acute accent affecting the whole statement, indicates denial. (MacColl 1906, p. 5)

For the expression of sentence negation MacColl uses an accent as well as the small minus. $(\alpha^{\beta})'$ expresses the same as $\alpha^{-\beta}$: As mentioned above, he does not introduce special negations for predicate terms; affirmation and negation are the only qualities for sentences in MacColl: He does not have the Kantian infinite judgements, i.e., judgements with negative predicates, as a special kind of affirmative judgements. As two-placed propositional connectives he introduces the classical disjunction $(A^C + B^D)$ and conjunction $(A^C B^D)$.

Based on this material one can compose different kinds of expressions for statements:

If α, β, γ are P-terms, then single occurrences of $\alpha, \alpha', \alpha^{*\beta}, \alpha_{*\beta}^{*\gamma}$ are statements, where $'$ expresses the denial of a statement, $*$ indicates the place where the small minus can be or not. If S_1 and S_2 are statements, then $S_1', S_1 S_2$ and $S_1 + S_2$ are statements.

The syntax introduced this way is loaded with semantic presupposition. Every use of $\alpha_{*\beta}$ in a statement presupposes $\alpha^{*\beta}$:

The symbol H_{C} ("The *caught horse*") *assumes* the statement H^{C}, which *asserts* that "The *horse* has been *caught*." Similarly $\mathrm{H}_{-\mathrm{C}}$ assumes the statement $\mathrm{H}^{-\mathrm{C}}$. (MacColl 1906, p. 5)

The fact that we are confronted here with presupposition is shown by the fact that H^C is assumed both if the whole sentence in which it occurs is affirmative (H_C^A) and if it is negative (H_C^{-A} or $(H_C^A)'$).

As a special class MacColl introduces the class of non-existing things:

The symbol 0 denotes *non-existence*, so that $0_1, 0_2, 0_3$, &c., denote a series of names or symbols which correspond to nothing in our universe of admitted realities. (MacColl 1906, p. 5)

This zero-class plays an important role for expressing quantified categorical propositions. According to MacColl, we can express the categorical judgments of traditional logic in the following way:

α_{β}^{0}	"No α is β"
α_{β}^{-0}	"Some α are β"
$\alpha_{-\beta}^{0}$	"Every α is β"
$\alpha_{-\beta}^{-0}$	"Some α are not β"

The existence presupposition mentioned above, according to which every use of $\alpha_{*\beta}$ in a statement presupposes $\alpha^{*\beta}$, clearly does not hold in the case of categorical judgements, because then "No α is β" (α_{β}^{0}) would assume "α is β" (α^{β}). This in some sense contradicts the overall stipulation of MacColl's that every use of H_C assumes H^C, and we should correct it in the following way: Every use of $\alpha_{*\beta}$ in a statement, the predicate of which does not equal 0, presupposes $\alpha^{*\beta}$. I mention this here because it is another example of MacColl's context-sensitive use of his symbolic language: Sometimes even simple substitutions have to be structurally treated in a way other than the expression in which the substitution was performed. I shall keep this in mind in undertaking to explicate the syntactic use and semantic role of the truth-predicate $^{\tau}$.

2.2. The modal distinctions in pure or abstract logic

For his propositional logic, MacColl uses the expression *pure* or *abstract* logic. He characterizes his modal distinctions as follows:

In *pure* or *abstract* logic statements are represented by single letters, and we classify them according to attributes as *true*, *false*, *certain*, *impossible*, *variable*, respectively denoted by the five Greek letters $\tau, \iota, \varepsilon, \eta, \theta$. Thus the symbol $A^{\tau}B^{\iota}C^{\varepsilon}D^{\eta}E^{\theta}$ asserts that A is *true*, that B is *false*, that C is *certain*, that D is *impossible*, that E is *variable* (possible but uncertain). The symbol A^{τ} only asserts that A is true in a particular case or instance. The symbol A^{ε} asserts more than this: it asserts that A is certain, that A is always true (or true in every case) within the limits of our data and definitions, that its probability is 1. The symbol A^{ι} only asserts that A is false in a particular case or instance; it says nothing about the truth or falsehood of A in other instances. The symbol A^{η} asserts more than this; it asserts that A contradicts some datum or definition, that its probability is 0. Thus A^{τ} and A^{ι} are simply *assertive*; each refers to only one case, and raises no general questions as to data or probability. The symbol A^{θ} (A is a *variable*) is equivalent to $\mathrm{A}^{-\eta}\mathrm{A}^{-\varepsilon}$; it asserts that A is neither *impossible* nor *certain*, that is, that A is *possible* but *uncertain*. (MacColl 1906, pp. 6 f.)

Based on the given modal characterizations, MacColl defines the strict implication between A and B as the impossibility of the conjunction between A and non B:

The symbol $\mathrm{A}^{\mathrm{B}} : \mathrm{C}^{\mathrm{D}}$ is called an *implication*, and means $(\mathrm{A}^{\mathrm{B}}\mathrm{C}^{-\mathrm{D}})^{\eta}$, or its synonym $(\mathrm{A}^{-\mathrm{B}} + \mathrm{C}^{\mathrm{D}})^{\varepsilon}$. It may be read in various ways, as (1) A^{B} implies C^{D}; (2) If A belongs to the class B, then C belongs to the class D; (3) It is impossible that A could belong to the class B without C belonging to the class D. (MacColl 1906, p. 7)

MacColl gives a sequence of "self-evident or easily proved formulae": these are of special importance for the explication of his semantic treatment of modalities (1906, p. 8; MacColl's numbering).[11]

$$(11)\ (A + A')^{\varepsilon};\ (12)\ (A^{\tau} + A^{\iota})^{\varepsilon};\ (13)\ (AA')^{\eta};\ (15)\ A^{\varepsilon} : A^{\tau};$$
$$(16)\ A^{\eta} : A^{\iota};\ (17)\ A^{\varepsilon} = (A')^{\eta};\ (18)\ A^{\eta} = (A')^{\varepsilon};\ (19)\ A^{\theta} = (A')^{\theta};$$
$$(20)\ \varepsilon : A = A^{\varepsilon};\ (21)\ A : \eta = A^{\eta};\ (22)\ A\varepsilon = A;\ (23)\ A\eta = \eta.$$

With some of these formulae, we are again confronted by the syntactic context sensitivity of MacColl's logical language. In (20)–(23) the modal signs ε and η occur in different syntactic and semantic functions: as symbols for statements that are certain and as symbols for the predicate "is certain" (analogously for "impossible"). However, we have to acknowledge that the context of MacColl's use is clear enough, so that we can, depending on the position of these signs, sharply discern these different functions. In this case, there is no reason to accuse MacColl by saying that his syntactic uniform use would lead to ambiguity.

2.3. Classical non-classical semantics for pure logic

At the beginning, we mentioned that, following MacColl, a statement could sometimes be true and sometimes false. It would be misleading to conclude from this that MacColl was a supporter or even a forerunner of paraconsistent logic. At the same time (or in the same situation), there is no possibility for the same statement to be true and false. A statement's being true and false is an impossibility: As sound formulae in MacColl we have $(AA')'$, $(AA')^{\eta}$ and $(A^{\tau}A^{\iota})^{\eta}$.

[11] As an abbreviation for $(A : B)(B : A)$ MacColl introduces $A = B$.

One remark of MacColl's in *Symbolic Logic and its Applications* gives the impression that he acknowledged that a statement A can be neither true nor false. He even introduces a symbol in order to express this: "ϕ^0 asserts that ϕ is a *meaningless* statement which is neither true nor false" (MacColl 1906, p. 10). In fact, such meaningless statements are easy to produce in the framework of MacColl: Because the use of H_C assumes H^C, any statement S containing H_C is meaningless if H^C is not true. In this case S is neither true nor false. However, no consequences follow from this for the logical system in the sense that there would be truth-value gaps or three values *true*, *false* and *meaningless*. Logic has to do only with admissible values and *no meaning* is not an admissible value for a statement. This follows from the statement preceding the above quotation:

> The symbol ϕ^ε asserts that ϕ is *certain*, that is, true for all admissible values (or meanings) of its constituents; the symbol ϕ^η asserts that ϕ is *impossible*, that is, false for all admissible values (or meanings) of its constituents; the symbol ϕ^θ means $\phi^{-\varepsilon}\phi^{-\eta}$, which asserts that ϕ is *neither certain nor impossible*. (MacColl 1906, p. 10)

Furthermore, MacColl takes $A + A'$, $(A + A')^\varepsilon$ and $(A^\tau + A^\iota)^\varepsilon$ to be sound formulae. According to this, MacColl admits in his logic only the values true and false for statements and propositions. We here have a close similarity between MacColl 1906 and Frege's *Begriffsschrift* from 1879. As arguments of his content-stroke (Inhaltsstrich) Frege admits only judgeable contents (beurteilbare Inhalte), i.e., true or false propositions. This picture changes with the Frege of the *Grundgesetze der Arithmetik*. Here, after the introduction of the horizontal-stroke (Waagerechter), all values are admissible.

MacColl explicates his intuitive semantic principles by examples and by pointing to formulae which should be sound in his treatment. I will try to systematize MacColl's semantic intuition and to build up a formal semantics for his pure logic. In this semantics, we shall have means to express the context-sensitivity of valuations and we will not speak just about the value of an expression but about the value of an expression in a context (or in a situation).

We will use as abbreviations, with k as an element of a set of possible contexts (situations, worlds, cases):

For "The value of A in k is *True*": $v_k(A) = T$.
For "The value of A in k is *False*": $v_k(A) = F$.

2.3.1. The classical non-modal principles:

P1. $$\text{Either } v_k(A) = T \text{ or } v_k(A) = F$$

P2. $$v_k(A') = T \Longleftrightarrow v_k(A) = F$$

P3. $$v_k(AB) = T \Longleftrightarrow v_k(A) = T \text{ and } v_k(B) = T$$

P4. $$v_k(A + B) = F \Longleftrightarrow v_k(A) = F \text{ or } v_k(B) = F$$

Here P1 states that in the sense of MacColl's we have only T and F as admissible values for statements.

Definitions:

D1. $$A \supset B =_{\text{df}} (AB')'$$

D2. $$A \leftrightarrow B =_{\text{df}} (A \supset B)(B \supset A)$$

2.3.2. The classical modal principles:

Since for MacColl certainty is intuitively explained just as truth in all possible situations, in the following I will not use relational semantics for the modal part. Other developments of MacColl's system would of course involve the introduction of relational semantics. I start with some clear and non-controversial principles explained by MacColl.[12]

PC. $$v_k(A^\varepsilon) = T \Longleftrightarrow \forall k(v_k(A) = T)$$

PI. $$v_k(A^\eta) = T \Longleftrightarrow \forall k(v_k(A) = F)$$

PV. $$v_k(A^\theta) = T \Longleftrightarrow \neg\forall k(v_k(A) = T) \text{ and } \neg\forall k(v_k(A) = F)$$

Definitions:

D3. $$A : B =_{\text{df}} (A \supset B)^\varepsilon$$

D4. $$A = B =_{\text{df}} (A : B)(B : A)$$

Soundness:

> A formula A is sound ("$\models A$") if and only if for every set of contexts in every possible valuation k: $v_k(A) = T$.

Because such distinctions play an essential role in MacColl's argument for the non-redundancy of the truth predicate, I will now formulate semantic rules for the modal distinctions in the position of statements or propositions. As we saw above, MacColl sometimes uses ε, η and θ in the position of statements. Thus, we find expressions like

[12]In the following, the signs $\neg$, $\forall$ and $\exists$ are used in the metalanguage.

$\varepsilon^{\varepsilon}$, η^{ε} and θ^{ε} or ε^{η}, η^{τ} and θ^{ι}, etc. It would be misleading to treat the statement-modalities as logical constants, standing for "The Necessity", "The Impossibility", "The Variable" or as special truth-values "Necessary", "Impossible", "Variable". These expressions should be interpreted as variables for statements or propositions of different sorts, namely ε as a variable for statements which are certain, η as a variable for impossible statements and θ as a variable for variable statements. Subscripts on these sort-variables indicate that we have different variables of the same sort. According to this we have:

PC*. $v_k(\varepsilon_i) = v_k(A_i)$, with $\forall m(v_m(A_i) = T)$

PI*. $v_k(\eta_i) = v_k(B_i)$, with $\forall m(v_m(B_i) = F)$ and

PV*. $v_k(\theta_i) = v_k(C_i)$, with $\exists m(v_m(C_i)) = T$ & $\exists m(v_m(C_i)) = F$

We have, e.g., $\models \varepsilon_i : \varepsilon_j$, $\models \varepsilon_i = \varepsilon_j$, $\models \eta_i : \eta_j$, $\models \eta_i = \eta_j$, $\models \theta_i : \theta_i$, or shorter $\models \theta : \theta$, but we don't have $\theta_i : \theta_j$ as a sound expression, because $v_k(C_i) = v_k(C_j)$ does not hold in every valuation k for different variable statements C_i and C_j. The logical difference between on the one hand $\varepsilon, \eta, \theta$ taken as variables for propositions, which are sorted according to their modal characterizations, and on the other hand the use of these symbols as modal operators in A^{ε}, A^{η} and A^{θ}, is underlined by the following: While we have as sound $\models A^{\theta} = (A^{\varepsilon})'(A^{\eta})'$, the expression $\theta = \varepsilon'\eta'$ is not sound. While θ stands for a proposition sometimes true and sometimes false, $\varepsilon'\eta'$ expresses a plain contradiction, which is never true.

There are sound expressions often referred to as "paradoxes of strict implication". Based on his classical attitude, MacColl argues convincingly for the soundness of such paradoxes:

> Symbolic logic too has its paradoxes, that is to say, formulae which appear paradoxical till they are explained, and then cease to be paradoxes. Such is the formula $\eta : \varepsilon$, which asserts that "an impossibility implies a certainty". As soon as we define the implication A : B, by which we symbolize the statement that "A implies B," to mean simply $(\mathrm{AB}')^{\eta}$, which asserts that the affirmation A coupled with the denial B$'$ contradicts our data or definitions, the paradox vanishes. For then $\eta : \varepsilon$ is seen simply to mean $(\eta\varepsilon')^{\eta}$, which is a clear truism. (MacColl 1906, p. 505)

Again, in accordance with our explication of the treatment of modal terms in statement-position as variables of special sorts, the sound expressions $\models \eta : \varepsilon$ and $\models (\eta\varepsilon')^{\eta}$ as well as $\models \theta : \varepsilon$, $\models \eta : \theta$ should not be confused with the respectively unsound ones $A^{\eta} : A^{\varepsilon}$, $(A^{\eta}A^{-\varepsilon})^{\eta}$, $A^{\theta} : A^{\varepsilon}$, $A^{\eta} : A^{\theta}$.

3. Semantics for the Truth-Predicate

One unsolved problem in explaining the semantic background of MacColl's system consists in the explication of the semantic features of the expressions A^τ (A is true) and A^ι (A is false), according to the intuition that A^τ should be equivalent but not synonymous with A (not substitutable with A in every occurrence):

It may seem paradoxical to say that the proposition A is not quite synonymous with A^τ, nor A' with A^ι : yet such is the fact. Let A=*It rains.* Then $\mathrm{A}' =$ *It does not rain*; $\mathrm{A}^\tau =$ *It is true that it rains;* and $\mathrm{A}^\iota =$ *It is false that it rains.* The two propositions A and A^τ are *equivalent* in the sense that each *implies* the other; but they are not *synonymous*, for we cannot always substitute the one for the other. In other words, the equivalence $(\mathrm{A} = \mathrm{A}^\tau)$ does not necessarily imply the equivalence $\phi(\mathrm{A}) = \phi(\mathrm{A}^\tau)$. For example let $\phi(\mathrm{A})$ denote A^ε; then $\phi(\mathrm{A}^\tau)$ denotes $(\mathrm{A}^\tau)^\varepsilon$ ... Suppose now that A denotes θ_τ, a variable that turns out true, or happens to be true in the case considered, though it is not true in all cases. We get

$$\phi(\mathrm{A}) = \mathrm{A}^\varepsilon = \theta_\tau^\varepsilon = (\theta_\tau)^\varepsilon = \eta;$$

for a variable is never a certainty, though it may turn out true in a particular case.

Again, we get

$$\phi(\mathrm{A}^\tau) = (\mathrm{A}^\tau)^\varepsilon = (\theta_\tau^\tau)^\varepsilon = \varepsilon^\varepsilon = \varepsilon$$

... In this case, therefore, though we have $\mathrm{A} = \mathrm{A}^\tau$, yet $\phi(\mathrm{A})$ is not equivalent to $\phi(\mathrm{A}^\tau)$. (MacColl 1906, p. 16)[13]

MacColl believes these distinctions between A and "A is true" to be of fundamental cultural importance:

It is a remarkable fact that nearly all civilized languages, in the course of their evolution, as if impelled by some unconscious instinct, have drawn this distinction between a simple affirmation A and the statement A^τ, that A is *true*; and also between a simple denial A' and the statement A^ι, that A is *false*. (MacColl 1906, pp. 17 and 513 f.)

Nevertheless, one gets into some trouble if one tries to give a fitting explication for this distinction in the framework for a formal semantics sketched above, because here we have a case where a system seems to be neither extensional nor intensional in the sense of Carnap, yet seems to work logically properly.

[13] An analogous result was developed by MacColl in relation to A' and A^ι. In the following we discuss the problem with A' and A^ι not separated from A and A^τ, but consider A^ι just as $(\mathrm{A}')^\tau$.

Following the line of argumentation from the introduction, I will discuss three promising attempts for such an explication: First, I will treat $^{\tau}$ as an actuality-operator which gives strict equivalence between A and A^{τ}. Second, stressing the cross-reference to the case considered, the non-synonymy between A and A^{τ} comes out. And third, semantic means will be developed which allow the first and the second treatments to be brought together. This results in the fulfilment of MacColl's demand, according to which we should have both strict equivalence and non-synonymy of A and A^{τ}.

3.1. A first attempt: Fixing the actual

One possible way to get fitting semantic stipulations for expressions with the truth-predicate could be expected by confining the reference of expressions in the scope of the truth predicate $^{\tau}$ (like A in A^{τ}) to a fixed actual context c_a. M. Davies and L. Humberstone give a similar treatment (not directed to MacColl) for the actuality operator:

> We need to extend our modal language by the addition of an operator 'A' corresponding to the adverb 'actually'. The main semantic feature of such an operator is that for any sentence σ, $\lceil A\sigma \rceil$ is true with respect to a given possible world just in case σ is true with respect to the actual world (that is, just in case σ is actually true). (Davies and Humberstone 1980, p. 221)[14]

In accordance with the core idea of 'actuality', we can give a first semantic explication of MacColl's truth predicate τ:

PT1. $$v_k(A^{\tau}) = T \Longleftrightarrow v_{c_a}(A) = T$$

However, this confinement to one distinguished actual situation, which is independent of the given context of valuation, seems to be contrary to the pragmatic normal language orientation of MacColl's. There should be different possible situations which can give different actual contexts (or cases considered), not only one context-fixing eternal actual context in which we can use "A is true". Clearly, MacColl's formal claims are not fulfilled in a treatment of the predicate $^{\tau}$ according to PT1: his intention to have A and A^{τ} be equivalent expressions in any case considered is not fulfilled, i.e., instead of the sound $A = A^{\tau}$, we have only $(A = A^{\tau})^{\tau}$.

In order to overcome this deviation concerning the strict equivalence between A and A^{τ}, one could revise PT1 in the sense that the idea of a fixed actual context is given up and the context of valuation taken as the actual context, which changes according to different valuations. This would lead to the semantical principle $v_k(A^{\tau}) = T \Longleftrightarrow c_a = k$ and $v_{c_a}(A) = T$. This can be simplified to PT2:

[14] Other relevant papers are Crossley and Humberstone 1977 and Davies 1981.

PT2. $$v_k(A^\tau) = T \Longleftrightarrow v_k(A) = T$$

Then we have

$$\text{For all } k : v_k(A^\tau) = v_k(A).$$

This seems to give what MacColl asserts: A^τ and A are materially and strictly equivalent. We don't relate A^τ to a special designated actual world, but to "the case considered", as MacColl demands. According to PT2, $\models_{PT2} A^\tau \leftrightarrow A$, and $\models_{PT2} A^\tau = A$ are sound expressions. However, what about the asserted non-synonymy of the statements A^τ and A? The given condition PT2 considers the operator $^\tau$ in A^τ as semantically redundant. Thus, for every context k we have the same value for A^τ and A. From this we can replace A^τ by A everywhere without changing the truth-value of the imbedding sentence. The expressions A^τ and A are then synonymous, contrary to MacColl's intuition.

3.2. Reference to the case considered

Evidently, with PT2 we did not succeed in giving an adequate explication of a semantics behind MacColl's intuition and his formal claims for the use of the expression "A is true" (A^τ). We caught only the unsurprising part, where A^τ is equivalent with A: In every context, A and A^τ are both true or both false. Accordingly, our task now should be to catch the surprising part, where A is not synonymous with A^τ, but A is nevertheless equivalent to A^τ. The breakdown of synonymy between A and A^τ is connected with possible occurrences of A and A^τ inside the scope of modal operators. Here the context-fixing function of $^\tau$ develops its logical significance: Similar to a rigid designation, with $^\tau$ we can produce a kind of *de-re*-valuation inside modal contexts. Then inside the scope of modal operators, there can be a difference between the values of A not connected with $^\tau$ (which has to be treated *de dicto*) and the values of A in A^τ (which has to be treated *de re*, taking its values from the valuation outside the scope of the modal operator).

In order to prepare for the semantic explication of the context-fixing role of the truth-predicate $^\tau$ we introduce a new syntactic device for the expression of a special context-fixing predicate, which refers to specific explicitly indicated contexts.[15] A^i means that "A is true in context i", according to the semantic rule:

PT$^\text{f}$. $$v_k(A^i) = T \Longleftrightarrow v_i(A) = T$$

[15]In the rest of this paper the symbols i, j, k etc. are used in the metalanguage as symbols for contexts; they are used in the object language as predicates governed by the semantic rule PT$^\text{f}$.

Introduced in this way A^i delivers a kind of context-invariant proposition, which we do not have in MacColl. We will use this context-fixing operator to determine A^τ, which is in a special sense context-invariant.

The actuality-predicate, as treated in PT1, is just one special case of the truth-predicates introduced with PT^{f}.

The context-independence of A^i given by PT^{f} leads to the following:

$$(*) \qquad v_k(A^i) = T \Longleftrightarrow v_m(A^i) = T$$

As mentioned in connection with the Russell–MacColl discussion and concerning the case where we do not have variable statements,[16] for such context-invariant statements the modal distinctions between being true and being certain (or necessary) have no logical significance:

$$v_k(A^i) = T \Longleftrightarrow v_m(A^{i\varepsilon}) = T.$$
$$v_k(A^{-i}) = T \Longleftrightarrow v_m(A^{i\eta}) = T.$$

Or, syntactically:

$$\models A^i = A^{i\varepsilon}.$$
$$\models A^{-i} = A^{i\eta}.$$

Nevertheless, we don't have: $\models A^\varepsilon = A$ (or $\models A' = A^\eta$).

So, A^i and A are not synonymous. However, they are not equivalent either, because we do not have

$$v_k(A^i) = T \Longleftrightarrow v_k(A) = T.$$

Accordingly, we have neither $\models A^i = A$ nor $\models A^i \leftrightarrow A$.[17] However, with the truth-predicates of the kind just introduced we did not connect a claim that MacColl's truth-predicate $^\tau$ would be one of those truth-predicates. Truth-predicates ^{i}, ^{j}, ^{k}, etc. will play a supporting role in the explication of the truth-predicate $^\tau$. They will allow us to combine the merits of PT1 and PT2 for the semantic explication of MacColl's $^\tau$ in a technically simple way: In a valuation of a formula H in a given context k the expressions A and A^τ should be equally evaluated in non-modalized contexts, but in modalized contexts the valuation of A^τ should be based on a fixed *de-re*-reference to the context k in which the

[16] See page 94.

[17] Of course (cf. the actuality-treatment with PT1), we have as sound formulae:

$$\models (A^i \leftrightarrow A)^i.$$
$$\models (A^i = A)^i.$$

whole formula H is evaluated (the "actual" world, which can change with every valuation of H). This leads to the following semantical rule: In evaluating a formula H, we fix all values of subformulae A which are governed by the truth predicate $^\tau$ to the value of A in the valuation considered for the formula H.

PT3. $$v_k(H) = T \Longleftrightarrow v_k(H^*) = T,$$

where we get H^* from H by replacing all occurrences of $^\tau$ with ^{k}.

In this treatment, the truth-predicate $^\tau$ is formally equivalent to the predicate "now" treated by Hans Kamp (1971).[18] The main difference concerns the framework of explication: While Kamp uses a device of double indication of the truth-value of a formula ("H is true relative to situation s_1 and relative to situation s_2"), we avoid the double indication of context-relatedness by the use of the context-fixing predicates ^{i}, ^{j}, ^{k}, etc. One can wonder whether the double indexing framework or our substitution framework is better suited for different tasks. We concentrate our efforts on the special task of giving an explication of MacColl's truth-predicate $^\tau$, which should be close to the intuitions and formal claims of MacColl's concerning this predicate. So, we shall examine the results of the level of explication now reached for expressions A^τ, which in fact (having in mind remarks of MacColl's tending to a time-logic interpretation of his modalities) could be read not only as "A is true in the case considered", but also as "A is true *now*".

Given PT1, the following semantic principle prevented the equivalence between A and A^τ:

$$v_k(A^\tau) = T \Longleftrightarrow v_m(A^\tau) = T.$$

This does not follow from PT3; however, we have:

$$v_k(A^\tau) = T \Longleftrightarrow v_k(A) = T, \text{ for all possible contexts } k.$$

Therefore, we have as sound the equivalence between A and A^τ,

(1) $$\models_{PT3} A^\tau \leftrightarrow A,$$

i.e., A^τ and A are materially equivalent in every valuation. And it holds, as claimed by MacColl, that A^τ and A are not synonymous. The modal characterizations of being true and of being necessary that it is true are strictly equivalent:

(2) $$\models_{PT3} A^\tau = A^{\tau\varepsilon}.$$

[18] Prior 1968, Burgess 1984, Fenstad et al. 1987, van Benthem 1988, 1991, Gundersen 1997.

From this, together with the unsoundness of

$$(3) \qquad \models_{PT3} A = A^{\varepsilon},$$

we obtain (in accordance with MacColl) that A and A^{τ} are not synonymous. But they are synonymous in their unmodalized occurrences. So, from the soundness of (1) and (2) we obtain the soundness of

$$(4) \qquad \models_{PT3} A = A^{\tau\varepsilon}.$$

3.3. The case considered and the Gödel-rule

The results mentioned so far witness to the appropriateness of our explication of the truth-predicate $^{\tau}$ according to PT3. But there is one serious problem unsolved: MacColl claims not only the soundness of the material equivalence between A and A^{τ}, but also the soundness of the strict equivalence between them. And this does not hold with PT3.

We do not have—contrary to the claim of MacColl's—that A^{τ} and A are strictly equivalent with each other, i.e.,

$$(5) \qquad \models_{PT3} A^{\tau} = A$$

does not hold. The soundness of $\models_{PT3} A^{\tau} \leftrightarrow A$ together with the unsoundness of (5) demonstrates that with PT^{f} and PT3 the Gödel-Rule

$$\text{if } \models_{PT3} H, \text{ then } \models_{PT3} H^{\varepsilon}$$

does not hold.

One way to overcome the trouble with the Gödel-Rule (and in this way to ensure the soundness of $\models A^{\tau} = A$, as demanded by MacColl) consists in differentiating expressions H in which the truth predicate $^{\tau}$ occurs *unmodalized* (outside every subformula of kind G^{ε}, G^{η}, G^{θ}, $G_1 : G_2$ and $G_1 = G_2$) from expressions where all occurrences of the truth predicate $^{\tau}$ occur *modalized* (located inside modal contexts). Principle PT3 will be replaced by the following principle PT4:

PT4. If there are unmodalized occurrences of $^{\tau}$ in a formula H, then, before applying other semantic rules, the valuation of H has to proceed according to the following scheme:

$$v_k(H) = T \Longleftrightarrow v_k(H^*) = T,$$

where we obtain H^* from H by replacing all (modalized and unmodalized) occurrences of $^{\tau}$ with ^{k}.
(Expressions of kind A^{ι} are treated as $(A')^{\tau}$.)

The difference between PT3 and PT4 lies in the condition for the replacement of $^\tau$ by the truth-predicate ^{k} corresponding to the given valuation k. With PT4 the substitution of ^{k} for all (modalized and unmodalized) occurrences of $^\tau$ only needs to be performed in formulae in which $^\tau$ occurs unmodalized. So, with PT3 we have a general *de-re*-treatment of $^\tau$, while with PT4 we distinguish the *de-re*-treatment of $^\tau$ in formulae with unmodalized occurrences of $^\tau$ from the *de-dicto*-treatment of $^\tau$ in formulae without unmodalized occurrences of $^\tau$. This leads to the result that formulae of type H^τ are sound in the treatment with PT3 if and only if H^τ is sound in the treatment with PT4:

(I) $$\models_{PT3} H^\tau \iff \models_{PT4} H^\tau.$$

With

(II) $$\models_{PT3} H^\tau \iff \models_{PT3} H$$

this gives

(III) $$\models_{PT3} H \iff \models_{PT4} H^\tau.$$

Characteristic differences between the PT3- and the PT4-treatment can be exemplified by the following comparison: While in the PT3-treatment we have both (1) $H \supset H^{\tau\varepsilon}$ and (2) $H^\tau \supset H^{\tau\varepsilon}$ sound, only (2) is in the PT4-treatment. However, in accordance with (II), from $\models_{PT3} H \supset H^{\tau\varepsilon}$ we get soundness of $\models_{PT4} (H \supset H^{\tau\varepsilon})^\tau$.

The specific possibility to express a *de-dicto*-treatment of $^\tau$ in the PT4-treatment validates the Gödel rule,

(IV) $$\models_{PT4} H \implies \models_{PT4} H^\varepsilon,$$

which does not hold with the PT3-treatment. There are formulae sound in PT4 (like $A = A^\tau$) which are not sound in PT3. This is the case because of the different treatments of $^\tau$ in $A = A^\tau$: With PT3 it is treated *de-re*, with PT4 *de-dicto*.

Because of the difference between the *de-re*-treatment and the *de-dicto*-treatment given with PT4, even given soundness of the strict equivalence between A and A^τ, there is no equivalence concerning the soundness of separate formulae A and A^τ: It is not the case that $\models_{PT4} H \iff \models_{PT4} H^\tau$. E.g., in accordance with (III) and the unsoundness of $H = H^\tau$ with PT3, $(H = H^\tau)^\tau$ is not sound with PT4, but $\models_{PT4} H = H^\tau$ is.

With the rule PT4 (as formerly with PT3) we connected the valuation for expressions of kind A^τ with the syntactic structure of such expressions, but also took into consideration the valuation context (or the actual context of use) for this expression. In some sense we have managed to bring together compositionality and context: If a formula H contains unmodalized occurrences of $^\tau$, then in a first step all such formulae are bound to the given evaluation context. Further calculations are carried out not with expressions of kind A^τ, which have been evaluated in a subformula according to special valuation contexts of this subformula, but with expressions of kind A^k, which are context-insensitive, as shown in the semantic relation $v_k(A^i) = T \Longleftrightarrow v_m(A^i) = T$ and in the corresponding sound formula $\models A^{ik} = A^{im}$.

With respect to the explication of MacColl's views, the treatment of τ according to PT4 brings about the following results:

STRICT EQUIVALENCE OF A^τ AND A. $\models A^\tau = A$.

Proof. For every m:

(1)	$v_m(A^\tau = A) = T \Longleftrightarrow \forall k(v_k(A^\tau \leftrightarrow A) = T)$	D2, D3, D4, PC
(2)	$v_k(A^\tau \leftrightarrow A) = T \Longleftrightarrow v_k(A^k \leftrightarrow A) = T$	PT4
(3)	$v_k(A^k \leftrightarrow A) = T \Longleftrightarrow v_k(A^k) = v_k(A)$	D1, D2, P2, P3
(4)	$v_k(A^k) = v_k(A)$	PT^{f}
(5)	$v_k(A^k \leftrightarrow A) = T$	3,4
(6)	$v_k(A^\tau \leftrightarrow A) = T$	2,5
(7)	$\forall k(v_k(A^\tau \leftrightarrow A) = T)$	6
(8)	$v_m(A^\tau = A) = T$	1,7 □

So, A^τ and A are materially and strictly equivalent.

NON-SYNONYMY OF A^τ AND A. *There are syntactic contexts in which A^τ and A are not replaceable one by the other without changing the truth-value of the resulting expression: From the sound*

$$\models A^\tau : A^{\tau\varepsilon},$$

by replacing A^τ with its equivalent A, we receive the unsound

$$\models A : A^\varepsilon.$$

Proof. 1. $\models A^\tau : A^{\tau\varepsilon}$. For every m:

(1)	$v_m(A^\tau : A^{\tau\varepsilon}) = T \Longleftrightarrow \forall k(v_k(A^\tau \supset A^{\tau\varepsilon}) = T)$	D3, PC
(2)	$v_k(A^\tau \supset A^{\tau\varepsilon}) = T \Longleftrightarrow v_k(A^k \supset A^{k\varepsilon}) = T$	PT4
(3)	$v_k(A^k \supset A^{k\varepsilon}) = T \Longleftrightarrow$ $(v_k(A^k) = T \Longrightarrow v_k(A^{k\varepsilon}) = T)$	D1, P2, P3

(4)	$v_k(A^k) = T \Longrightarrow v_l(A^k) = T$	PT$^{\mathrm{f}}$
(5)	$v_k(A^k) = T \Longrightarrow \forall l(v_l(A^k) = T)$	4
(6)	$v_k(A^k) = T \Longrightarrow v(A^{k\varepsilon}) = T$	PC
(7)	$v_k(A^\tau \supset A^{\tau\varepsilon}) = T$	2, (3,6)
(8)	$\forall k(v_k(A^\tau \supset A^{\tau\varepsilon}) = T)$	7
(9)	$v_m(A^\tau : A^{\tau\varepsilon}) = T$	1,8

2. Disproof of $A : A^\varepsilon$. We take the following value-stipulation:

$$v_k(A) = T, v_m(A) = F.$$

Then we have

$$v_k(A \supset A^\varepsilon) = F.$$

So, $A : A^\varepsilon$ is not sound. □

To sum up: With the above semantic explanation of the symbol $^\tau$, MacColl's claims concerning the expression "is true" are met: A and A^τ are strictly equivalent and they are not synonymous in the sense of being replaceable in all contexts.

3.4. Scope-relatedness

Despite the seemingly convincing results reached with PT4 so far, there are unusual—and maybe unwanted—features connected with the way in which the soundness of the Gödel-rule is ensured by PT4 along with the hyperintensionality of $^\tau$. With the breakdown of replacability for logically equivalent formulae, there are other unsound formulae and rules one could expect to be sound in accordance with usual extensional or intensional logics. It is not only the replacement rule for logically equivalent expressions, which (as desired) breaks down, but we get into trouble (maybe unwanted by MacColl) with the substitution rule and modus ponens too. E.g., we have the sound (1) along with the unsound (2), which is produced by substitution from (1):

(1) $$\models_{PT4} B \supset (A = A^\tau).$$

(2) $$\not\models_{PT4} B^\tau \supset (A = A^\tau).$$

In addition, we are in trouble with modus ponens: We have $\models_{PT4}$ $(A = A^\tau) \supset (B^\tau \supset (A = A^\tau))$ and $\models_{PT4} A = A^\tau$ as sound formulae, but again we don't have (2) $B^\tau \supset (A = A^\tau)$ as a sound formula.

Connected with the problems concerning the use of modus ponens, the transitivity of implication fails to be generally sound: E.g., we have $\models_{PT4} A^\tau : A^{\tau\varepsilon}$ and $\models_{PT4} A^{\tau\varepsilon} : A^\varepsilon$, but we do not have $\models_{PT4} A^\tau : A^\varepsilon$.

Such problems emerge, because, according to PT4, formula (2), containing unmodalized occurrences of ${}^\tau$, has to be evaluated in a valuation k in its substitution (2*), where all ${}^\tau$ are replaced by ^{k}. Because (1) has no unmodalized occurrences of ${}^\tau$, for the evaluation of (1), according to PT4, no such substitution has to be carried out. So, we get, according to PT4, essentially different formulae, before applying the usual semantic rules as given in subsection 2.3. After applying PT4 we have to evaluate in context k:

(1) $$B \supset (A = A^\tau)$$

(2*) $$B^k \supset (A = A^k)$$

and (1) is not equivalent with (2*).

In fact, as treated by PT4, the ${}^\tau$ in the subformula $A = A^\tau$ of the formula $B^\tau \supset (A = A^\tau)$ has different semantical features compared to the ${}^\tau$ in a separate formula $A = A^\tau$. This comes from the quantifying-in-power of unmodalized occurrences of ${}^\tau$ for the whole formula in which such free ${}^\tau$ occur, with the result that all occurrences of ${}^\tau$ in such a formula are bound to be *de-re*-evaluated with respect to the given basic valuation. In this sense an unmodalized occurrence of ${}^\tau$ binds all occurrences of ${}^\tau$ to the given valuation in which this unmodalized occurrence of ${}^\tau$ has to be evaluated. However, the formula $A = A^\tau$ is sound only in the *de-dicto*-treatment.

In the PT4-treatment, the scope of the binding power of an unmodalized ${}^\tau$ is the whole formula in which this unmodalized ${}^\tau$ occurs. In order to limit the scope of this binding power and to be able to have *de-dicto*- and *de-re*-treated occurrences in the same formula, one can introduce special scope-delimiters '{' and '}', inside which *de-re*-binding of occurrences of ${}^\tau$ is blocked. Unmodalized occurrences of ${}^\tau$ will give *de-re*-treatment of ${}^\tau$ only in formulae scoped by '{' and '}'.

We add to the formula-definition:

If H is a formula, then $\{H\}$ is a formula.

In accordance with the purpose of the scope delimiters '{' and '}', we revise PT4 in the following way:

PT4$^{\#}$. If there are unmodalized occurrences of ${}^\tau$ in a formula $H^{\#}$, then, before applying other semantic rules, the valuation of $H^{\#}$ has to proceed according to the scheme

$$v_k(H^{\#}) = T \Longleftrightarrow v_k(H^*) = T,$$

where

(1) $H^{\#}$ is formula $\{H\}$ or formula H, and
(2) we obtain H^* from H by replacing all (modalized and unmodalized) occurrences of $^\tau$ outside scoped subformulae G with ^{k}.

If there are no occurrences of scope-delimiters in a formula H, then

$$\models_{PT4} H \Longleftrightarrow \models_{PT4^{\#}} H,$$

i.e., the treatment of such a formula by PT4$^{\#}$ is just the same as with PT4.

A restricted version of modus ponens then holds in the following form:

MP$^\tau$. $$\models_{PT4^{\#}} G^{\#} \supset H^{\#},\ \models_{PT4^{\#}} G \implies \models_{PT4^{\#}} H,$$

where $G^{\#}$ ($H^{\#}$) is $\{G\}$ ($\{H\}$), if G (H) contains unmodalized unscoped occurrences of $^\tau$ and H (G) contains modalized unscoped occurrences of $^\tau$.

A restricted substitution rule works with the following conditions:

SR$^\tau$. $$\models_{PT4^{\#}} H \implies \models_{PT4^{\#}} H[p/G]_s,$$

where $H[p/G]_s$ is obtained by substituting all occurrences of the statement-variable p in H by the expression G, if $^\tau$ does not occur unscoped in G, and by substituting p by the expression $\{G\}$ otherwise.

The transition from PT3 to PT4 allowed not only formulae with *de-re*-occurrences of the truth-predicate $^\tau$ to be handled, but also formulae where all occurrences of $^\tau$ are *de-dicto*-treated. After the introduction of scope delimiters with PT4$^{\#}$ it is possible to handle mixed formulae containing *de-re*- and *de-dicto*-occurrences of $^\tau$, and to express a sound analogue of the Gödel-rule for the truth-predicate $^\tau$,

GR$^\tau$. $$\models_{PT4^{\#}} H \implies \models_{PT4^{\#}} \{H\}^\tau,$$

which without the use of scope-delimiters holds only with the restriction that H contains unmodalized occurrences of $^\tau$.

The main deficiency of the PT3-treatment of MacColl's $^\tau$ was that, because of the unsoundness of the Gödel-rule, MacColl's claim concerning the soundness of strict equivalence between A and A^τ was not fulfilled even though soundness for material equivalence was. This was so because the truth-predicate $^{\tau}$ was taken in a *de-re*-use in all places of occurrence. Equipped with scope delimiters we can look back to PT3 in order to explicate the claims of MacColl concerning the truth-predicate, and with the following revision of PT3 to PT3$^{\#}$ it is possible to handle *de-re*- and *de-dicto*-occurrences of $^{\tau}$ in the same formula:

PT3$^{\#}$. Before applying other semantic rules, the valuation of $H^{\#}$ has to proceed according to the scheme

$$v_k(H^{\#}) = T \Longleftrightarrow v_k(H^{*}) = T,$$

where

(1) $H^{\#}$ is the formula $\{H\}$ or H, and
(2) we obtain H^{*} from H by replacing all (modalized and unmodalized) occurrences of $^{\tau}$ outside of scoped subformulae G with ^{k}.

Now it is possible to formulate a sound restricted version of the Gödel-rule in the PT3$^{\#}$-treatment:

GR3$^{\#}$. $$\models_{PT3^{\#}} H \implies \models_{PT3^{\#}} \{H\}^{\varepsilon},$$

i.e., if H is sound in PT3$^{\#}$, then the certainty of the *de-dicto*-treated H is sound in the PT3$^{\#}$-treatment.

In accordance with GR3$^{\#}$, the strict implication between A and A^{τ} is sound in the following form:

$$\models_{PT3^{\#}} \{A \leftrightarrow A^{\tau}\}^{\varepsilon}.$$

Unlike the PT4$^{\#}$-treatment, in the PT3$^{\#}$-treatment we do not have

$$\models \{H\}^{\varepsilon} \implies \models \{H^{\varepsilon}\}.$$

Because of this, we cannot secure *de-dicto*-treatment of $A = A^{\tau}$ by putting the scope-delimiters around it: While $\{A = A^{\tau}\}$ is sound with PT4$^{\#}$ independently of its place of occurrence, $\{A = A^{\tau}\}$ is not sound with PT3$^{\#}$.

With PT4, PT4$^{\#}$ and PT3$^{\#}$ we have alternatives for the explication of MacColl's intuitive and formal claims concerning the truth-predicate $^{\tau}$. A significant difference between PT4 on the one hand and PT3$^{\#}$ and PT4$^{\#}$ on the other hand consists in the fact that PT4 does not require symbolic tools not found in MacColl 1906.

The crucial point for the explication of MacColl's claims is the fact that the soundness of strict equivalence between A and A^{τ} presupposes a *de-dicto*-treatment of the truth-predicate $^{\tau}$ inside the formula $A = A^{\tau}$, which is given with every treatment of this formula according to PT4, PT4$^{\#}$ and PT3$^{\#}$. In *de-dicto*-contexts the replacement-rule for sound material and sound strict implications works unrestricted. In the *de-dicto*-use, there is no problem with replacing, e.g., A by A^{τ} and vice versa. Differently for contexts in which, with the help of $^{\tau}$, local *de-re*-uses of the kind H^{τ} are constituted, these *de-re*-contexts lose their *de-re*-character at least partially if they are replaced by expressions G without unscoped occurrences of $^{\tau}$, even if H^{τ} and G are logically equivalent formulae. Despite the logical equivalence between A and A^{τ}, in (1) $A^{\tau} \supset A^{\tau\varepsilon}$, for instance, A in the subformula $A^{\tau\varepsilon}$ is treated *de-re*, while in (2) $A^{\tau} \supset A^{\varepsilon}$, which is produced from (1) by the replacement of A^{τ} by the logically equivalent A, the A in A^{ε} is treated *de-dicto*. This is the reason why (1) is sound and (2) is not, even though (3) $A = A^{\tau}$ is.

With the truth-operator $^{\tau}$, we have an extremely instructive example of the context-sensitivity of MacColl's symbolic language and the importance of context for the determination of features of the signs in his formalism, one which has strong non-classical consequences. The kinds of explicit differentiation between *de-re*- and *de-dicto*-occurrences of $^{\tau}$ introduced make it possible to give explicit characterizations of the difference between its *de-re* and *de-dicto* uses, a distinction which is hidden in *Symbolic Logic*. This demonstrates that MacColl was not only the pioneer of modern modal logic. With his truth-predicate, MacColl introduced a sophisticated tool into his logical framework which finds its logical foundation not in ambiguities but in the aim to capture special context-sensitive logical features of the way in which modal statements and statements in modal contexts are evaluated *de re* or *de dicto* in natural and formal languages.

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MACCOLL ON JUDGEMENT AND INFERENCE*

The first part of the paper presents a framework of distinctions for the philosophy of logic in which the interrelations between some central logical notions, such as judgement(-act), judgement (made), proposition(al content), consequence, and inference are spelled out. In the second half the system of MacColl is measured against the distinctions offered in the framework.

The theme of our conference is that of Hugh MacColl and the logical tradition. From any point of view, surely, judgement and inference are (possibly *the*) central components of the logical tradition. However, they do not occur as such in MacColl's *Symbolical reasoning*(s). What we find are statements, assertions and applications of the symbol $\therefore$. Accordingly, I begin with a rational reconstruction of what I see as the pivotal moment in the 19th century logical tradition, namely Bolzano's introduction of a novel form of judgement, which will be used to take the measure of the early MacColl with respect to judgement and inference.

I. A Logical Framework

Hilary Putnam and, following him, George Boolos have, on different occasions, taken exception to Quine's dictum that

"*Logic is an old subject, and since 1879 it has been a great one*",

with which he opened the first editions of his *Methods of Logic*.[1] In their opinion, Quine's implicit preference for Frege's *Begriffsschrift* does an

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[1] Putnam 1982, Boolos 1994, and Quine 1950.

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injustice to Boole (Boolos and Putnam) and the Booleans, of whom Peirce in particular (Putnam). Ten years ago, in an inaugural lecture at Leyden, I too argued that Quine presented too narrow a view of logic, and that as far as the nineteenth century was concerned the crucial date in the development of logical doctrine was not 1879 (nor 1847, I would add today, disagreeing with Boolos's stimulating paper), but 1837, the year in which Bernard Bolzano published his treatment of logic in four hefty volumes.[2]

Why does this Bohemian priest deserve pride of place over and above such luminaries as Boole, Peirce and Frege? For more than two thousand years, logic has been concerned with how to effect valid acts of inference from judgements known to other judgements that become known through the inference in question. Basically, these judgements take the subject/copula/predicate form [S is P]. Bolzano now has the courage to break with this traditional pattern and uses instead the unary form

(1) A is true,

where A is a *Satz an sich*, or a *Gedanke*, in the later alternative terminology of Frege. The latter term was translated into English as *proposition* by Moore and Russell, with an unusually confusing ambiguity as a result: prior to 1900 a "proposition" stood for a judgement (made), whereas later it came to stand for the propositional content of such a judgement. For Bolzano, logic was very much concerned with knowledge; his critical examination and exposition of *logic* is called *Wissenschaftslehre* [an approximate translation might be *The Theory of (Scientific) Knowledge*]. Just as his main target, Kant, he holds that a correct (*richtig*) judgement is a piece of knowledge (*eine Erkenntnis*).[3] To my mind, he is perfectly right in doing so. After the "linguistic turn", in place of judgements, one can consider instead the proper form, and relevant properties, of their linguistic counterparts, namely assertions. An assertion is effected by means of the assertoric utterance of a declarative sentence. This explanation must be supplemented with a criterion of assertoric force, on pain of a vicious circularity. Such a criterion is provided by means of the question:

(2) How do you know? What are your grounds?

[2] *Oordeel en Gevolgtrekking. Bedreigde Species?*, an inaugural lecture delivered at Leyden University, September 9, 1988, and published in pamphlet form by that university.

[3] Bolzano 1837, § 34.

which is legitimate as a response to an assertion. In case the utterance was assertoric, the speaker is obliged to answer, and if he cannot do so the assertion was *blind*. The assertion issued by an act of assertion, but for what is stated, also contains an illocutionary claim to knowledge. Thus, I am able to make public my knowledge that snow is white through the assertoric utterance of the declarative

(3) 'Snow is white'.[4]

The explicit form of the assertion thus made would then be:

(4) I know that snow is white.

A sole utterance of the nominalization

(5) that snow is white,

which expresses the propositional content, on the other hand, will not so suffice. A phrase sufficient for the making of assertions is reached by appending 'is true' to the nominalization in question. An assertoric utterance of the declarative

(6) that snow is white is true

does suffice to effect an assertion with (4) as the assertion made. The fully explicit form, with indication of knowledge, and truth of content, accordingly becomes:

(7) I know that that snow is white is true.

This grammatically necessary, but hardly idiomatic, iteration of *that* can be avoided here through the transformation:

(8) that S is true = it is true that S,

which yields

(9) I know that it is true that snow is white

as the explicit form of the assertion made through an utterance of (3).

I do not mean to imply that this was the route that Bolzano actually took to his novel form of judgement: it was not. I have used various linguistic considerations concerning the form of assertions when viewed as reports of knowledge, whereas Bolzano insisted that his *Sätze an sich*

[4] The example 'snow is white' is taken from Boole (1854, p. 52).

were completely independent of all matters linguistic and cognitive. Be that as it may, the argument given provides a rationale for why correct judgements made are pieces of knowledge, and why the proper form of judgement is "truth ascribed to propositional content".

There remains the problem of choosing appropriate terminology for entities in the expanded declarative form (6). Frege held that declarative sentences expressed *propositions*, that is, for example, the declarative *snow is white* expresses the proposition that snow is white. I prefer not to join Frege in this. Wittgenstein used the terminology *Satz* and *Satzradikal*. The latter, clearly, is the proposition(al content), but the former has to do double duty for declaratives and what they express. For my purposes the best choice here might well be *statement*.[5] Sentence, statement and proposition then serve in different logical roles:

(10) a declarative sentence expresses
a statement with a proposition as content.

Thus,

(11) the declarative 'snow is white' expresses that snow is white.

When one steps over to the expanded statement-form an iteration of *that* occurs,

(12) 'snow is white' expresses that that snow is white is true,

which can be removed using the transformation (8)

(13) the declarative 'snow is white' expresses
that it is true that snow is white.

[5] *Enunciation* and *declaration* are other alternatives. The latter has a certain high-sounding ring to it, but might otherwise have served very well. My discussion could then have been pithily summarised:

The assertoric utterance of a declarative makes an assertion that claims knowledge of the declaration expressed (declared?) by the declarative in question.

My preference for statement is, i.a., based on the fact that *statement* is the English term which is applied to reports by witnesses. This is followed also in German: *Aussage, Zeugnis*, and Swedish: *utsaga*, whereas Dutch: *Declaratie, verklaring* uses *declaration* instead. Also MacColl (1880, p. 53) links his use to the legal one, for which reference I am indebted to Shahid Rahman. This advantage has to be weighed against the drawback that since Cook Wilson—*Statement and Inference*—the term has been in constant Oxford use, where it has served in many roles, among which those of propositional content (with indexicality taken into account; perhaps, after E. J. Lemmon (1966), the most common current use), declarative sentence, and state of affairs, the act of saying, and what is said, the act of asserting, and the assertion made.

Consider now an act of assertion made through an assertoric utterance of the declarative sentence 'snow is white'. With respect to the assertion made the discussion above can be summarised in the following table:

Assertion made (explicit form)	I know that it is true that snow is white.
(Illocutionary) Knowledge-claim	I know that
Statement that is asserted	that snow is white is true = it is true that snow is white
Propositional content	that snow is white

Note here also that judgement is often used instead of *statement* and *assertion*, with respect to both the act and the object. Thus propositions have truth-conditions, whereas statements (judgements) have assertion-conditions.

The implication A implies B, in symbols $A \supset B$, between two propositions A and B, is another proposition, which accordingly is a candidate for truth. Classically $A \supset B$ is true when A is false or B is true, whereas its constructive truth consists in the existence of a suitable proof-object. It should be stressed that 'implies' can only join propositions, but not statements: the proposition that grass is green implies that snow is white is fine from a grammatical point of view, whereas an attempted connection between statements yields the nonsensical 'grass is green implies snow is white', which, as Quine noted (1940), contains too many verbs. Propositions can also be joined into a relation of consequence, which yields a generalisation of propositions:

(14) the consequence from A to B,

in (Gentzen-like) symbols $A \Rightarrow B$.[6]

The consequence, or sequent, $A \Rightarrow B$ holds precisely when the corresponding implication $A \supset B$ is true (also constructively). Much to his credit, Bolzano considered also this notion of consequence—he called it *Ableitbarkeit*—whereas today one is only interested in the *logical* holding of the consequence. (A consequence holds logically when the corresponding implication is a logical truth, that is, is true come what may, independently of what is the case.) It should be clear that the inference

$$\frac{A \Rightarrow B \text{ holds} \quad A \text{ is true}}{B \text{ is true}}$$

[6]Consequences between statements will not work for the Quinean reasons. Cf. the preceding footnote.

is perfectly valid as it stands; one does not need the logical holding or the logical truth in the premises in order to be allowed to conclude that B is true. (Similarly, we do not need the *logical* truth of $A \supset B$ in order to draw the conclusion that B is true from the premise that A is true.)

Just as we can combine propositions into both implications, which are propositions, and consequences, which are not, statements can be combined into conditionals, which are statements, and inferences, which are not. For example, a *conditional statement* results from applying, not categorical, but hypothetical truth

(15) $\ldots$ is true, provided that A is true,

to a proposition:

(16) B is true, provided that A is true.

The *proviso* can also be expressed in other ways: on condition that, under the hypothesis that, assuming that, etc., will all serve equally well here. Conditional statements can be obtained also in other ways; for example, by joining statements by means of *If-then*:

(17) If A is true, then B is true.

The assertion-conditions for the three statements

$A \supset B$ is true,

$A \Rightarrow B$ holds,

B is true, provided that A is true, or, in another formulation,

If A is true, then B is true

are different (we do not have the same statement three times over), but if one is entitled to assert any one of them, the requirements for asserting the others can also be met.

Finally, an inference is, in the first instance, a mediate act of judgement, that is, (taking the linguistic turn) an act of asserting a statement on the basis of other statements being already asserted (known). So the general form of an inference I is:

$$\frac{J_1 \ldots J_k}{J.}$$

The inference I is valid if one is entitled to assert J when one knows (has asserted) $J_1 \ldots J_k$. Accordingly, in order to have the right to draw the inference I must possess a chain of immediately evident axioms and inferences that link premises to conclusion.[7] After Bolzano it has been common to conflate the validity of the inference I′

$$\frac{A_1 \text{ is true}, \ldots, A_k \text{ is true}}{C \text{ is true.}}$$

with the logical holding of the consequence $A_1, \ldots, A_k \Rightarrow C$. That is, one reduces the validity of the inference to the logical holding of a relation of consequence between the propositional contents of statements that serve as premises and conclusion, respectively, of the inference in question. Bolzano also reduced the correctness of the statement that the rose is red is true to the rose's *really* being red. In both cases, the reduction gives rise to what Brentano called *blind judgements*: a judgement can be correct, by fluke, even though the judger has no grounds, and similarly for blind inference.

Bolzano's other notion of consequence—that of *Abfolge*—is less clear, but can perhaps be understood in the following way. Consider the inference

(18) S_1. Therefore: S_2.

In expanded form it becomes:

(19) That S_1 is true. Therefore: that S_2 is true.

When this inference is drawn and made public through an utterance of (18), we have assertions of (i) the premise that S_1 is true, (ii) of the conclusion that S_2 is true, and (iii) of the inferential link between them. Instead of considering the validity of the inference, Bolzano's *Abfolge* involves a *propositional operator* ... entails ... such that

(20) The proposition that S_1 entails that S_2 is true =
the inference (18) is valid, and
the premise *that S_1 is true* is correct.

II. The Early Measure of MacColl

How does Hugh MacColl stand with respect to the novel Bolzano-form of judgement? How do his writings fare when measured against the above battery of distinctions? The early parts of his *Symbolic(al)*

[7]This notion of validity is age-old. Compare Quine and Ullian (1970, p. 22) for a recent formulation: 'When a ... truth is too complicated to be appreciated out of hand, it can be proved from self-evident truths by a series of steps each of which is itself self-evident—in a word it can be deduced from them.'

Reasoning(s) are in many ways as fascinating and as difficult as the corresponding passages in Frege's *Begriffsschrift*.[8] The logical key-term on which they rest is that of a *statement*: 'My system, which adopts full and complete *statements* as the ultimate constituents into which any argument can be resolved, steers clear of the discussion altogether.'[9] Some would expect a careful definition, or (perhaps better) elucidation, of the notion in question, according to which other claims could then be made evident owing to the meaning assigned to the term *statement*. Such, for instance, is, *mutatis mutandis*, Frege's procedure in the *Grundgesetze*. Frege, indeed, is one of the foremost proponents of the paradigm that Jean van van Heijenoort (1967) has dubbed "Logic as Language" and Hintikka (1988, 1996) has transformed into "Language as the Universal Medium". It is hard to determine MacColl's position with respect to this paradigm: like the Booleans he seems to appreciate the calculus aspects more than the language aspects, but, on the other hand, his remarks concerning pure and applied logic (1880, pp. 48, 58) point in the direction of the Logic as Language conception. Also the remarks concerning the *Law of Implication* (1880, p. 52), with their strong epistemological slant, tend in the direction of Logic as Language, as do the considerations on logical methodology (1880, p. 59). For sure, MacColl does not offer an explicit definition of the notion of statement. Instead he confines himself to examples, and other indications, of the roles in which his statements serve. (In this, of course, he is no worse than, say, Bolzano with respect to his *Sätze an sich*.) Statements are the denotations of the *'temporary symbols'*, that is, variables, or, perhaps even better, 'statement letters' in modern terminology. Temporary symbols can be joined into more complex ones by means of certain 'permanent symbols', which denote relations in which statements stand, and accordingly play the roles of logical constants in current logical formalisms. Examples (1880, p. 49) of statements are: "He is tall", "He is dark", and "He is German"; when a, b and c denote these statements respectively, their logical product abc denotes the statement "He is a tall, dark German." Furthermore, statements are *made*, or so MacColl informs us in definitions 1 and 2 (1880, pp. 49–50).

[8]In preparing my talk I have had to confine myself largely to MacColl's series of articles in *Mind*, owing to the difficulty in obtaining copies of further works. It should be noted that its book-length compilation from 1906 differs substantially from the earlier articles of the series.

[9]MacColl 1880, p. 59. The discussion in question concerns Hamilton's quantification of the predicate. The point concerning the use of statements is reiterated in MacColl 1902, p. 352, where propositions are also explained as subject-predicate statements.

Complex statements are obtained from other statements by means of (iterated) applications of

$$\times \text{ and } +,$$

which give, respectively, *compound statements* and *disjunctive statements*. Which notion, if any, of *my* proposition, statement and assertion corresponds best with MacColl's notion of a statement? MacColl's use of the "permanent symbols" $\times$, $+$, $'$, as well as many of the resulting formulae (1880, p. 53), is strongly reminiscent of modern uses of the corresponding propositional operators that are well known since Boole and Frege. Thus, the statements of MacColl would correspond to the *Sätze an sich/Gedanken* of Bolzano and Frege, and through *Principia Mathematica*, to the wff's of all of modern mathematical logic.

On the other hand, MacColl tells us,

> [t]he disjunctive symbol $a + b + c$ asserts that one of the three events named will take place, but it makes no assertion as to whether or not more than one will take place.

Here a disjunctive symbol, which "denotes" (expresses?) a disjunctive statement, appears to function as an *assertion*. Similarly, a denial a' is held to *assert* in definition 5 (1880, p. 52), and in the formulae (1.) $aa' = 0$ and (2.) $a + a = 1$, the accent seems to function as a propositional negation-sign. In formula (3),

$$(abc\dots)' = a' + b' + c' + \dots ,$$

on the other hand, if we read the $+$ and $'$ with assertoric force, according to their explanations, we get manifold violations of Geach's (1965) *Frege-point*: $(abc)'$ is assertoric and stands in the antecedent of an implication: $(a = b) =_{\text{def.}} (a : b) \times (b : a)$. Furthermore, each of the terms $a', b', c', \dots$ asserts, since the denial is assertoric, and, in formula (3.), is also part of the disjunctive statement $a' + b' + c' + \dots$. But a disjunction cannot have assertions as part. A proposition, or a statement, in my opinion, does not assert, but says or states. If MacColl does indeed miss the Frege-point here, he is not the only logician to do so: as Kenny (1963, p. 228) observes, even the theory of judgement proposed by the main promulgator of the Frege-point, to wit Peter Geach in his *Mental Acts* (1957), violates the Frege-point and is really a theory of propositional content.

Matters become more obscure in

> Def. 3—The symbol :, which may be read "implies", asserts *that the statement following it must be true, provided the statement preceding it be true.*

Thus, the expression $a : b$ may be read "a implies b," or "If a is true, b must be true," or "Whenever a is true, b is also true".

Expressions of the form $a : b \ldots$ (involving the symbol :) are called *Implications* or *Conditional Statements*. The statement to the left of the sign : is called the *Antecedent*, and the statement to the right of the sign : is called the *Consequent*.

How should *statement* be understood in definition 3? The obvious alternative is that the statements of MacColl are propositions. This works well with the three readings offered:

$$\text{That } S \text{ implies that } T;$$
$$\text{If that } S \text{ is true, that } T \text{ must be true;}$$
$$\text{Whenever that } S \text{ is true, that } T \text{ is true as well;}$$

all make grammatical sense (even though they will read more pleasantly after an application of transformation (8)).

However, when they are taken in the sense of proposition, according to the third reading "Whenever ...", the implication A implies B is true, when B is a logical consequence of A, where the latter notion is defined in the style of Bolzano and Tarski. That is, not only must the consequence $[A \Rightarrow B]$ hold, it has to hold logically in all cases. The second reading ('must be true'), on the other hand, which is what one would start with in explaining the validity of inference, seems to turn implication into an "entailment-connective" between propositions corresponding to the rendering of Bolzano's *Abfolge* that was suggested above.

In definition 4 (1880, p. 51) statement-identity is explained as implication (in the sense of MacColl, that is) in both directions; on the reading I have offered, this turns out to be the same theory as that offered by Wittgenstein in the *Tractatus*, namely that which identifies logically equivalent propositions. According to MacColl

$$a : b = (a = ab).$$

In modern terminology this would be:

$$A \models B \text{ is logically equivalent to the logical truth of } A \equiv A \,\&\, B.$$

From the point of view of standard modern logic, be it classical or constructive,

$$A \supset B \text{ is logically equivalent to } A \equiv A \,\&\, B.$$

This, however, is not what MacColl gets. His result is weaker owing to his very strong reading of implication.

A further difficulty lies in his terminology: both *Implication* and *Conditional Statement* can be used for symbols of the form $A : B$. (I prefer to use capital statement letters.) But ... implies ... only takes (my) propositions, whereas If ..., then ... only accepts (my) statements. Accordingly, the symbol : seems to do double duty, both for my implication $\supset$ and for my consequence $\Rightarrow$. In fact, as far as terminology is concerned, the sign : is also used as a symbol for inference:

=, the symbol of equivalence, and ... :, the symbol of *inference*, or implication. (MacColl 1880, p. 53, my emphasis)

So the statements of MacColl cover both my propositions and my statements. In fact, MacColl's statements serve as the minimal constituents of arguments, as we saw in the quote offered above. But in an argument assertions occur; otherwise no argumentative power is present. Accordingly, MacColl seems to use statements also as my asserted statements, or judgements made. If MacColl is guilty of this triple conflation, he is not alone: Frege took Peano to task for overburdening his $\supset$-sign with four or even five meanings, whereas the *consequentiae* of Scholastic logic had to allow for four different readings: implication, consequence, inference and causal grounding.

MacColl's definition (1880, p. 55) of the symbol $\therefore$, therefore, takes the form

$$A \therefore B = A \times (A : B).$$

But this again seems to ignore the Frege-point: in the locution

$$A.\ \text{Therefore}\ B$$

the statement A carries assertoric force and cannot be put in antecedent position, whereas the right-hand side of MacColl's equation does not suffer from these liabilities.

Again, my insisting upon this letter of the logical law might be niggardly and uncharitable. MacColl makes the clear observation:

The statement $A \therefore B$ is stronger than the conditional statement $A : B$ and implies the latter. The former asserts that B is true *because* [emphasis added] A is true; the latter asserts that B is true *provided* that A be true. (MacColl 1880, p. 55)

The conditional statement does not, of its own, assert anything, but it can be asserted. An assertion made by means of it does assert that B

is true, provided that A is true (where A and B are propositions in my sense).

If I am right in the above, MacColl *does* ignore a number of basic distinctions in the foundations of logic, in particular concerning the differences between the implication(al proposition)

$$\text{that } R \supset \text{that } S,$$

the conditional statement

$$\text{that } S \text{ is true, on condition that it is true that } R,$$

the consequence from that R to that S

$$\text{that } R \Rightarrow \text{that } S,$$

and the inference

that R is true. Therefore: that S is true.[10]

However, at this stage in the tradition of logic, almost everybody does so. Frege, certainly, had seen the Frege-point, but the difference between proposition and statement he did not have, and, in particular his *Begriffsschrift*-system suffers from this on a number of scores.[11]

In the examples I have considered from MacColl, invariably he hit upon something interesting or true, and sometimes both; there is more to be found, especially concerning the logic of epistemic notions (rather than epistemic logic) in the later instalments. Hugh MacColl was a pioneer and it is greatly to his credit to have pinpointed so surely such a wealth of crucial logical notions and issues.

[10]This was still the case in 1906a, where, in particular, the treatment of *therefore* in §§ 76–78, at pp. 80–83, continues to beg the Frege point.

[11]Note added in proof: The referee rightly observed that my stern reading of MacColl might not do him justice at this point; these conflations seem to have been at least partly resolved in later works, for instance, the 1906 book version of his symbolic reasonings. In that work, the statements are clearly best read as propositions. Furthermore, MacColl claims it to be an advantage of his system, rather than a serious drawback, that it allows for many readings; see his letter to Russell, May 15, 1905: "[the] enormous superiority of my system is due in great measure to the very principle which you find so defective, namely the principle of leaving to context everything in the reasoning or symbolical operations which it is not absolutely necessary to express." (I am indebted to Prof. Astroh for putting the letters of MacColl to Russell at my disposal.)

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MACCOLL ON MODALITIES

This paper tries to reconstruct modal principles advanced by MacColl. It is argued that he had the basic rules of the modal square of opposition. On the other hand, his proofs of contradictions stemming from iterating modalities are incorrect.

Frege and Russell, the fathers of mathematical logic, were not very much interested in modalities and relations between them.[1] For Frege:

> The apoidictic judgment differs from the assertory in that it suggests the existence of universal judgements from which the proposition can be inferred, while in the case of the assertory one such suggestion is lacking. By saying that a proposition is necessary I give a hint about the grounds for my judgments. *But, since this does not affect the conceptual content of the judgment, the form of the apoidictic judgment has no significance for us.*
>
> If a proposition is advanced as possible, either the speaker is suspending judgment by suggesting that he knows no laws from which the negation of the proposition would follow or he says that the generalization of this negation is false. In the latter case we have what is usually called a *particular affirmative judgment* … "It is possible that the earth will at some time collide with another heavenly body" is an instance of the first kind, and "A cold can result in death" of the second. (Frege 1879, p. 13; Frege's italics)

This quotation shows that Frege located modalities outside the domain of pure logic.

Russell 1903 offers no treatment of modalities. Appendix C of *Principia Mathematica*, on truth-functions and other propositional forms, mentions epistemic operators (assertion, belief), but contains nothing about alethic, that is, proper modal propositions. In Russell 1905, we find a form '$C(x)$', where x is a free variable, as a general scheme of a proposition. Further, Russell considers the phrases '$C(x)$ is always

[1] See Rescher 1974 and Dejnožka 1999 on Russell and his objections to MacColl and modal logic.

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true' and '$C(x)$ is sometimes true', which are often (Russell does not mention this) considered as connected with modalities. For Russell, these phrases mean, respectively, 'C(everything)' and 'C(something)'. Thus, we can say that Russell reduced the logical meaning of modalities to quantifiers. This is confirmed by the following passage from his *Introduction to Mathematical Philosophy*, which is probably an allusion to MacColl:

> Another set of notions as to which philosophy has allowed itself to fall into hopeless confusions through not sufficiently separating propositions and propositional functions are the notions of "modality": *necessary, possible,* and *impossible* ... In fact, however, there was never any clear account of what was added to truth by the conception of necessity. In the case of propositional functions, the three-fold division is obvious. If "ϕx" is an undetermined value of a certain propositional function, it will be *necessary* if the function is always true, *possible* if it is sometimes true and *impossible* if it is never true. (Russell 1919, p. 165; Russell's italics)

Not even more traditional logicians were involved in formal studies of modalities. This becomes clear if we inspect the logical treatises of Sigwart, Erdmann and other authors of the turn of the 20th century. Thus, the great tradition in modal logic going back to Aristotle, and successfully continued in the Middle Ages, was almost entirely neglected until the 1930s. Of course, almost every logician considered so-called modal sentences: problematic (expressing possibility) and apoidictic (expressing necessity), but almost everything discussed was limited to analysis of various meanings of modal concepts, and not of formal relations between modal sentences. We can find something in Höfler, who described relations from the square of oppositions for modals in 1917. Even Lewis 1912 contains nothing about the logic of modalities, which started with Lewis 1918, really a pioneering work in the field. Causes of this situation seem to be these. Firstly, analysis of modalities was burdened by very obscure epistemological and psychological considerations. Secondly, the dogma of extensionalism accepted by Frege, Russell and the majority of formal logicians of that time was responsible for the neglect of modal logic.

Hugh MacColl is a notable exception in this respect. He gave an analysis of modalities, established some connections between them and stated some problems. The question must have been important to him, because he considered it in his papers, his letters to Russell and in his main book published in 1906. The treatment in MacColl 1906 is the most extensive and I will use this source. I will try to reconstruct MacColl's ideas concerning modalities using his terminology, but not his symbolism.

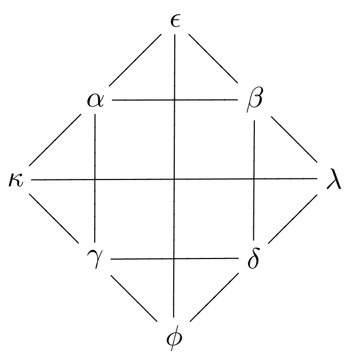
The importance of modalities for MacColl was evidently connected with his understanding of implication (see MacColl 1906, p. 7). Let $T(A)$ mean 'A is true'. Thus, '$T(A)$ implies $T(B)$' means (a) if A belongs to the set of truths, then B belongs to the set of truths, (b) it is impossible that A belongs to the set of truths without B belonging to the set of truths, (c) it is certain that either A does not belong to the set of truths or B belongs to the set of truths. The locutions are not only equivalent for MacColl, they are even synonymous. I do not discuss whether he is right or not. I mention this view of MacColl's just in order to show that modalities were important to him for fundamental reasons, having obvious relations, speaking in a more contemporary manner, with strict implication and many-valueness.

MacColl distinguishes five attributes of statements considered in pure or abstract logic: truth (T), falsity (F), certainty (N), impossibility (I) and variability (C). Let me quote relevant explanations (adopted in my symbolism):

> ... the symbol ... $C(A)$ asserts that A is *variable* (possible, but uncertain). The symbol $T(A)$ only asserts that A is true in a particular case or instance. The symbol $N(A)$ asserts more than this: it asserts that A is *certain*, that A is always true (or true in *every case* within the limits of our data and definition, that its probability is 1). The symbol $F(A)$ only asserts that A is false in a particular case or instance; it says nothing as to the truth or falsehood of A in other instances. The symbol $I(A)$ asserts more than this; it asserts that A contradicts some datum or definition, that its probability is 0. Thus, $T(A)$ and $F(A)$ are simply *assertive*; each refers only to one case, and raises no question as to data and probability. The symbol $C(A)$ (A is variable) is equivalent to $\neg I(A) \wedge \neg N(A)$; it asserts that A is neither *impossible* nor *certain*, that is, that A is *possible* but *uncertain*. In other words, $C(A)$ asserts that the probability of A is neither 0 nor 1, but some proper fraction between the two. (MacColl 1906, pp. 6–7; MacColl's italics)

There are certain interpretative problems concerning probability, or contradicting some datum or definitions which I will not enter into here. However, we can derive from MacColl's explanations clear formal ideas. Let's think about instances or cases as possible worlds, or temporal points. Thus, truth *simpliciter* means truth in some possible world (at a certain temporal point), falsity *simpliciter*—falsity in some possible world (at a certain temporal point), certainty—truth in all possible worlds (at all temporal points), impossibility—falsity in all possible worlds (at all temporal points), and variable—truth in some possible world (at a certain temporal point) and falsity in some possible world (at a certain temporal point). In any case, we are entitled to treat certainties as necessary statements and variables as contingent statements. Further, MacColl notes that $FI(A)$ is not generally equivalent to $IF(A)$, which is of course a correct observation, and extends his definition of implication to other modalities—this point is not relevant for my further considerations. MacColl states (pp. 12–19) the following theorems on modalities: (a) $N(A \vee \neg A)$, (b) $N(T(A) \vee F(A))$, (c) $N(T(A) \wedge F(A))$, (d) $N(N(A) \vee I(A) \vee C(A))$, (e) $N(A) \Rightarrow T(A)$, (f) $I(A) \Rightarrow F(A)$, (g) $N(A) \Leftrightarrow I(\neg A)$, (h) $I(A) \Leftrightarrow N(\neg A)$, (i) $C(A) \Leftrightarrow C(\neg A)$, (j) $\neg C(A) \Leftrightarrow (N(A) \wedge I(A))$.

Thus, (a)–(c) assert (roughly speaking) that modal logic is an extension of classical logic, (d) that every statement is necessary, impossible or contingent, (e) that necessity implies truth, (f) that impossibility implies falsity, (g) and (h) establish the mutual definibility of necessity and impossibility *via* negation, (i) that $C(A)$ and $C(\neg A)$ are equivalent, and (j) that non-contingency is equivalent to necessity or impossibility. There is a problem with the definition of possibility in MacColl. In one place he says that possibility (M) is defined by $M(A) \Leftrightarrow \neg I(A)$. This suggests the standard understanding of $M(A)$ as $\neg N(\neg A)$. If we take this route, MacColl's formal ideas on modalities can be summarized by the following diagram.



We have the following dependencies:

(a) $\alpha \Rightarrow \gamma$,	(g) $\alpha \Rightarrow \kappa$,	(m) $\alpha \Rightarrow \epsilon$,
(b) $\neg(\alpha \Leftrightarrow \delta)$,	(h) $\beta \Rightarrow \lambda$,	(n) $\beta \Rightarrow \epsilon$,
(c) $\neg(\beta \Leftrightarrow \gamma)$,	(i) $\kappa \Rightarrow \gamma$,	(o) $\phi \Rightarrow \gamma$,
(d) $\neg(\alpha \wedge \beta)$,	(j) $\lambda \Rightarrow \delta$,	(p) $\phi \Rightarrow \delta$,
(e) $\beta \Rightarrow \delta$,	(k) $\epsilon \Leftrightarrow \alpha \vee \beta$,	(r) $\neg(\epsilon \Leftrightarrow \phi)$,
(f) $\gamma \vee \delta$,	(l) $\phi \Leftrightarrow \gamma \wedge \delta$,	(s) $\alpha \wedge \beta \wedge \phi$.

Now interpret α as $N(A)$, β as $I(A)$, γ as $M(A)$, δ as $M(\neg A)$, κ as $T(A)$, λ as $F(A)$, and ϕ as $C(A)$. We get MacColl's modal logic as an interpretation of a square of oppositions extended (by adding κ, λ, ϵ and ϕ) for modal sentences. If this interpretation of MacColl is correct, I think that he was the first who was conscious of that in modern times, regardless of his unclarities about possibility. He notes (see MacColl 1906, p. 105) that four modalities of the traditional logic are represented by the formula $N(A) \vee I(A) \vee T(A) \wedge C(A) \vee F(A) \wedge C(A)$. It is a conjunction of (b) and (d) and a theorem. One can guess that $T(A) \vee C(A)$ represents possibility (M) and $F(A) \vee C(A)$ non-necessity ($M\neg A$). However, this hypothesis is inconsistent with the standard definition of $M(A)$ as $\neg N(\neg A)$ and $M(\neg A)$ as $\neg N(A)$. Unfortunately, MacColl does not say exactly which "traditional" modalities he had in his mind. A hint for understanding his views we find in the following explanations (pp. 14–18). Let $P(A)$ mean 'A is probable' (the likelihood of A is greater than 1/2), $Q(A)$ means 'A is improbable' (the likelihood of A is less than 1/2), and $U(A)$ means 'A is uncertain'; other modals have their already explicated meanings. Now MacColl stipulates: (a) the denial of truth is an untruth, and, conversely, (b) the denial of probability is an improbability, and, conversely, (c) the denial of certainty is an impossibility, and, conversely, (d) the denial of variable is a variable, and (e) the denial of possibility is uncertainty, and conversely. The stipulation (a) is obvious (MacColl obviously identifies here untruth and falsity), (b) is unclear, because we do not know whether the probable includes certain or not, (d) is obvious, but (c) and (e) contradict the standard understanding of modalities. MacColl explains why the denial of possibility is uncertainty and not impossibility. Consider, he says, the statement (i) 'It will rain tomorrow'. Now the statement (ii) 'It will not rain tomorrow' is its denial. The statement (i) is a possibility and (ii) merely uncertain, not an impossibility. In particular, in order to prove that a denial of a possibility is an uncertainty we have to prove that the possibility in question implies the uncertainty of this possibility.

The problem with MacColl's explanations is connected with the fact that he passed from an analysis of modalized statement to the status of unmodalized ones. His example expresses a typical future contingency ('It will rain tomorrow'). It is fairly obvious that here MacColl confused possibility and contingency, because he constructed his example as expressing possibility and non-necessity (possibility not), that is, just contingency. Moreover, he also confused denials of modalized statements with denials of arguments of modal operators in the situation in which their modal status is determined; for example, the denial of $M(A)$ with the denial of A itself, provided that we know that A is a possibility. Assuming our diagram, a proper analysis of 'it will rain tomorrow' is that it is located at the point ϵ. Thus, the denial of $C(A)$ is $N(A) \vee I(A)$, but if we know that A is a contingency, $\neg A$ is also a contingency. Hence, MacColl should say that if A is uncertainty and possibility, its denial is the same. Of course, he is right that in order to prove that A is a contingency, we must prove that this fact implies that $\neg A$ is uncertainty, but without further ado his explanations are burdened by an ambiguity of 'possible' and 'uncertain'. It is not blocked by a remark (MacColl 1906, p. 15, footnote) that we should understand the denial of certainty as a denial of a certain statement. Of course, the denial of a tautology (a certainty) is a contradiction (an impossibility), which legitimizes (c), but it also leads to ambiguity. It is also possible that this second treatment of modalities is more consistent with many-valued logic than with extensions of classical logic.

MacColl constructs an antinomy concerning so-called second-degree modal statements. A statement of the form $S(A)$, where S expresses a modality, is called a first-degree modal statement. Now a statement $SS(A)$ is second-degree, a statement $SSS(A)$ third-degree, and so on. Take a statement $CC(A)$. We can assume that any statement is a certainty, an impossibility or a variable. Assume that A is a certainty. This means that A belongs to the set of certainties. On the other hand, provided that A is a certainty, $C(A)$ means that A is a variable (contingency). Thus, we arrive at a conclusion that a certainty is a variable which is impossible. So $IC(A)$. But, in this situation $CC(A)$ means that an impossibility is a variable; in the terminology of this paper a contingency is an impossibility. This is a contradiction. Similarly, we prove that if is A is an impossibility, $CC(A)$ is an impossibility too. Thus, a variable is an impossibility. Finally, assume that A is a variable. In this situation the formula $C(A)$ is self-evidently true and certain. But the formula $CC(A)$ asserts that a certainty is a variable, which leads to a contradiction, that is, an impossibility. On the other hand, take any set of arbitrary statements which consists of certainties, impossibilities and variables. We can check the probability that a statement A taken from this set at random is a certainty, a variable or an impossibility. Thus, the sentences $N(A)$, $C(A)$ and $I(A)$ are variables. Then, $CC(A)$ is always true.

MacColl solves the problem in the following manner:

After some reflexion, I found that the second of these antinomies (namely that $CC(A)$ is *not* self-contradictory) is the true one. Where then is the error in the first argument? It consists in this, that it tacitly assumes that A *must* either be *permanently* a certainty, or *permanently* an impossibility, or *permanently* a variable—an assumption for which there is no warrant. On the second assumption, on the contrary—a supposition which is perfectly admissible—*A may change its class.* In the first trial, for example, A may turn out to represent a certainty, in the next a variable, and in the third an impossibility. When a certainty or an impossibility turns up, the statement $C(A)$ is evidently false; when a variable turns up, $C(A)$ is evidently true;

and since (with the data taken) each of these events is possible, and indeed always happens in the long run, $C(A)$ may be false or true, being sometimes the one and sometimes the other, and is therefore a variable. That is to say, on perfectly admissible assumptions, $CC(A)$ is possible; it is not a *formal* impossibility.

But, *with other data*, $C(A)$ may be either a certainty or an impossibility, in either of which cases $CC(A)$ would be an impossibility. For example, if all the statements from which A is taken at random be exclusively variable, ... then, evidently, we should have $NC(A)$, and not $CC(A)$. On the other hand, if our universe of statements consisted solely of certainties and impossibilities, with no variables, we should have $IC(A)$, and not $CC(A)$. Thus the statement $CC(A)$ is *formally* possible; that is to say, it contradicts no definition or linguistic or symbolic convention; but whether or not it is *materially* possible depends upon our special or material data. (MacColl 1910, pp. 197–198; MacColl's italics)

The distinction between formal and material possibility is of little help here. On the other hand, MacColl is almost right about the status of $CC(A)$. By definition, this formula means $MC(A) \wedge M\neg C(A)$. The second conjunct, that is, $M\neg C(A)$ is equivalent to $M(N(A) \vee I(A))$, which gives that $CC(A)$ is equivalent to $MC(A) \wedge M(N(A) \vee I(A))$. Thus, $CC(A)$ says that it is possible that A is contingent and it is possible that A is necessary or impossible. Now it is evident that the formula $CC(A)$ is either true or false, depending on the status of A. If A is a possibility, then it is possible (not excluded) that $C(A)$ and $C(\neg A)$, so $MC(A)$ is true. Since $M(A)$ does not exclude $N(A)$, then if A is possible, it is possible that A is necessary and the second conjunct is also true. On the other hand, if A is either necessary or impossible, the formula $MC(A)$, that is, the first conjunct of $MC(A) \wedge M(N(A) \vee I(A))$ is false, and the whole formula $CC(A)$ is false. It seems that complications introduced by MacColl are caused by his confusing contingency and possibility. This confusion seems to me more important than other unclarities pointed out by Shearman (1906, pp. 152–161), who argued that MacColl did not observe that certainty implies truth, that he confused events and statements as well as propositions and propositional functions, and that he misinterpreted other logicians as far as relations between particular modalities were concerned. I will not discuss Shearman's objections (they were directed at papers preceding MacColl 1906, which clarified some points), because the formal connections between modals that MacColl noted are fortunately independent of a particular interpretation of modalities and the distinction of propositions and propositional functions. Thus, MacColl's work can be rightly regarded as a predecessor of the formal logic of modalities.

Finally, I express my gratitude to the referee, who suggested important improvements.

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MACCOLL'S EVOLUTIONARY DESIGN OF LANGUAGE

The account of logical form underlying MacColl's modal system is not only due to his mathematical approach to logics. Algebra, analysis and probability theory provide the formal context in which it develops. Likewise, however, MacColl's dualistic understanding of natural and human evolution in terms of purpose and chance shape his account and layout of a modal logic. His metaphysical beliefs and, especially, the conception of language they comprise articulate his religious reaction to major progress in the empirical sciences of his century. Statements of ever increasing complexity articulated with the help of conventional signs are the genuine subject of MacColl's logic. His concept of a statement essentially recapitulates the notion of a root in 19th century linguistics.

1. Introduction

In one of his last letters to Bertrand Russell Hugh MacColl sums up the important stages of his later intellectual development. His report confirms what the dates of publication of his major works already indicate. After the public discussion of his "Calculus of Equivalent Statements" MacColl refrains from investigating basic issues in logic and mathematics for more than ten years. The letter to Russell written on the 17th of May 1905 comments on this lengthy period:

When, more than twenty-eight years ago, I discovered my Calculus of Limits, or as I then called it, my "Calculus of Equivalent Statements and Integration Limits", I regarded it at first as a purely mathematical system restricted to purely mathematical questions. … When I found that my method could be applied to purely logical questions unconnected with the integral calculus or with probability, I sent a second and a third paper to the *Mathematical*

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> *Society*, which were both accepted, and also a paper to *Mind* (published January 1880). These involved me in a controversy with Venn & Jevons, of which I soon got tired, as I saw it would lead to no result. — I sent a fourth paper (in 1884) to the *Math. Soc.*, on the "Limits of Multiple Integrals", which was also accepted. This I thought would be my final contribution to logic or mathematics, and, for the next twelve or thirteen years, I devoted my leisure hours to general literature. Then a friend sent me Mr. Dodgson's ("Lewis Carroll's") *Symbolic Logic*, a perusal of which rekindled the old fire which I thought extinct. My articles since then I believe to be far more important from the point of view of general logic than my earlier ones; but unfortunately the views which they express are far more subversive of the orthodox or usually accepted principles in symbolic logic. I feel myself an Ishmael among logicians, with my hand against every man, and every man's hand against me; but it is hardly my fault; I follow the natural development of my method in the direction of truth, and according to my lights, whatever be the consequences. (MacColl 1905)

It is easy to misunderstand the regretful heroism of these lines. They are not just articulating the professional disappointment of an elder man who spent more than half of his life as a private teacher of mathematics and languages at Boulogne-sur-Mer. The desperate pride has more pertinent reasons than a want for personal recognition:

On various occasions MacColl presents himself as a "peacemaker" (1880, p. 47), who intends to "bridge the gulf between Symbolic Logic and the Traditional" (1906a). However, a close inspection of his writings soon reveals a different stance. The principal assumptions and basic intuitions on which his mature system relies are not set forth as instruments for tolerant cooperation in theoretical matters. He rather presents them as natural proposals that require neither detailed exposition nor diligent justification. In a sense this trouble-free radicalism seems justified. For all in all he sets out from then fairly well received views on man's evolution and, in particular, on the development of language and communication. However, their forthright application to the fundamentals of logic was by no means a self-evident move. Without any discussion of traditional or modern alternatives MacColl promoted a new conception of logical form. Apparently he borrowed it from comparative linguistics, and in fact it proved itself a useful means to integrate an epistemic conception of logic with a teleological understanding of man's natural condition. Most likely, MacColl finally acquired his philosophical convictions during these twelve or thirteen years in which he "devoted [his] leisure hours to general literature."

The present paper is meant to assess the influence of this basic credo on his mature outline of symbolic logic. My investigation assumes that at least in his later works MacColl developed and kept a rather uniform understanding of human communication. By his later works I understand the articles and books on logic and philosophy as well as the novels he published after 1889. In this year his science fiction novel *Mr. Stranger's Sealed Packet* appeared. Here, and more explicitly in *Ednor Whitlock*, a second literary work he managed to publish two years later, MacColl promotes extravagant, philosophical speculations in rather poor, literary disguise. Towards the end of his life, he revised, extended, and published these philosophical convictions in a number of articles and, finally, in a book on *Man's Origin, Destiny and Duty*.

If one wants to offer a concise picture of MacColl's intellectual development and its impact on his logical system his literary works are an indispensable source. They manifest the fact that the formation of his metaphysical beliefs preceded or at least accompanied the design of the kind of symbolic logic on which he started to publish shortly before the turn of the century. Apparently, the order of his scientific or philosophical publications does not mirror this evolution accurately. MacColl's philosophical position, in particular his understanding of language and communication developed before he started to publish on his strictly propositional account of modal logic. The limited quality of his literary works minutely evidences their impact on his later works on logic.

Man's biological and cultural evolution was among the major issues European academics and their educated public were discussing in the second half of the 19th century. Darwin's revolution of biology as well as the rise of comparative studies in various disciplines, and especially in linguistics and religious sciences, were current topics of the intellectual magazines with which MacColl was familiar or to which he liked to contribute; in particular *Chambers Edinburgh Journal*, *The Westminster Review*, *The Quarterly Review*, *The Edinburgh Review*, *The Educational Times*, *The Athenaeum* or *The Hibbert Journal*. Foremost, the theological impact of Darwin's naturalisation of the human being was anxiously discussed, especially during the sixties and the seventies of the century. In his works beyond logic and mathematics MacColl takes up this issue. He addresses himself to the general, well-educated public—in particular to those interested in the dilemma of faith and modern science. MacColl unreservedly participated in the Victorian *Zeitgeist*, apparently with the self-confidence of the educated layman.

Ednor Whitlock was not a literary success. The critics naturally opposed MacColl's unbalanced presentation of theoretical issues in a work of fiction. One cannot avoid getting the impression that this description of a young man's difficulties in establishing his professional and social standing was but an unsuitable means to contribute to almost out-dated discussions on scientific limitations of the Christian faith. The present interest in MacColl's theological views does not result from historic curiosity. His views matter and thus will be presented in so far as they relate to his account of the linguistic form of an elementary logical structure.

Certainly, MacColl's attempt to reconcile Darwinian evolution theory with Paley's natural theology scarcely has genuine theoretical value. But any comprehensive and yet scientific understanding of man's being a part of nature affects the possibility of acknowledging language as a specific difference between man and brutes. Eminent linguists such as August Schleicher conceived of their science as a natural science, and read *The Origin of Species* as a confirmation of their stance. As we know, MacColl was familiar with this issue at least through the *Lectures on the Science of Language* by Max Müller. He, too, conceived of linguistics in contrast to philology as a physical and not as an historical science. In contrast to Schleicher, however, he still accepted Christian Theology as an appropriate basis for a theory of language.

On various occasions MacColl's later writings comment on anthropological implications in Müller's presentation of comparative linguistics. His at times critical remarks and their reference to the constitutive elements of his logical system will throw light on both the biological and the linguistic context in which MacColl conceives of *statements* as basic logical units. How their conception relates to this two-fold background is understood best if a short presentation of the system's guiding principles precedes its exposition.

2. The Conventional Articulation of Statements

In several places MacColl puts forward two philosophical principles on which the entire set-up of his account of logic relies. I quote from *Symbolic Logic and its Applications*:

> The first is the principle that there is nothing sacred or eternal about symbols; that all symbolic conventions may be altered when convenience requires it, in order to adapt them to new conditions, or to new classes of problems. The symbolist has a right, in such circumstances, to give a new meaning to any old symbol, or arrangement of symbols, provided the change of sense be accompanied by a fresh definition, and provided the nature of the problem or investigation be such that we run no risk of confounding the new meaning with the old. The second principle which separates my symbolic system from others is the principle that the complete statement or proposition is the real *unit* of all reasoning. Provided the complete statement (alone or in connexion with the context) convey the meaning intended, the words chosen and their arrangement matter little. (MacColl 1906b, pp. 1–2)

The second of these principles indeed mentions a most characteristic feature of MacColl's system. At first glance his strictly propositional founding of logic seems to be in line with Frege's position that a *Begriffsschriftsatz*, as the adequate articulation of either a true or a false *Gedanke*, is the genuine object of all logical investigation. On closer inspection, however, striking discrepancies between these two perspectives will quickly become evident. I will discuss them subsequently.

The first principle has not been advocated exclusively by MacColl. Its revisionism mildly echoes the famous variety of linguistic anarchy Humpty Dumpty sets forth in Lewis Carroll's *Through the Looking Glass* (1960, p. 269):

> "When *I* use a word," Humpty Dumpty said, in rather a scornful tone, "it means just what I choose it to mean—neither more nor less."
>
> "The question is," said Alice, "whether you *can* make words mean so many different things."
>
> "The question is," said Humpty Dumpty, "which is to be master—that's all."

Interestingly enough, Carroll's *Symbolic Logic*, which "rekindled the old fire", seems to admit of the same kind of conventionalism MacColl's first principle concedes:

> ... I maintain that any writer of a book is fully authorised in attaching any meaning he likes to any word or phrase he intends to use. If I find an author saying, at the beginning of his book, "Let it be understood that by the word '*black*' I shall always mean '*white*', and that by the word '*white*' I shall always mean '*black*'," I meekly accept his ruling, however injudicious I may think it. (Carroll 1896, p. 166)[1]

In MacColl's earlier outline of a "Calculus of Equivalent Statements", both principles are already at work. From his first publications in the *Proceedings of the London Mathematical Society*, he aims at an altogether propositional presentation of logical structures. Without hesitation he modifies a given notation like Boole's or his own previous conventions if only a theoretical purpose calls for a more adequate form of expression. Up to his last publications this style of writing mirrors the evolutionary prospect his first principle applies to matters of logic and language. When from 1896 onwards he presents his mature account of symbolic logic its anthropological presuppositions are made explicit. They are precisely those which are meant to justify his second principle.

[1]This quotation belongs to the introduction to a paragraph on *The "Existential Import" of Propositions*. In his comment on Humpty Dumpty's idiosyncratic stance, Martin Gardner quotes this introduction at length, and discusses the literary impact of Carroll's "nominalistic attitude". Cf. Carroll 1960, pp. 268–269.

> I define a statement as any sound, sign, or symbol (or any arrangement of sounds, signs, or symbols) employed to give information; and I define a proposition as a complex statement, which, as regards form, may be divided into two parts, respectively called subject and predicate. ... The sound of a signal gun, the national flag of a passing ship, and the warning "Caw" of a sentinel rook are by this definition statements, but not propositions; whereas "We are in danger," " This is a British ship," "A man is coming with a gun," are propositions, and therefore statements as well. ... In thus taking statements as the ultimate constituent units of symbolic reasoning I believe I am following closely the gradual evolution of the human language from its primitive prehistoric forms to its complex developments in the languages, dead or living, of which we have knowledge now. There can be little doubt that the language or languages of primeval man, like those of the brutes around him, consisted of simple elementary statements indivisible into subject and predicate, but differing from that of even the highest order of brutes in being uninherited—in being more or less conventional, and therefore capable of indefinite development. (MacColl 1906b, p. 2, or MacColl 1903, p. 131)

This comment evidences that MacColl's guiding principles have a common root. Both reflect an evolutionary concept of man.

The second principle accounts for the natural preconditions of man's intellectual ascent. The basic form of his means of communication is supposed to match those of his natural partners. Just like human beings, brutes are taken to communicate by an exchange of statements. In MacColl's view man transcends the realm of natural life. But still the means of his intellectual sovereignty are conceived of in such a way that man's interaction with brutes can be described in terms of their dispositions to produce or grasp statements.

As statements are the form in which information is transferred, this principle will be subsequently referred to as MacColl's *information principle*. The first one, however, will be labelled as his *semiotic principle*. For it accounts for man's semiotic disposition to raise himself above his natural condition.

MacColl actually assumed the existence of beings intellectually superior to man. So bold an assumption is pointless unless it presupposes man's possibility to know of them accordingly—if not to communicate with them. He firmly intended to protect religious belief against various epistemological or metaphysical incentives. However, his trust in science let him likewise acknowledge that religious beliefs should be sufficiently rational as to comply with possible scientific progress: No one should believe what, in principle, cannot be known.

When MacColl stresses that "there is nothing sacred or eternal about symbols" (1903, p. 131), he is not just advocating that their coherent choice is entirely at our disposition. His comments on his first guiding principle evidence its constitutive sense. Man's ability to replace a given statement with an equivalent, though not synonymous, rendering foremost allows for a future increase in structural knowledge:

> ... if words were always restricted to their primary meanings no human language could ever have been developed, abstract ideas could never have been formed, and science and philosophy would never have come into existence. Words are mere symbols to which we may assign any convenient meaning that suits our argument, provided we make it perfectly clear, by definition or context, what that meaning is. (MacColl 1910, p. 198)

In the present context I have to refrain from discussing the relevant kind of epistemic progress in a more detailed manner. His various comments on the philosophical relevance and purpose of his logical system confirm the present consideration of its guiding principles. In a review of Alfred North Whitehead's *A Treatise on Universal Algebra with Applications*, Vol. I, MacColl writes:

> The ultimate units of expressed thinking, whether those units be individually communicated to ear or eye by single symbols or by many, are *statements*; and in no sphere or region of investigation can reasoning be expressed *without* those units. Since, therefore, statements, and statements alone, constitute the ever indispensable elements of all expressed reasoning, we should, in my opinion, first investigate the mutual relations of these statements, representing each by its own independent symbol, and call this process of investigation *Pure Logic*. The moment we begin (as in mathematics and in the traditional logic) to represent *things*—things which are not statements—by separate symbols, we are no longer in the domain of *Pure* (or Abstract) Logic, but in that of *Applied* Logic. A system of Symbolic Logic thus built up wholly of statements has one great advantage which no other system can possibly possess, namely, the advantage of *homogeneity of matter*. (MacColl 1899, p. 109)

In a résumé of his contribution to the *Ier Congrès International de Philosophie* at Paris he equally points to the need of a uniform set-up of logic in terms of its propositional basis:

> ... quel que soit le sujet de recherche, tout raisonnement, pour pouvoir s'exprimer, demande des propositions. Donc, pour rendre notre raisonnement parfaitement général, et nos formules universellement applicables, nous devons prendre la classification des différentes espèces de proposition et les rapports entre elles comme le premier but de notre recherche, et appeler ce travail la Logique pure. (MacColl 1901, p. 135)[2]

Man's evolution under natural conditions induces the unity of MacColl's principles. In contrast to Frege's *Begriffsschrift* a symbolic

[2] "... whatever be the subject of research, in order to articulate itself all reasoning requires propositions. Hence, in order to make our reasoning perfectly general and our formulae universally applicable, the first goal of our research must be the classification of the different kinds of propositions and the relationships between them, and we have to call this work Pure Logic." (translation M.A.)

language in MacColl's sense is by no means a "Formelsprache des reinen Denkens" under the constraints of human intuition.[3] Even as the language of Pure Logic it is an authentic record of man's intellectual evolution. MacColl never refers to a realm of pure thought. In the first instance his theory of logic is meant to identify and to investigate the invariant constituents of any form of expressed thought; i.e. statements and the inferential order in which they matter. And if, moreover, all ramifications of logic are but accurate renderings of man's epistemic development their presentation should not involve more than applications of MacColl's semiotic principle.

3. Historical Contexts

Discussing MacColl's basic design of logical form will not be sufficiently fruitful unless the historical contexts of its motivation have been introduced properly. The two principles on which his mature system relies stand in for an evolutionary conception of man's logical competence: In former periods of natural history man was indistinguishable from brutes. Slowly—by a process which Darwin's evolution theory describes in terms of natural selection—the human being has overcome the limits in reasoning and communication to which his natural peers are definitely confined. The information principle accounts for man's continuity with his natural past and present. The semiotic principle accounts for his possibilities to improve even now his future means of rational orientation. In order to understand MacColl's outline of symbolic logic we have to know what he understood by evolution and, in particular, how he conceived of the origin and development of human language.

When it comes to matters of evolution MacColl scarcely mentions and never criticizes Charles Darwin. He fiercely attacks Haeckel's monism and tries to defend Paley's creationism. But his attitude to the author whose writings instigated the debate on man's natural history remains impartial. We do not know whether chance or purpose is responsible for this reservation. However, with MacColl's literary publications man's natural and cultural evolution becomes a major subject of his writings beyond logic. He thus participates in the Victorian debate on biological evolution by natural selection and Divine Providence in Creation.[4]

In MacColl's science fiction novel *Mr. Stranger's Sealed Packet* this issue essentially conditions the imaginary plot. On Mars, Mr. Stranger

[3] Cf. Frege 1882, in particular p. 56.

[4] For a detailed presentation of this controversy cf. Roppen 1956, pp. 1–63.

encounters a form of culture morally superior to his own and the reader's familiar context. As we learn, these Martians are in fact human beings who during the earth's glacial period in a still inexplicable way were transferred to Mars.

> I found to my surprise that they now lived very happily under a form of socialism; but a socialism very different from what we commonly hear advocated, and which will only be possible on earth when science has learnt to place the means of subsistence and comfort within the reach of all. These conditions existed on this planet. Here there was no struggle for existence. There was no necessity for the sowing of corn or the slaughter of animals for the support of human life. Their science, if behind ours in some respects, was far in advance of it in others. (MacColl 1889, pp. 102–103)

In this passage Darwin's key metaphor of a struggle for existence occurs in a context of political economy. It reads as a reference to T. R. Malthus's *Essay on the Principle of Population* in which the phrase originally was coined. The Martian exception from this rule of human existence does not pertain to others living on the planet. Martian nature and the menacing tribes in the neighborhood of Mr. Stranger's hosts are still competing for the survival of the fittest.

> I soon found that here also, as on earth, cruel blots marred the beauty of nature. Here, also, the inexorable law prevailed that life must be sacrificed to sustain life: the life of many for the sustenance of one. (MacColl 1889, p. 59)

The following illustration, however, is not confined to the relation between a predator and its prey, but includes a desperate combat between rival predators: " … the long grey tiger was still growling over his sickly meal, … when I saw stealthily creeping up to the scene from another cluster of bushes another of the same species of carnivora" (1889, p. 60). Both animals die in the desperate combat they cannot avoid. This example clearly refers to Darwin's understanding of a "Struggle for Existence". His introduction of the metaphor in *The Origin of Species* stresses that competition is most severe between individuals and varieties of the same species (Darwin 1998, p. 59). More explicitly the discussion on evolution theory in *Ednor Whitlock* refers to Darwinian subjects. Mr. Manning, the atheist participant, considers the development of the eye as an exemplary case of accidental adaptation:

> Still higher in the scale of animal life we find the same rudimentary organ in a more forward stage of development, and conferring upon its possessor some slight advantage in the struggle for existence over animals closely resembling it in other respects but destitute of this one incipient faculty. … there is a general balance in favour of modifications and variations that tend towards the

> improvement of useful organs. This improvement going steadily on through many generations, and for ages upon ages, *though each infinitesimal onward step is purely accidental*, attains at last such an approximation to perfection in the higher types of animals that the unthinking multitude, marvelling at the grand result and ignorant of the true explanation, not unnaturally attribute the whole to a mighty and intelligent Creator. (MacColl 1891, p. 59)

Obviously, the passage refers to Darwin's discussion of *Organs of extreme Perfection and Complication*. Here, too, the visual organ serves as a possible counterexample against the working of natural selection:

> When it was first said that the sun stood still and the world turned round, the common sense of mankind declared the doctrine false; but the old saying of *Vox populi, vox Dei*, as every philosopher knows, cannot be trusted in science. Reason tells me, that if numerous gradations from a simple and imperfect eye to one complex and perfect can be shown to exist, each grade being useful to its possessor, as is certainly the case; if further, the eye ever varies and the variations be inherited, as is likewise certainly the case; and if such variations should be useful to any animal under changing conditions of life, then the difficulty of believing that a perfect and complex eye could be formed by natural selection, though insuperable by our imagination, should not be considered as subversive of the theory. (Darwin 1998, pp. 227–228)

The similarities between these texts indicate that MacColl's presentation of the atheist's position was most likely inspired by Darwin's seminal book.

MacColl's strong interest in man's natural and prehistoric past includes a continuous concern with the origin and development of human language. Even before Mr. Stranger learns about the terrestrial origin of his Martian friends he infers its probability from the form of their language:

> I had learnt something of philology at a college, and was now much surprised to find that the Marsian language had much in common with the Indo-European languages. This was particularly noticeable in their numbers ... Their language had only two genders; but nouns and adjectives had four cases, distinguished by inflections, which were generally at the beginning, and not at the end of words. Their verbs were very simple and regular, and had only three tenses, present, past, and future. The perfect, plusperfect, and future perfect were expressed by circumlocutions. (MacColl 1889, p. 101)

Considerations of this sort could have been inspired by Max Müller's *Lectures on the Science of Language*. In some of his later works MacColl refers to this comprehensive presentation of linguistics which occasionally presents the history of language in quasi-Darwinian terms. Müller like Schleicher refuses to speak of comparative linguistics as a branch of *philology.* Neither of them conceives of linguistics as a historical science. Müller's text indicates, however, that *Comparative Philology* was a common, though in his view misleading, description of his subject.[5]

MacColl's attention to linguistic issues was not merely due to his writing science fiction. In 1884 he participated in a public debate on the grammatical classification of English phrases. The controversy took place in the *Educational Times* to which he contributed from 1864 until his death.[6]

Both novels refer to the religious perspectives in which MacColl conceives of man's evolution. On most occasions he defends the self-asserted stance of a natural theologian trained in logic and mathematics, but still tries to hold up the existential conviction of a fervent Christian believer. Neither in his earlier literary nor in his later metaphysical writings does he seem to aim at a reconciliation of the two apparently conflicting attitudes with one another. For his evolutionary conception of language and logic only his adherence to philosophical theology matters.

4. The Argument from Design

4.1. Paley's natural theology

Darwin discusses the "Extreme Perfection and Complication" of visual organs as a possible counterexample to natural selection. Paley's famous argument from design relies on this very case as an exemplary indication of God's purposeful creation of living beings. However, in contrast to both Darwin's discussion and MacColl's quoted reference to his argument, Paley's presentation does not cover the evolutionary aspects of the case. He merely compares the functional perfection of the eyes of vertebrates with the set-up of man-made telescopes. Interestingly enough, Darwin criticizes Paley's reasoning by analogy from a theological point of view:

> It is scarcely possible to avoid comparing the eye with a telescope. We know that this instrument has been perfected by the long-continued efforts of the highest human intellects; and we naturally infer that the eye has been formed by a somewhat analogous process. But may not this inference be presumptuous? Have we any right to assume that the Creator works by intellectual powers like those of man? (Darwin 1998, pp. 227–228)

Here, Darwin points to a weakness in Paley's presentation of the argument from design that a more sophisticated defender of a similar theological claim, for instance MacColl, certainly would like to avoid:

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[5] Cf. Müller 1994, pp. 20–24, as well as Schleicher 1863, pp. 6–7.

[6] Cf. MacColl 1884a, and MacColl 1884b.

Paley's argumentation by analogy starts from man-made objects whose mechanical features exhibit contrivance and design in view of an end of their usage in daily life. The unity of their instrumental features is taken to indicate by necessity their intentional production:

In crossing a heath, suppose I pitched my foot against a stone, and were asked how the stone came to be there: I might possibly answer, that, for anything I knew to the contrary, it had lain there for ever; nor would it perhaps be very easy to show the absurdity of this answer. But suppose I had found a watch upon the ground, and it should be inquired how the watch happened to be in that place; I should hardly think of the answer which I had before given, — that, for any thing I knew, the watch might have always been there. Yet why should not this answer serve for the watch as well as for the stone? Why is it not as admissible in the second case, as in the first? For this reason, and for no other, viz. that when we come to inspect the watch, we perceive (what we could not discover in the stone) that its several parts are framed and put together for a purpose, e.g. that they are so formed and adjusted as to produce motion, and that motion so regulated as to point out the hour of the day; ... This mechanism being observed (it requires indeed an examination of the instrument, and perhaps some previous knowledge of the subject, to perceive and understand it; but being once, as we have said, observed and understood), the inference we think is inevitable, that the watch must have had a maker: that there must have existed, at some time, and at some place or other, an artificer who formed it for the purpose which we find it actually to answer: who comprehended its construction, and designed its use. (Paley 1807, pp. 1–2)

In the sequel, living beings, and foremost their various parts, are subjected to this kind of instrumental reasoning:

... every indication of contrivance, every manifestation of design, which existed in the watch, exists in the works of nature; with the difference, on the side of nature, of being greater and more, and that in a degree which exceeds all computation. I mean that the contrivances of nature surpass the contrivances of art, in the complexity, subtility, and curiosity of the mechanism; and still more, if possible, do they go beyond them in number and variety; yet, in a multitude of cases, are not less evidently contrivances, not less accommodated to their end, or suited to their office, than are the most perfect productions of human ingenuity.

I know no better method of introducing so large a subject, than that of comparing a single thing with a single thing; an eye, for example, with a telescope. As far as the examination goes, there is precisely the same proof that the eye was made for vision, as there is that the telescope was made for assisting it. (Paley 1807, pp. 19–20)

In a later passage of his book Paley confirms explicitly that he conceives of design foremost in mechanical terms:

My object ... has been to teach ... that the *mechanical* parts of our frame, or, those in which this comparison is most complete, although constituting, probably, the coarsest portions of nature's workmanship, are the most proper to be alleged as proofs and specimens of design. (Paley 1807, pp. 99–100[7])

MacColl's presentation of an argument from design does not repeat this most inadequate propensity. Its probabilistic rendering avoids any pragmatic preconception of natural design. A more detailed account of Paley's original account of "natural history applied to the proof of an intelligent Creator" (1807, p. 372) will allow for a comprehensive understanding of MacColl's *broader* notion of design and its relevance for his evolutionary outlook on language and logic. Some essential aspects of Paley's concept of design can be summarized as follows:[8]

Neither ignorance as regards an object's producer, the act and method of its production nor inability to produce or at least to reproduce the relevant object can hinder the recognition of its design (p. 4). Likewise, neither imperfections of the object itself nor an incomplete understanding of its functioning nor its partial misidentification can preclude the grasp of its purposeful composition (pp. 4–6).

In Paley's view it is pointless to account for an object's design in terms of a contingent regularity or by an assumption of creative principles. Any order the object's composition exhibits is supposed to be nothing but an intentional application of the laws of nature by its intelligent producer (pp. 6, 77–79, and 452–453). Order and design, however, are not universal:

In the forms of rocks and mountains, in the lines which bound the coasts of continents and islands, in the shape of bays and promontories, no order whatever is perceived, because it would have been superfluous. No useful purpose would have arisen from moulding rocks and mountains into regular solids, bounding the channel of the ocean by geometrical curves; or from the map of the world, resembling a table of diagrams in Euclid's Elements, or Simpson's Conic Sections. (Paley 1807, p. 79)

Natural history relies on reproduction. Hence the primarily mechanical model adopted in Paley's argument has to incorporate features which will allow that his "argument from design remains as it was" (p. 7). In fact it does only if an object's disposition to mechanical reproduction is embodied in the original design all reproductions inherit and pass on. In contrast to objects bringing about their reproduction their genuine producer is taken to be the author of their design, i.e. the cause of the relation of their parts to their use (p. 6). Production and reproduction are thus distinguished. The production of an

[7]On the relationship between law and mechanism, cf. p. 453.
[8]Page numbers in the remainder of this section refer to Paley 1807.

object or of its reproducible kind includes the design their reproduction presupposes. Whether its repeated realization is brought about by the reproduced objects themselves or by some other cause does not affect the basic distinction between *creative* and *executive* authorship (pp. 54–60). The number of intermediate reproductions, being finite or infinite, cannot abolish the difference between these two kinds of authorship (pp. 13–15).

Paley distinguishes between an object's design and the laws of nature to which it conforms in having this design. All design consists in an application of laws. There are no laws whose pertinence did not presuppose agents that proceed in accordance with them. Foremost, God's creation of living beings is an application of the laws of nature. In their intentional production of useful things, human creatures—or agents of higher ranks—may follow his very example, and being designed in this way they are able to recognize and praise him as the designer of all that lives, and, in particular, as their own creator (pp. 7 and 42–46).

Paley's natural theology assumes that God has made the laws of nature and by doing so has limited his creative powers to their application. God's reason for this twofold creation is a didactic one. He thus enables his creatures to recognize him as their thoughtful creator (p. 43). Whereas the design of individuals or of their species answers to their own needs as well as to their use for others the creation of animate nature as a whole aims at God's recognition and praise by his creatures.

Chance as an origin of evil is a major threat to any natural theology which conceives of nature as the purposeful creation of a benevolent deity. At the end of his book Paley approaches the decisive issue: At first sight his remarks on chance seem to suggest an ontological account of contingency. Chance seemingly occurs where designs interfere (pp. 558–559). However, this does not mean that they actually leave room for a contingent course of events. It rather says that they interfere for man's grasp of things. Once an observer properly knows the design of things no room for real chance is left. What is usually called chance is but apparent chance. All uncertainty about an event's occurrence or non-appearance is due to a lack in information about the design of the involved objects. Paley subscribes to an epistemic concept of chance or contingency (pp. 559–560).

The theological relevance of Paley's threefold, epistemic reference of modalities—contingency, certainty, and uncertainty—lies in his account of evil. Natural theology results in theodicy. For Paley evil is but a passing phenomenon. It is the way things *appear* to us on behalf of our intellectual and, presumably, emotional deficiencies. To the extent that man succeeds in understanding natural design he will refrain from complaining about the world's imperfection. This intellectual exercise is by no means futile: God speaks to man through the phenomena of natural design. Man's irritation is meant to challenge him. The natural life to which he is bound is a form of *probation*. Moral improvement ultimately consists in an acquisition of knowledge (pp. 570–573).

In this final respect it is no surprise that Paley's argument presupposes a very optimistic account of man's epistemic capacities and their realistic conditions. An object's composition never counts just as a motive for believing in its intentional production, but at any rate is a proof thereof. Mere knowledge about an object's purpose is sufficient for the recognition of its being someone's product (p. 7).

4.2. MacColl's probabilistic parable

In all his later writings MacColl advocates his teleological account of evolution in a literary form frequently used for religious and moral purposes. He chooses a "parable" (1906–1907, p. 385) to present his view. Between the first presentation of this illustration in Ednor Whitlock and its final occurrence in *Man's Origin, Destiny and Duty* its content does not vary much. This last version offers the most explicit rendering of MacColl's metaphysics and thus should be quoted here:

> Every mathematician who has studied the theory of local probability and averages will admit, and even tyros in mathematics can prove by actual experiment, though the experiment would in general be long and laborious, that chance, working within the limits of prescribed conditions, can be made to evolve with almost perfect accuracy in every detail, foreseen, designed, predetermined figures of various forms, sizes and shadings. . . . The advanced mathematician who prescribes the law or conditions, which the random points constituting the future shaded figure must not transgress, knows beforehand almost every detail of this figure as regards size, shape, distribution of shading; but the mathematical tyro who laboriously carries out the random, or seemingly random, process by which the figure is slowly evolved, point by point, from an apparent chaos into its final foreordained form and shading, may foresee nothing of this final and (to him) astonishing result. If two mathematical tyros carry out the random pointing independently, and the process be continued long enough, they will finally evolve two figures almost exactly alike in size, shape, and shading—provided, of course, the random points of which they are composed be subjected to the same restrictions as to laws and limits. (MacColl 1909, pp. 101–102)

If one considers that MacColl subscribes to a substance dualism of physical and psychic entities,[9] the sense of his literary image becomes

[9] Cf. MacColl 1907–1908 pp. 167–168, and MacColl 1909, pp. v and 1–27.

accessible rather easily. The advanced mathematician is to be taken as a divine designer. The laws or restrictions he sets forth are meant to specify the general frame to which any course of events in a bodily world has to conform. Divine creation consists foremost in a wilful specification of these laws. The calculating activity of a mathematical tyro stands for a possible course of natural events. As a result of his calculations and in accordance with a uniform method the person will put a series of points on a sheet of paper. Each configuration of points stands for a particular situation in a possible course of natural history. A random point his calculation allows him to add is supposed to represent a contribution to evolution. Points he may not add stand for changes that do not contribute to the articulation of the intended development. In each case the calculations allow a decision to be made about the value of the change at issue. Irrespective of the order in which the value of the points is assessed, nearly the same configuration will finally present itself—provided the procedure is carried out for a sufficient amount of time. With reference to a mathematical theory, though certainly not with reference to this theory alone, MacColl claims the following: Random alternatives in the course of natural history are not relevant unless a set of laws to which they equally conform is presupposed. Owing to their enormous length these alternative courses of events will finally lead to converging results. Each of them will *cum grano salis* exhibit the same purposeful design. Natural selection in the struggle for existence is based on a random procedure. Hence evolution theory and a teleological account of nature are not inconsistent with one another. MacColl joins the side of those naturalists, philosophers and theologians, for instance A. Gray, C. Kingsley, J. S. Mill, G. Mivart or J. M. Wilson, who argued for a reconciliation of scientific biology and the Christian Faith.[10]

The Darwinian account of natural history does not explain why biological individuals vary from their parents. The theory focuses on the transmutation of species, and explains it in terms of natural selection. Individual variation merely counts as a necessary condition of the species' instability. For the present context Alvar Ellegård's discussion of this fundamental issue is most instructive:

> . . . though several passages in the *Origin* were liable to obscure the issue, those who really followed the argument of the book could hardly be in doubt as to the nature of the new theory. It explained all the phenomena of adaptation as due to differential preservation of random variations. It is true that Darwin did not use the word *random*, and that whenever he employed such terms as *chance* or *accidental* he was careful to explain that he meant thereby that the

[10] Cf. Roppen 1956, pp. 31–34 as well as pp. 62–63.

causes were unknown: they could not be connected with any specific internal or external conditions. Still, though Darwin—like Huxley—might profess that he believed in strict determinism for all natural phenomena, it could hardly be denied that, in the ordinary sense of he word, he was ascribing the production of the variations to chance. If Darwin had admitted this—as he might have done—he would have brought out more clearly the revolutionary nature of his explanation. But he refused to admit it, and thus laid himself open to one of the commonest criticisms of his theory. It was declared to be incomplete as long as the real causes of the individual variations from parents to offspring were not indicated. Since the causes were known neither to Darwin nor to anybody else, why not then concede that they might be above the reach of mere science? ... His reluctance to entertain and to come to grips with the idea of randomness appears from the fact that he never published the results of any investigation to show that the variations were in fact wholly indefinite. He probably would not have known how to carry out such an investigation: statistical techniques were not available to him. (Ellegård 1956, pp. 186–187)

In his parable MacColl tacitly accepts Darwin's predominant concern with the transmutation of species. Individual variation is accounted for in terms of the initial values from which the mathematical tyro starts each of his calculations. Each of them determines the occurrence of a point on the sheet of paper and thus stands for an instance of natural selection. Under the presupposition of his dualistic metaphysics MacColl accepts natural selection as a means for the realization of a pre-established design. Unfortunately, he says very little about the "laws or restrictions" regulating the course of natural history. His presentation of the parable does not tell us how strict they are, or how indeterminate they may be. Otherwise it would have been easier to compare MacColl's understanding of chance and design with C. S. Peirce's account of evolution. Apparently in contrast with MacColl he intends to conceive of the natural laws themselves in statistical terms.

The main element of habit is the tendency to repeat any action which has been performed before. It is a phenomenon at least coëxtensive with life, and it may cover a still wider real realm. Imagine a large number of systems in some of which there is a decided tendency toward doing again what has once been done, in others a tendency against doing what has once been done, in others elements having one tendency and elements having the other. Let us consider the effects of chance upon these different systems. To fix our ideas suppose players playing with dice, some of their dice are worn down in such a way that the act of losing tends to make them lose again, others in such a way that the act of losing tends to make them win. The latter will win or lose much more slowly, yet after a sufficient length of time they will be in danger of being ruined and if the game is quite even, they will eventually be ruined and destroyed. Those whose dice are so worn as to reproduce the same effects, will be divided into two parts, one of which will quickly be destroyed, the other made stronger and stronger. For every kind of an organism, system, form, or compound, there is an absolute limit to a weakening process. It ends in destruction; there is no limit to strength. The result is that chance in its action tends to destroy the weak & increase the average strength of the objects remaining. Systems or compounds which have bad habits are quickly destroyed, those which have no habits follow the same course; only those which have good habits tend to survive.

May not the laws of physics be habits gradually acquired by systems. (Peirce 1992, pp. 223)

Peirce and MacColl knew each other. In view of Peirce's high esteem for MacColl their intellectual relationship deserves further investigation.

It would be misleading to discuss the genuine value of MacColl's contentions any further. Naturally, they provoke the kind of reservation Hume expresses in his *Dialogues Concerning Natural Religion.*[11] Here, they matter only in so far as his evolutionary account of language and logic depends on a concept of design. This goal requires a number of comments on the metaphysical presuppositions of MacColl's parable. They will make clear to what extent and in what way MacColl's teleological stance differs from Paley's argument.

Just like his theological predecessor, MacColl describes animals or their parts as being well equipped for the struggle for existence. However, references to an object's pragmatic value in daily life do not enter MacColl's attempt to argue for a teleological account of natural history.

Already in 1882 MacColl firmly advocates an objective account of probability,[12] and to my knowledge he never changed his view. Even so the mathematical tyros of his parable are supposed to be "astonished" about the gradually appearing design of a figure. The illustration should be understood primarily as a parable on nature, and not as an illustration of man's way of recognizing the laws to which it conforms. The configurations of points are to be taken as an articulation of design *in things*.

In contrast to Paley, MacColl is not concerned with individuals or species showing a particular design, but with a natural course of events in which an overall design of objects gradually articulates itself. In view of the continuity of this process neither imperfections of particular objects or kinds of objects nor deficiencies in man's grasp of their purposeful set-up, are of any particular relevance.

MacColl identifies the design of objects with the laws to which they conform. In contrast to Paley, he does not conceive of design and

[11] Cf. Hume 1990, especially chapter 8.
[12] Cf. MacColl et al. 1882.

contrivance in terms of an *application* of laws. Finally, man's grasp of design consists in his discovery and mathematical rendering of the laws of nature. Consequently, MacColl does not restrict design to animate objects. His theology aims at a thorough reconciliation of science and religion.

As MacColl does not distinguish between law and design, it is not relevant for his version of creationism to distinguish between the executive authorship of natural beings and the creative authorship of a divine designer applying his laws in accordance with their particular needs and tasks. The only creative act MacColl admits of consists in the choice of laws of the Supreme Being (1909, pp. 89, 105). Nevertheless, he repeats Paley's mechanistic view on natural reproduction and even adapts it to an evolutionary account of varied reproduction (pp. 128–129). However, neither of these considerations enters his holistic account of design.

As regards the ultimate purpose of nature MacColl agrees at least in principle with Paley's didactic understanding of God's creation (MacColl 1909, p. vi). In MacColl's view the human body, i.e. the condition of man's natural existence, is an "instrument of education", a means for his gradual intellectual and moral ascent to a higher form of existence (1907–1908, pp. 167–168). Paley concedes the possibility of higher ranks of agents between man and God. MacColl is convinced of their existence and assumes corresponding forms of bodily and mental life. Although he refuses to conceive of the difference between physical and psychic entities in terms of material and immaterial units, he holds that they can exist independently of one another (1907–1908, p. 165). In his view, a soul, i.e. an entity "which is not always unconscious" (1907–1908, p. 158), is neither an abstraction from states of other, physical objects, nor is it identical with a set of special kinds of events. In this respect MacColl's understanding of "the Ego, the Soul, the real Person" obviously follows the metaphysical tradition of Joseph Butler and Thomas Reid, which in contemporary philosophy is maintained primarily by Richard Swinburne.[13]

Just like Paley's natural theology, MacColl's metaphysics is designed as a theodicy. The hazards of nature are inevitable if man or any other being is supposed to have a chance of learning by its own experience. The lasting knowledge that humans or other beings can acquire *themselves*, but only under the temporary condition of phys-

[13] Cf. MacColl 1909, pp. 74–75. For historical and contemporary debates on personal identity, cf. Perry 1975 and Shoemaker and Swinburne 1984. On contemporary discussions of the argument from design, in particular with regard to Swinburne's position, cf. Garcia 1997 as well as Taliaferro 1998, pp. 365–369.

ical or moral evil, ascertains its instrumental value not only for the improvement of the sentient individual but also for the universe of its existence.[14]

In contrast to Paley and Christian theology, MacColl approves of reincarnation. He assumes that human souls may have different lives in which they can even attain superhuman forms of existence. His understanding of evolution exceeds natural evolution. This presumably theosophist aspect of his metaphysics pertains to his understanding of language. To some extent it depends on MacColl's overall conception of *life*[15] or bodily existence. On the one hand, MacColl advocates the same traditional view as Paley and conceives of living bodies as machines. Accordingly, animal bodies are taken to be automata.[16] On the other hand, he conceives of bodies as "instruments of education" whose usage is essential for a being's intellectual and moral ascent:

> This body its guardian the ego loses sooner or later, in childhood through illness or accident, or in old age through decay. Then it receives another instrument of education, whether human or superhuman may depend upon the ego's fitness and development. This, in due course, or through accident, it loses in its turn, after which it receives another, and so on for ever—always rising in the long-run (though not always steadily and continuously) from higher to higher, and from better to better. (MacColl 1907–1908, p. 168)

For MacColl, evolution—be it natural or supernatural—is bound to forms of bodily existence, i.e. to lower or higher forms of life. Purposeful change and progress presuppose guided bodily movement. The didactic sense of God's creation depends on it, especially as regards any acquisition and transfer of information or knowledge which is part of a being's intellectual ascent. A soul may not be able or willing to express what it thinks or feels. However, all higher forms of communication, and foremost God's communication with man, finally depend on an ability to *express* information under bodily conditions. Psychic and physical phenomena as a whole exhibit God's design of an evolving and improving universe.

> These phenomena may be regarded in one sense as God's language to reveal his purpose and his will—a language which it is man's duty to study, and which he will understand more and more as the years roll on. As one generation succeeds another, each passes on the knowledge which it has acquired to the generations which follow. ... Man learns this divine language as the child

[14] Cf. for instance MacColl 1909, pp. 38–39, 77–78, 83 and MacColl 1906–1907, p. 387.

[15] Cf. for instance MacColl 1909, pp. 3–4, 10, and MacColl 1906–1907, p. 387.

[16] Cf. for instance MacColl 1909, p. 128.

learns its mother's tongue, by observation, experiment, and slow, inductive reasoning. (MacColl 1907–1908, pp. 77–78)

Obviously, MacColl draws on a traditional metaphor. Nevertheless, this *façon de parler* is compatible with his concept of a statement and the information principle that underlies his account of logic. The subsequent considerations will present MacColl's linguistic conception of logical form as being coherent with his teleological metaphysics.

5. A Linguistic Account of Logical Form

In MacColl's view man's "faculties both of symbolisation and of introspection" (1907–1908, p. 115) definitely establish his superiority over all other animals.

> ... the difference between the lowest human savage and the highest animal of any other species is a chasm which no evolutionary theory hitherto enunciated can adequately explain. It is not merely a difference of degree; it is a difference of kind. (MacColl 1909, p. 107)

As MacColl explains, this difference consists in man's ability to invent, acquire or develop "a conventional code of representative sounds or symbols" (1909, p. 114) in order to express and convey thoughts or feelings. Abstraction and reasoning presuppose a disposition of this kind. It is most likely that man's earliest varieties of language were indistinguishable from the means of communication of which higher animals dispose. Nevertheless, MacColl contends that man's form of communication is not a fruit of natural selection or, to say the least, of natural selection alone.

> ... then, as now, his language was not instinctive and inherited. *It was of his own formation.* The first real man (or woman) was the first of human or humanlike shape and structure who possessed the faculty not merely of speech but of conscious speech-development—the faculty of representing ideas (in order to remind himself or give information to others) by arbitrary sounds or symbols. (MacColl 1909, p. 115)

In MacColl's view, human language merely *occurs* in natural history. The means of communication of animals, however, *evolve* in its course. Despite the evolutionary dualism of his metaphysics, MacColl calls these means a language, and even accounts for its form as follows: Animals inherit the kind of sounds or symbols which they produce instinctively. None of these units is composed of more elementary ones. They lack any explicit internal structure. Owing to their simplicity, statements of this sort convey information rather vaguely

(1909, p. 113). Each statement an animal brings about is a *datum* from which other animals can draw elementary conclusions. Although MacColl says in other places that statements "constitute" or "form" data (ibid.), the following quotations make me assume that he conceived of data as statements:

> With birds and brutes ... each separate sound or symbol is a complete statement—subject and predicate being, as it were, rolled into one. The warning "caw" of a sentinel rook is a *datum* expressly given to the rest that they may therefrom conclude that danger of some kind is at hand. (MacColl 1909, p. 112)

> The first of human form who barked a tree or erected a heap of stone or other simple monument that he might afterwards remember by its suggestion where he had buried or otherwise hidden a provision of food for himself or his family, performed therein an act which (whatever may have been the material constitution of his brain) stamps him at once as human. ... The barked tree or stone monument (like a knotted handkerchief in modern times) was a symbol or *datum* in order to give information ... (MacColl 1909, p. 115–116)

In the sense outlined any physical or psychic phenomenon experienced by human beings can be regarded as a statement or datum that a Supreme Being expressly gives to human beings. And if it is man's duty to study and learn from natural phenomena, then he is certainly bound to draw conclusions from them. In MacColl's view all means of communication are data, thus possibly composite statements contributing to an inferential structure. Divine language is no exception to this rule. Pure logic being nothing but the logic of statements therefore may be regarded as a universal kind of logic. MacColl explicitly confirms this interpretation by his remark that logical consistence limits divine omnipotence (1909, p. 38).

His most general account of language obviously includes all phenomena that relate to the overall disposition of an animal, of a human or of a superhuman being to receive and to provide data. Accordingly, MacColl presents his most general outlook on means of communication in terms of *information* processing, and he relies on the same conceptual tools in order to introduce epistemic modalities:

> The symbol A^ϵ asserts ... that A is *certain*, that A is *always* true (or true in *every case*) within the limits of our data and definitions, that its probability is 1. ... The symbol A^η asserts ... that A contradicts some datum or definition, that its probability is 0. ... The symbol A^θ (A is a *variable*) is equivalent to $A^{-\eta}A^{-\epsilon}$; it asserts that A is neither *impossible* nor *certain*, that is, that A is *possible* but *uncertain*. In other words, A^θ asserts that the probability of A is neither 0 nor 1, but some proper fraction between the two. (MacColl 1906b, pp. 6–7)

Already in his novels he is using the term *datum* in an inferential sense. When reading a work like *Ednor Whitlock* one can scarcely avoid the impression that at least its main characters have been attending seminars in modal logic à la MacColl. The novel clearly witnesses that MacColl's epistemic account of modalities and his theological preoccupations coincide:

> "The conclusion is not far wrong; but on what grounds did you build it?" "They are very simple. Firstly, the handwriting is that of a gentleman, for I saw the address; secondly it is a very long letter, for which the sender had to pay double postage; thirdly, your eyes brightened and you got red when you received it; fourthly the reading of it absorbed your whole soul; and fifthly, you sighed a deep but not an unhappy sigh when you had finished it. Those are my data, five in number. The inference is inevitable: it is a love-letter. ... " (MacColl 1891, p. 50)

The theist, the atheist and the agnostic whose discussion on adaptation, evolution and design young Ednor may follow are nothing but allegories for MacColl's basic modalities. They represent certainty, impossibility and variability in theological matters (MacColl 1891, pp. 54–79). Likewise *Mr. Stranger's Sealed Packet* results in a "Critical Discussion" about the *scientific* validity of Mr. Stranger's report on his voyage to Mars. Here, too, the arguments turn around the probability of the story's "fundamental data" and the consequences of their assumption (MacColl 1889, pp. 332–338).

MacColl does not offer an explicit reason as to why his further account of logical form depends on the results of comparative linguistics. However, an argument for his choice becomes evident, if one takes into account that eminent linguists of his time conceived of their discipline as a natural or physical science. Accordingly, MacColl's evolutionary dualism excludes the possibility that the phonetic and grammatical development of natural languages has to mirror any progress in man's intellectual evolution. The linguistic misunderstandings to which man's natural condition gives rise rather veil than exhibit the genuine form of expressed thought, and the language of mathematics or of science in general is not immune to such perils. However, by systematic comparison between various natural languages and by a detailed reconstruction of their ongoing development the science of language has been able to identify a uniform linguistic structure on which all natural variations in the history of language depend. Moreover, at least one language, that is Chinese, has even preserved this order. Therefore it may serve as a measure for any detailed set-up of the form in which expressed thought should present itself and accordingly should document its slow, but continuous, evolution. In Müller's *Lectures on the Science of Lan-*

guage MacColl found all the details required in order to develop a corresponding account of logical form. As Müller's general outlook on science, evolution and language converged in important respects with MacColl's own views, he could rely with little reservation on this summary of linguistic research in the first half of the 19th century.

Müller was convinced that science by a systematic gathering and classification of information finally leads to a metaphysical account of its subjects. Although he used evolutionary terminology in his presentation of linguistic developments (1994, pp. 368–371), he often defended his view on science against Darwinian evolution theory. Like MacColl he was convinced of a teleological conception of nature, and conceived of the human being as God's crowning work (pp. 17–18 and esp. p. 327). Müller equally shared MacColl's view of language as the decisive indication for man's superiority over all animals, a phenomenon evolution theory could explain.

> Language is our Rubicon, and no brute will dare to cross it. This is our matter of fact answer to those who speak of development, who think they discover the rudiments at least of all human faculties in apes, and who would fain keep open the possibility that man is the only favoured beast, the triumphant conqueror in the primeval struggle for life. Language is something more palpable than a fold in the brain, or an angle in the skull. It admits of no cavilling, and no process of natural selection will ever distil significant words out of the notes of birds or the cries of beasts. (Müller 1994, p. 340[17])

However, in several respects his account of language as the object of comparative linguistics differs from MacColl's understanding of human language. Müller concedes that natural languages are a product of human activity. He denies, however, that man invented language as "a conventional code of representative sounds or symbols" (1994, p. 331). Likewise, he rejects theories on the origin of human language which set out from man's imitation of animals or his spontaneous articulation of emotions. In Müller's view human language is of divine origin. God invested human nature with a unique creative faculty. It let prehistoric man produce all elementary constituents of all languages. But once this goal was achieved man lost this faculty.

> Man, in his primitive and perfect state, was endowed not only, like the brute, with the power of expressing his sensations by interjections, and his perceptions by onomatopoieia. He possessed likewise the faculty of giving more articulate expression to the rational conceptions of his mind. That faculty was not of his own making. It was an instinct, an instinct of the mind as irresistible as any other instinct. So far as language is the production of that instinct, it belongs to the realm of nature. Man loses his instincts as he ceases

[17] Cf. equally pp. 13–14, 333, 355, 369.

to want them. His senses become fainter when, as in the case of scent, they become useless. Thus the creative faculty which gave to each conception, as it thrilled for the first time through the brain, a phonetic expression, became extinct when its object was fulfilled. (Müller 1994, pp. 370–371)

Originally, "in the spring of speech", an immense amount of these radical elements were at man's disposition. By a process of "natural selection", their number reduces to a few hundred (ibid.). Out of these, by combination and iteration, the human mind develops the whole variety of natural languages. Müller allows at least for the possibility that all natural languages derive from one single source. The present state of appearance of most languages is due to considerable phonetic and morphological transformations. They hide their original set-up. Comparative linguistics, however, is able to discover this order. All modified and modifying components of words and utterances are shown to result from constituent elements of the same kind.

... the whole, or nearly the whole, grammatical framework of the Aryan or Indo-European languages has been traced back to originally independent words, and even the slightest changes which at first sight seem so mysterious, such as *foot* into *feet*, or *I find* into *I found*, have been fully accounted for. This is called comparative grammar, or a scientific analysis of all the formal elements of a language preceded by comparison of all the varieties which one and the same form has assumed in the numerous dialects of the Aryan family. ... The result of such a work as Bopp's "Comparative Grammar" of the Aryan languages may be summed up in a few words. The whole framework of grammar—the elements of derivation, declension, and conjugation—had become settled before the separation of the Aryan family. Hence the broad outlines of grammar, in Sanskrit, Greek, Latin, Gothic, and the rest, are in reality the same; and the apparent differences can be explained by phonetic corruption, which is determined by the phonetic peculiarities of each nation. On the whole, the history of all Aryan languages is nothing but a gradual process of decay. After the grammatical terminations of all these languages have been traced back to their most primitive form, it is possible, in many instances, to determine their original meaning. This, however, can be done by means of induction only; and the period during which, as in the Provençal *dir vos ai*, the component elements of the old Aryan grammar maintained a separate existence in the language and the mind of the Aryans had closed, before Sanskrit was Sanskrit or Greek Greek. That there was such a period we can doubt as little as we can doubt the real existence of fern forests previous to the formation of our coal fields. (Müller 1994, pp. 221–222)

Those constituent elements that the historical analysis of comparative linguistics cannot reduce to other units are called *roots*. Each of them expresses a *general* idea. There are but two irreducible categories of roots. The only constituent elements of language are *predicative* and *demonstrative* roots.

. . . we must admit of a small class of independent radicals, not predicative in the usual sense of the word, but simply pointing, simply expressive of existence under certain more or less definite, local or temporal prescriptions. (Müller 1994, p. 255)

In Müller's view thought and language depend on one another. Language is merely the outward manifestation of human reasoning. He conceives of man's rational competence solely in terms of abstraction and classification. Accordingly, he finally identifies roots with general terms, primarily used as subjects, predicates or attributes. Originally, utterances thus consisted in series of roots whose logical relationship with one another was indicated by the order of their succession.

The analysis of language in terms of its radical constituents allows for a morphological classification of languages and of the stages of their gradual formation. During a certain phase of its development the roots of a language either occur as separate units of speech or they are joined together and result in words whose components either keep or lose their independence. All Indo-European languages show a genuine morphological evolution. Chinese, however, is a language that has preserved its radical set-up.

There is one language, the Chinese, in which no analysis of any kind is required for the discovery of its component parts. It is a language in which no coalescence of roots has taken place: every word is a root, and every root is a word. It is, in fact, the most primitive stage in which we can imagine human language to have existed. It is language *comme il faut*; it is what we should naturally have expected all languages to be. (Müller 1994, pp. 259–260)

The radical constitution of language on which Müller reports does not depend on the philosophical predilection he shares with MacColl. Almost the same view and equally with reference to Chinese is proposed by Schleicher, who in an open letter to Haeckel proposed a monistic, and to some extent Darwinian, conception of comparative linguistics.

Der Bau aller Sprachen weist darauf hin, dass seine älteste Form im wesentlichen dieselbe war, die sich bei einigen Sprachen einfachsten Baues (z. B. beim chinesischen) erhalten hat. Kurz, das, wovon alle Sprachen ihren Ausgang haben, waren Bedeutungslaute, einfache Lautbilder für Anschauungen, Vorstellungen, Begriffe, die in jeder Beziehung, d. h. als jede grammatische Form fungieren konnten, ohne dass für diese Functionen ein lautlicher Ausdruck, so zu sagen, ein Organ, vorhanden war. Auf dieser urältesten Stufe sprachlichen Lebens gibt es also, lautlich unterschieden, weder Verba noch Nomina, weder Conjugation noch Declination u. s. f. (Schleicher 1863, pp. 21-22)[18]

[18] "The construction of all languages points to this, that the eldest forms were in

The linguistic identification of an elementary form in which human communication and reasoning originally presented itself is essential to MacColl's evolutionary dualism. The prehistoric occurrence of this form marks the threshold where man's intellectual evolution meets the natural conditions under which it proceeds. Seemingly, MacColl agrees with Müller and Schleicher when he writes about prehistoric man:

> ... his language consisted of simple, independent, unrelated elementary sounds, each a complete statement in itself (a subject and predicate, as it were, rolled into one) and conveying its own separate information. (MacColl 1909, p. 114)

However, and the reasons for this difference have been set out previously, MacColl does not conceive of language in terms of roots, but in terms of statements. Accordingly, he criticizes Müller for his Aristotelian account of man's basic form of explicit reasoning, and recommends conceiving of a root's basic logical role not in terms of subject or predicate, but in terms of predication. In close analogy to Müller he writes:

> The fact that every word was originally a predication or statement, which, like the cries of the lower animals, first conveyed information either about a personal wish or emotion, or else about some external object, and which was afterwards employed to give similar information about similar objects, is one of the most important discoveries in the science of language. (MacColl 1909, p. 117)[19]

Applied to the results of comparative linguistics MacColl's information principle thus points to the fact that any use of signs presupposes *inferential* contexts. Müller's account of human language as incarnate thought fails to notice this basic aspect of thinking.

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reality alike or similar; and those less complex forms are preserved in some idioms of the simplest kind, as, for example, Chinese. In a word, the point from which all languages had their issue were significant sounds, simple sound-symbols of perceptions, conceptions, and ideas, which might assume the functions of any grammatical form, although such functions were not denoted by any particular expression, although they were not organized, as we might say. In this remote stage of the life of speech, there is consequently no distinction in word or sound* [* lautlich.—T.] between verbs and nouns; there is neither declension nor conjugation." (Schleicher 1869, pp. 50-51, here quoted after Koerner 1983.)

[19] Müller originally wrote, "The fact that every word is originally a predicate, that names, though signs of individual conceptions, are all, without exception, derived from general ideas, is one of the most important discoveries in the science of language" (1994, p. 369). The difference between the two texts shows clearly that MacColl rejected not only Müller's adherence to traditional term logic, but likewise his speculations on the "instinctive" generation of roots.

The *Lectures on the Science of Language* evidence that the morphological and phonetic development of natural languages scarcely depends on man's intellectual disposition. Nothing more is required than his ability to combine formerly separate units of speech. MacColl's semiotic principle, however, accounts for an aspect of man's linguistic disposition that contributes to his intellectual development. Saying the same in different ways can make noticeable what is worth repeating. Man's intellectual evolution under natural conditions depends essentially on this semiotic strategy of identification or constitution. But still it cannot be applied unless basic linguistic forms are taken for granted. Even though MacColl rejects the linguistic concept of a root he approves of the conception of a form of language to which this notion has led. In close analogy to the idea of a language without "coalescence of roots" he contends that composite statements are nothing but combinations of statements. His conception of a proposition in which subject and predicate or, additionally, attribute are separate units closely follows Müller's and Schleicher's Chinese model. In the introduction to his *Symbolic Logic* MacColl presents his concept of a proposition as follows:

> Let us suppose that amongst a certain prehistoric tribe, the sound, gesture, or symbol S was the understood representation of the general idea *stag*. ... The symbol S, or the word *stag*, might have vaguely and varyingly done duty for "It is a stag," or "I see a stag," or "A stag is coming," &c. Similarly, in the customary language of the tribe, the sound or symbol B might have conveyed the general notion of *bigness*, and have varyingly stood for the statement "It is *big*," or "I see a *big* thing coming," &c. By degrees primitive men would learn to combine two such sounds or signs into a compound statement, but of varying form or arrangement, according to the impulse of the moment, as SB, or BS, or S_B, or S^B, &c., any of which might mean "I see a *big stag*," or "The *stag* is *big*," or "A *big stag* is coming," &c. In like manner some varying arrangement, such as SK, or S^K, &c., might mean "The *stag* has been *killed*," or "I have *killed* the *stag*," &c. Finally, and after many tentative or haphazard changes, would come the grand chemical combination of these linguistic atoms into the compound linguistic molecules which we call *propositions*. The arrangement S^B (or some other) would eventually crystallize and *permanently* signify "The *stag* is *big*," and a similar form S^K would *permanently* mean "The *stag* is *killed*." These are two complete propositions, each with distinct subject and predicate. On the other hand, S_B and S_K (or some other forms) would *permanently* represent "The *big stag*" and "The *killed stag*." These are *not* complete propositions; they are merely qualified subjects waiting for their predicates. On these general ideas of linguistic development I have founded my symbolic system. (MacColl 1906b, pp. 3–4)

This introduction of the basic form of compound statements derives *à la lettre* from Müller's example of radical composition in Chinese. At the very end of the quoted passage MacColl himself stresses that the further set-up of his system presupposes this linguistic conception of logical form. Müller's examples match exactly MacColl's explanation:

In some languages, and particularly in Chinese, a predicative root may by itself be used as a noun, or a verb, or an adjective or adverb. Thus the Chinese sound *ta* means, without any change of form, great, greatness, and to be great.* (Endlicher, Chinesische Grammatik, § 128.) If *ta* stands before a substantive, it has the meaning of an adjective. Thus *ta fu* means a great man. If *ta* stands after a substantive, it is a predicate, or as we should say, a verb. Thus *fu ta* would mean the man is great. (If two words are placed like *fu ta*, the first may form the predicate of the second, the second being used as a substantive. Thus *fu ta* might mean the greatness of man, but in this case it is more usual to say *fu tei ta*.) Or again, *ǵin ngǒ*, *li pǔ ngǒ*, would mean, man bad, law not bad.

Here we see that there is no outward distinction whatever between a root and a word, and that a noun is distinguished from a verb merely by its collocation in a sentence. (MacColl 1906b, pp. 255–256)

In MacColl's *Symbolic Logic* there is no outward distinction whatever between a statement, a subject, a predicate or an attribute. These components are distinguished from one another merely by their collocation in a statement. In one of his articles in *Mind* there is a passage on relations between statements that points to this basic feature of MacColl's system and its linguistic prototype.

To meet the requirements of logic, especially of symbolic logic, I propose the following: Let $\phi(x, \alpha, \beta, \pi)$ and $\psi(y, \beta, \alpha, \pi)$, or their abbreviations ϕ and ψ, denote two equivalent* statements which nevertheless differ in three things: (1) that (in *position*) x in the former corresponds to y in the latter; (2) that α in the former corresponds to β in the latter; and (3) that β in the former corresponds to α in the latter—the remaining constant portion π occupying the same position in both.

*"Equivalent" in the sense that each implies the other. The statements are supposed to be expressed in some non-inflectional language, symbolic or other, in which the value, effect, or meaning of a word or symbol generally varies with its position. Algebra and Chinese are good examples. (MacColl 1902, p. 360)

The impact of MacColl's understanding of logical form did not pass unnoticed. An anonymous review of *Symbolic Logic and its Applications* in *The Educational Times* points out the significance of his opposition to the traditional account of subject and predicate in terms of general nouns.[20] On 16 November 1906 Peirce writes to MacColl on the same issue:

[20] Cf. Anonymous 1906, p. 261.

Although my studies in symbolic logic have differed from yours in that my aim has not been to apply the system to the working out of problems, as yours has, but to aid in the study of logic itself, nevertheless I have always thought that you alone, so far as I know, except myself, have understood how the matter ought to be treated by making the elements propositions or predicates and not common nouns. (Peirce 1906)

6. Conclusions

In contrast to all other pioneers of modern logic MacColl proposed an evolutionary theory of symbolic reasoning. As is well known his system developed under the influence of logic's early algebraic tradition. However, the design of logical form on which the system relies stems from other sources. The guiding principles of his theory are due to an evolutionary substance dualism which was meant to reconcile theistic religion with scientific progress. The epistemic modalities of MacColl's system account for the conceptual needs of his theology. The form of predication in which they present themselves is designed with explicit reference to major results in 19th century linguistics.

The fundamentals of MacColl's modal system allow for a systematic ambiguity. What a statement means when occurring within a statement depends on the position in which it contributes to the relevant propositional context. On one occasion, for instance, the constituent "ϵ" of his symbolic language might be used as a predicate. In this case it would stand for "is necessary". Used in subject position it would stand for "the necessity" or "a necessity", and used as an attribute it would stand for "necessary". This structural ambiguity has be proven to be an intended feature of his system—and not a result of conceptual negligence. It illustrates MacColl's attempt to develop a system that acknowledges the contextual condition of human reasoning.

MacColl's theory of logic mirrors with clarity the metaphysical beliefs of his author. MacColl's literary writings evidence in detail that their formation preceded or coincided with the elaboration of his logic. The material at our disposition does not allow for a more definite reconstruction of the historical development. It should be clear, however, that MacColl's interest in logic was not a purely formal one. His work in this discipline was the focus of a comprehensive intellectual endeavor.

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THE METAPHYSICAL FOUNDATIONS OF HUGH MACCOLL'S RELIGIOUS ETHICS*

"How can it be otherwise, since the unknown is infinite and the known infinitesimal in comparison?"
Hugh MacColl, *Man's Origin, Destiny, and Duty*, p. 96.

This paper attempts to give a systematic exposition, interpretation and evaluation of Hugh MacColl's view on the ultimate meaning of life as expounded in his last book *Man's Origin, Destiny, and Duty* (1909). MacColl's religious ethics is a version of what Elizabeth Anscombe calls a divine-law conception of ethics. However, the essential doctrines of the Christian religion on which the divine authority of morality is based can be unmiraculously proved by means of a scientific methodology alone. MacColl justifies the theistic doctrines of the separateness of the Soul from the body, the survival (and transmigration) of the Soul, the existence of Superhuman higher intelligences, and the existence of a Supreme Being in terms of a version of Platonic dualism. In his defence of these metaphysical and theological foundations MacColl not only resists but also attacks the pretensions of an overall Darwinian evolutionary explanation, especially the monism of Ernst Heinrich Haeckel.

Hugh MacColl's (1837–1909) view on the ultimate reality and meaning of human life in the universe consists of a set of theological, ethical, metaphysical and anthropological doctrines. In this paper, I try to give a systematic exposition, interpretation and evaluation of this *Weltanschauung* as expounded in MacColl's last book, *Man's Origin, Destiny, and Duty*, published in the year of his death (1909).[1] To begin with, I sketch MacColl's metaphysical project in the light of my identification

*I thank Ivor Grattan-Guinness, Stephen Read and especially Gerd Van Riel for their useful comments and valuable information.

[1]MacColl had already dramatically expressed his world-view in the novel *Ednor Whitlock* (Chatto & Windus, London, 1891). The following abbreviations are used to refer to works of MacColl's. CP: "Chance or Purpose?" (1907a); WWS: "What and Where is the Soul?" (1907b); MODD: *Man's Origin, Destiny, and Duty* (1909). Both papers from *The Hibbert Journal* are reprinted as an Appendix in the latter book.

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of MacColl's religious ethics as a version of the divine-law conception of ethics. Subsequently, I interpret this project as a form of Platonic dualism and set out its three major tenets: mind-body dualism, the existence of a psychic hierarchy and the existence of God. Finally, I briefly evaluate these metaphysical foundations of MacColl's divine command ethics in the context of "serious" naturalistic metaphysics, especially Ernst Heinrich Haeckel's evolutionary monism.

I. The Divine-Law Conception of Ethics and MacColl's Metaphysical Project

Ethics and religion are intimately related in MacColl's world-view. Although it is possible to imagine a virtuous non-religious person and, conversely, a religious vicious person, religion and morality are not mutually independent. "It is evident *a priori*, ... " MacColl claims, that "the belief that an invisible Being or Beings take note of all we do, and can even read our most secret thoughts, must affect our conduct either for good or evil" (MODD, p. 149). This dependency of human conduct on religious belief is not only a psychological fact given in commonsensical and historical experience, but also a necessary truth, according to MacColl. Religious fundamentalism, for example, amply attests to this essential aspect of human nature.

Whether the influence on human conduct is benign or malign depends upon the content of the particular religious belief. In MacColl's opinion, African voodooism, impure Buddhism and ancient Greek and Roman polytheism have bad effects, whereas Hebraic and Christian monotheism have good consequences. The intermediate conclusion of MacColl's comparative philosophy of religion is that "the belief in one supreme, directing, all-powerful, and beneficent Being constitutes the best philosophical basis for a practical code of morality ... " (MODD, p. 154). From the standpoint of morality, then, Christian monotheism is superior to Hebraic monotheism because the former involves the ideas of punishment and reward in the life hereafter, spiritual inwardness and extreme altruism ("Love your enemies!"), whereas the latter is still too much bound to ritualistic outwardness and "this-worldliness". The final conclusion of MacColl's comparative study is that "if it were a mere question of choosing a religion whose moral precepts recommend themselves instinctively to our conscience, the Christian religion as Jesus taught it, and as epitomised in his Sermon on the Mount, would be an ideal religion" (MODD, p. 161).

Apart from the psychological dependency of moral conduct on monotheistic belief, MacColl also argues for the stronger constitutive thesis that an ethical system must depend upon a divine Superhuman Power. Non-religious ethics based on human authority alone will never command the respect of the average man. For want of ultimate foundations, an atheistic system of ethics has no stability. The final authority of a stable system of ethics must, therefore, be superhuman and theistic (MODD, pp. 71–72, 129). The respect for and efficacy of a practical code of morality are best served by the belief in an all-seeing God. An unshakeable ethical system depends upon a powerful Deity not only for its authority and efficacy, but also for its content.

In view of MacColl's comparative philosophy of religion, it is not surprising that the specific content of such an ethical system is delivered by the superior moral code of Christendom. The fundamental code of Christian (and Hebraic) monotheism is given with the Mosaic law and the ten commandments. God as the Supreme Ruler is, consequently, the ultimate source of moral obligation because His commands create duties. So, whatever is done in accordance with God's will and His commands is good; and whatever is done in opposition to these is evil: "What *he* approves must, by express definition, be *right*; what *he* disapproves must, by express definition, be *wrong*" (MODD, p. 72).

MacColl's religious ethics is, in my opinion, a clear version of what Elizabeth Anscombe (1958) calls *the divine-law conception of ethics*. Such a conception entered Western civilisation after Greek and Roman antiquity with the rise of Christianity. The notion of "moral obligation" and the strong notion of "morally ought" are, in this conception, made intelligible in terms of being bound by divine law. This Christian conception of ethics is, of course, conditional on the belief in God, as Anscombe observes: "Naturally it is not possible to have such a conception unless you believe in God as a law-giver; like Jews, Stoics and Christians" (ibid., p. 30). Accordingly, the authority and efficacy of the divine-law conception of ethics fully depends upon the conviction that there is one Supreme Ruler of the universe. In short, divine command ethics rests upon the belief in the existence of God. "No restraining power on earth," MacColl claims, "can equal that of the full conviction, *when the full conviction exists*, that an invisible superhuman eye ... is watching every deed ... " (MODD, p. 72). The stability of a system of morality is guaranteed by divine authority, but " ... this superhuman authority cannot be effectively appealed to till the educated and uneducated alike are firmly convinced that it really exists" (MODD, p. 129).

There are, according to MacColl, three obstacles to establishing—or re-establishing—the belief in the existence of God in the hearts of all men (MODD, pp. 130–31, 161). First, against theism stands Darwin's theory of evolution as an alternative explanation of every phenomenon in the universe, and even of the universe itself. I will deal with atheistic evolutionism in the final section. Second, God is not given in sense-experience, since he has a non-empirical existence. And God seldom, if ever, interferes directly with the normal course of nature. Hence, theistic belief cannot be empirical belief. Third, the main obstacle is the *miraculous* character of the Christian religion. Faith in the fundamental Christian doctrines is essentially bound up with faith in Jesus' miracles, and especially with his miraculous resurrection. For modern men, however, it is hard, perhaps even impossible, to believe in biblical miracles; but "without the miracles, they consider that the whole body of Christian doctrines, with the morality founded thereon, must lack divine authority" (MODD, pp. 161–62). So, if modern men lose faith in miracles, they lose faith in the Christian doctrines, and without these doctrines—especially the existence of God—morality loses its foundation and life its meaning. According to MacColl's line of thinking, if God does not exist, everything goes and nothing has any meaning whatsoever. This position is reminiscent of Ivan's thesis in Dostoyevsky's *The Brothers Karamazov*: "without God everything is permitted".

I am now in a position to state and clarify *MacColl's metaphysical project* in *Man's Origin, Destiny, and Duty*. In order to safeguard the meaning of life and secure the basis for morality, MacColl wants to establish "the fundamental and essential doctrines of the Christian religion . . . on which alone a durable, logical, and satisfactory code of morality can be founded . . . independently of and without any appeal to these miracles" (MODD, p. 162). To dissipate the modern scepsis he will try to prove unmiraculously three doctrines common to all Christian denominations, namely the existence of One Supreme Being, the existence of Superhuman Beings and the survival of the Soul after death. That is to say, MacColl will try to prove these doctrines by means of a *scientific* methodology, i.e. on the basis of " . . . the modern evidence afforded by undeniable experiments and observations in psychology, physiology, and other branches of science . . . " (MODD, p. 163). MacColl wants to relieve many anxious hearts by scientifically establishing the kernel of Christianity on which morality and the meaning of life are founded.

MacColl's project is very classical in Western philosophy, especially after the Enlightenment. The classical project of the rationalization of religious doctrines essentially includes the construction of positive proofs on the basis of pure reason and scientific evidence independent from the testimony of and historical evidence for miraculous divine revelation (MODD, pp. 69, 163). This philosophical rationalization of religion converts theological doctrines into *metaphysical* ones. MacColl thinks that such an "enlightened" religion based on metaphysical principles is perfectly possible, and that it is " ... a religion that will satisfy both the logical demands of the intellect and the yearning aspirations of the human heart" (CP, p. 386). Once the metaphysical principles are established by means of the scientific methodology, these reasonable assumptions can be employed as the *foundations* of a stable and authoritative system of ethics. Clearly, in MacColl's project of grounding morality and guaranteeing the meaning of life, religion harmonizes with science and metaphysics:

> Between honest, truth-seeking religion and honest, truth-seeking science there need never be any conflict. ... True religion, founded on pure Theism, must, like science, be progressive, and adapt its tenets to changing conditions and new discoveries. Science, accepting the same pure Theism, must, like true religion, tread softly and reverently, and regard nature as a divine book which it is man's privilege and bounden duty to study. (CP, p. 396)

Before looking at the realization of MacColl's metaphysical project in somewhat more detail, I will make three comments on his divine-law conception of ethics. First, MacColl's apologia for religious ethics, and in particular for Christian morality, is reactionary. At the beginning of this century and even much earlier, Kantian morality on the continent and utilitarian ethics on the Anglo-Saxon islands had both attempted to emancipate modern moral philosophy from theological domination. MacColl says nothing about these laudable attempts to ground morality on the rational autonomy of man. He does not, however, go back to an Aristotelian-Thomistic virtue-ethics, but only reaffirms a rationalized version of (Protestant) moral theology in which "duty" and "responsibility" are the focal moral concerns. Second, MacColl's divine-law conception of ethics is a version of moral externalism. The reasons for being moral do not come from within the moral practice, but from without. Moral life is motivated by the fear of an all-seeing and punishing God "in heaven". Conversely, if God were dead, everything would be permitted. Christianity is, moreover, deemed superior because a person can never escape his future punishment even for secret crimes in the life hereafter. Divine command ethics is, consequently, a legalistic ethics of deterrence. However, by making morality extrinsically dependent on the belief in the existence of a Supreme Ruler, morality is perhaps more unstable than when it is intrinsically motivated by autonomous rational obligation or the good life. When a person loses his faith in God, he does not necessarily lose his faith in moral values and the meaning of life. Third, the rationalization of the Christian divine-law conception of ethics discards the very basis of the Christian religion, namely the miraculous mystery of Jesus' resurrection. As a consequence, faith in a living personal God who reveals himself in history through His Son is replaced by belief in a postulated God—the "God of the philosophers". Yet the essence of Christianity seems thereby to be lost, and the notion of the meaning of life as a religiously informed journey to God made unintelligible.

II. MacColl's Platonic Dualism

In the light of his project, MacColl advances three fundamental metaphysical hypotheses:

(1) the distinctness of mind (soul) and body (*mind-body dualism*);

(2) the existence of non-human intelligences (*psychic hierarchy*); and

(3) the existence of One Supreme Being (*God*).

Notwithstanding his eclecticism there is, I think, a strong Platonic tendency present in his metaphysical system. In my interpretation, then, MacColl's metaphysics is a version of Platonic dualism. Theism is, of course, a component of every Platonic scheme, and mind-body dualism is a prerequisite for the Platonic thesis of the soul's survival after bodily death. In contradistinction to Cartesian dualism, however, MacColl's metaphysical hierarchical scheme includes infrahuman as well as superhuman souls in addition to human souls. And in comparison with the Aristotelian hierarchy, MacColl's metaphysical system is more dynamic and directed towards perfection. Overall mind-body dualism in combination with the ascending psychic universe justifies, to my mind, the label "Platonic", or perhaps more accurately "*neo*-Platonic".[2] Although mind-body dualism and theism are not exclusively (neo-)Platonic, the transmigration of the soul through a psychic hierarchy—the second hypothesis—is specifically Platonic. In addition, MacColl's general rationalistic methodology, as well as his ethics of duty and responsibility also indicate a tendency towards Platonism. It is, on the whole, not uncommon for mathematicians and logicians to have strong Platonist sympathies. Whether or not MacColl was in any way influenced by the "Cambridge Platonists" of the 17th century (Benjamin Whichcote, Ralph Cudworth, Henry More) or later British Platonists such as

[2]It would be interesting to compare and contrast MacColl's metaphysics with the Platonic scheme of Plotinus or Proclus.

J. McT. E. McTaggart, I do not know. In any case, "Platonic dualism" is a convenient label to systematize MacColl's eclectic metaphysics.

In the pursuit of truth, MacColl puts his speculative hypotheses forward on the basis of "*scientific* instinct" (CP, p. 389). Before going into detail concerning his defence of these metaphysical theses and what exactly they mean, I will say something more about the scientific methodology he uses to defend them. MacColl's Platonic metaphysics relies on three methodological strategies: the use of (i) data, solid facts and scientific evidence " ... afforded by undeniable experiments and observations in psychology, physiology, and other branches of science ... " (MODD, p. 163); (ii) induction and analogy (MODD, p. 69); and (iii) the mathematical theory of probability on which the modern system of logic is founded (MODD, pp. 133–34).[3] MacColl's rational methodology is thus partly *empirical*, partly *formal*. For example, the emphasis on the existence of God as an explanatory cosmological hypothesis—the third hypothesis—elucidates MacColl's preoccupation with having sufficient evidence and proofs for this hypothesis. Apart from the three formal strategies, MacColl also employs a fourth, more substantial methodological principle, which is in a way the converse of Occam's Razor, and which I call (iv) the principle of *pluriformity and complexity*. MacColl's ontology includes "unknown immaterial substances" besides known material ones such as measurable matter, as well as "unknown spiritual forces" besides known physical forces such as attraction, electricity and magnetism. According to MacColl, the probabilities are entirely against those scientists who conclude that " ... the substances existing, and the forces operating, in our universe are few in number and expressible in simple formulae" (MODD, p. 85). Because uniformity and simplicity hinder good science, "the sooner scientists get rid of the superstitious dogma called the 'uniformity of nature' the better it will be for science" (MODD, p. 86).

Hypothesis (1) states that " ... the Soul and body are two separate entities" (MODD, p. 9). In MacColl's mind-body dualism " ... the *Soul* or *Ego* (in man or in the sentient lower animal)" is defined "as simply *that which feels*, or is *conscious* ... " (MODD, p. 4). This definition, in order to include non-human souls, does not limit the soul, in a Cartesian fashion, to "that which thinks". MacColl casts his net wider: "By express definition, the *Soul* or *Ego* is the entity that feels, and, in its higher development, thinks and reasons" (MODD, p. 11). In the case of

[3] As to the latter, MacColl observes in a footnote that "in my *Symbolic Logic and its Applications* I have shown ... the fallacy of the subjective theory of probability which values a chance at *one-half* when ... there are no data for a calculation" (MODD, pp. 134–35).

humans the soul (or ego) is equated with *"the real person"* (MODD, p. 10). Remarkably, MacColl reduces the mental to the conscious. Everything that is mental comes under the category of sensations. One can agree that the notion of "unconscious sensation" is self-contradictory (MODD, p. 3; WWS, p. 159), but there is nothing self-contradictory about the notions of "unconscious judgement" and "unconscious reasoning" (WWS, p. 168), for mental states and processes can function perfectly well without being conscious, as contemporary cognitive science abundantly attests. The categories of propositional attitudes and mental dispositions seem to be null and void in MacColl's philosophy of mind.

MacColl's argument for mind-body dualism is based upon "physiological facts" about the insensibility of the body (MODD, pp. 5–7; WWS, p. 161). When we ask what the *real seat* of consciousness is, we cannot answer that it is the body or a part of the body. The toes, the limbs, the eyes, the ears, and even the nerves are not seats of consciousness, but only insensible instruments or channels of transmission. Although we project, for example, sensations of pain to parts of the body, it is not these parts themselves which feel pain and are conscious. So-called "phantom limb pain"—i.e. still feeling pain in an amputated limb immediately after the operation—is sufficient evidence for this physiological fact. Some physiologists conclude from this corporeal insensibility that it is the brain, and the brain alone, that feels, sees, hears, and is conscious. The brain is, according to these scientists, an exception to the general rule of corporeal insensibility. Furthermore, "physiologists assert, and probably with truth, that every thought, every sensation, is accompanied by some change in the substance of the brain ... " (WWS, p. 165). These two elements, the brain as the real seat of consciousness and the systematic correlation between mental and brain activity, are " ... supposed in some way to support the atheist's contention that the brain and the soul (or ego) are identical, or, at any rate, inseparable, so that the ultimate dissolution of the former necessarily leads to the extinction of the latter also" (WWS, p. 165).

However, MacColl claims that "*no physiologist has as yet brought forward any trustworthy data which would warrant the conclusion that the brain is an exception*" (MODD, p. 5) to the general rule of corporeal insensibility. Moreover, the brain and the soul (or ego) cannot be identical, for " ... the same conscious-thinking ego may be said to work with absolutely different brains at different periods of its existence"[4]

[4]MacColl's reason for saying this is based upon the empirical assumption that "this substance [the brain] is passing away continually" (WWS, p. 165). He observes

(WWS, p. 165). There is, according to MacColl, no valid reason to accept the atheistic physiologist's claim that the brain is the final and ultimate terminus of all channels of sensory transmission. On the contrary, the principle of corporeal insensibility should, by analogy, be applied to the case of the brain as well:

> But why not carry the principle further? Is it quite certain that all sensation *is* in the brain—or in the material body at all? If, in spite of the *direct* evidence of our sensations, we draw a wrong inference when we locate a certain feeling in our toes, may we not also be wrong when, trusting to the *indirect* evidence of our sensations and to our fallible reason, we locate that feeling or any other in the brain, or, for the matter of that, in any *fixed* position anywhere, whether in the body or out of it? (CP, p. 395)

If the brain is not an exception to the principle of corporeal insensibility, then the inevitable conclusion follows that "the whole body, brain and nervous system included, has in itself no more feeling, consciousness, will, thought, or initiative than a plant, phonograph, or calculating machine, or the inanimate apparatus in wireless telegraphy. Like the last, the brain neither feels nor understands the sensations and intelligence which it transmits" (MODD, p. 12). The material chunks of matter, called *body and brain*, are only unconscious automata. In light of the fact that the brain, like all other parts of the body, is a mere insensible link in a chain of sensory transmission, MacColl concludes further: "Yet, since feeling or consciousness is admittedly an ultimate fact of nature incapable of analysis, something—an intangible something, which here I call *soul*—does unquestionably feel" (WWS, p. 167). The real seat of consciousness is, accordingly, the soul (or the ego) which is separated from the body.

As to the nature of the soul, MacColl affirms the doctrine of the soul's immaterial substantiality: " ... the Soul, though invisible and imponderable, and therefore immaterial, is nevertheless composed of some substance different both from ordinary matter and from the hypothetical ether, and may thus have a spatial form, on some part of

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that "The material brain with which our ego did its thinking a year ago has already passed clean away, and has been replaced by fresh material particles, forming a new brain with which it does its thinking now" (CP, p. 393). The ego is not reducible to the brain because the ego remains the same thinking thing, whereas the brain does not remain the same substance. Clearly, MacColl has in mind the numerical identity of the brain (token) and not its qualitative identity (type). In order to sustain or realize the same thinking, the brain does not need to stay numerically identical—does not need to have exactly the same 'material particles'—but it needs to have the same qualities or functional organisation. MacColl's idea that new 'fresh material particles' can realize the same old thoughts is comparable with the ideas about the non-reducibility of the mind to the brain in contemporary *functionalism*. For these latter ideas, see Cuypers 1995.

which is permanently registered (as on the human brain) a record of its whole past" (MODD, pp. 66–67). Accordingly, an immaterial substance " . . . is not a mere abstraction, like a thought or idea, which has no form and occupies no position in space or in the material universe" (MODD, pp. 85–86). This immaterial substance, the soul, causally interacts with the material substance, the brain. The idea of mental causation is kept very vague and implicit: "the impressions communicated by the Soul's thoughts to the material human brain" (MODD, pp. 72–73) constitute molecular changes in the brain which give rise to bodily movements. MacColl's mind-body dualism is, quite naturally, connected to his ego-theory of personal identity: "While the material brain [and body] changes, *it*—the seeing, hearing, feeling, thinking *soul*—remains" (WWS, p. 166). He rejects not only the body and brain theory but also the bundle theory of personal identity.[5] The thoughts themselves are not the thinkers. The stream of consciousness is not the person: "The *Soul*, by express definition, is *'that which feels'*; it is not the *feeling*. In its higher developments, it is *'that which thinks'*, or the *'thinker'*; it is not the *thought* or the *succession of thoughts*" (MODD, p. 74).

Because the immaterial substance has "a spatial form" according to MacColl, it makes sense to ask the question *Where is the Soul or Ego?* "Is it in the body, or near the body, or far away from the body?" (MODD, p. 26). Even if the soul and the body were two separate entities, it would still be possible that they are necessarily connected in a way that excludes the soul's survival of bodily death. MacColl answers the question whether the soul can exist without the body by offering "the hypothesis of the Soul being external to and possibly far away from its own bodily mechanism" (MODD, p. 28). This hypothesis of the soul's *externality* is a hypothesis about the position of the soul: " . . . its position may be fixed or variable. It may, at one instant, be in the body, or near the body, and, the instant after, it may be millions of miles away from the body" (MODD, p. 12). The soul is, therefore, not necessarily connected to the body. It is external to the body and does not reside inside the body, or does not form a unity with the body: " . . . the Soul or Ego may be conscious while far away from its ever unconscious body, and may from a distance, while believing itself in close contact, control the ordinary movements of that body, and even influence others . . . " (MODD, p. 14). MacColl's model for this somewhat startling hypothesis is wireless telegraphy: "Do not the phenomena of wireless telegraphy make it plain that certain mechanisms, wonderfully

[5]For these theories, see Cuypers 1998.

suggestive of the nervous system, can be operated upon by conscious Beings from afar, and by these made to transmit thoughts and sensations which the mechanisms themselves neither feel nor understand?"[6] (WWS, pp. 162–162).

MacColl's main evidence for the hypothesis of the soul's externality and independence comes from psychopathology and even parapsychology. A certain class of nervous maladies proves that the conception of a disembodied soul is not incoherent (MODD, pp. 16–22). The telepathic theory which says that " ... a human being, voluntarily or involuntarily, may, without any electrical apparatus (as in wireless telegraphy), or other visible means of communication, instantly convey a sensation to another person many miles distant ... " (MODD, pp. 13–14) makes the hypothesis of the Soul's externality easily credible. MacColl accepted the possibility of telepathy on empirical grounds and believed that "no one who has seriously studied the theory of probability will regard this [materially unsupported communication] as a mere coincidence" (MODD, p. 29). The contemporary reader may be surprised by MacColl's belief in parapsychology, yet at the end of the last century and the beginning of this one it was not uncommon among intellectuals to take parapsychology, mesmerism, spiritism and the like very seriously.[7] MacColl widens the scope of his parapsychological soul-metaphysics even further: "If a human soul, as many eminent scientists now believe, can instantly (by 'brain waves' or otherwise) act upon another human soul, when their bodies are thousands of miles apart, where lies the difficulty of believing that a human soul can be similarly but more profoundly influenced by a superhuman soul, and from a still greater distance?" (MODD, p. 32). The possibility of such a communicative universe presupposes MacColl's second grand hypothesis.

Let me turn to the somewhat extraordinary hypothesis (2): the existence of non-human intelligences. Although the belief in the existence of non-human animal forms of consciousness and intelligence is quite common, the belief in the existence of non-human higher intelligences is rather extravagant by contemporary standards. According to MacColl, there exists a hierarchical psychic universe, starting from infrahuman consciousness and intelligence just above the level of the senseless plant, and ascending above the human level through superhuman higher forms of psychic life in infinite gradations, until reaching

[6] Although Daniel Dennett is a contemporary materialist philosopher of mind, he also answers the question *Where am I?* by giving a *functionalist* hypothesis of the externality and independence of the mind. See Dennett 1978.

[7] I owe this point to Ivor Grattan-Guinness. See, for example, Grattan-Guinness 1983.

the upper level of consciousness and intelligence in the One Supreme Being. The classical religious icon or metaphor for the superhuman higher forms of consciousness and intelligence is "the community of angels". MacColl's argument for the existence of a superhuman psychic universe is inductive and analogical: "Is it not more reasonable to infer, by induction and analogy, that man is but a link in the ascending evolutionary chain of intelligence?" (MODD, p. 31). Just as there is a descending series of infrahuman sentient and intelligent beings, there is an ascending series of superhuman sentient and intelligent beings. And just as numberless infrahuman beings lack the requisite faculties to perceive man's existence directly, man himself lacks the suitable organs in the present stage of his development to obtain direct empirical evidence of superhuman existence. "But that which man cannot perceive *directly* through his outward senses," MacColl says, "he is capable of learning *indirectly* by the exercise of his slowly developing reason—a faculty which was given him expressly that he might so apply it" (CP, p. 387).

Although "man is but a link in the ascending evolutionary chain of intelligence", the difference in psychic faculties between humans and infrahuman animals is not merely one of degree, but of kind: "Man, however low his type, possesses one faculty of which no other animal, not even the highest ape, shows the most elementary rudiment. For want of a better name, let us call it the faculty of *symbolisation*" (MODD, p. 107). Man not only possesses the faculty of articulation, as, for example, a parrot does, but also the faculty of " … conscious speech-development—the faculty of representing ideas (in order to remind himself or give information to others) by arbitrary sounds or symbols" (MODD, p. 115). Although higher animals understand commands, they are incapable of understanding the meaning of propositions (classifications and abstractions). And though non-human animals can draw elementary inductive inferences, they are incapable of abstract deductive reasoning or drawing a necessary conclusion from given premises (MODD, p. 123).

The superhuman psychic universe is filled with higher forms of consciousness and intelligence in infinite gradations. Probably all degrees of consciousness and intelligence are represented, from just above the hypersensitive and genial human level until the omniscience, omnipotence and omnibenevolence of the One Supreme Being. The superhuman psychic universe is at the same time a *moral* universe. And just as consciousness and intelligence are distributed in different degrees, good and evil (pleasure and pain) are likewise distributed. However, good and evil coexist on all levels (except on the divine upper level) but in different proportions: in ascending the hierarchy " ... the evil *in proportion* to the good will eventually diminish, and the good in proportion to the evil will increase" (CP, p. 388; MODD, pp. 33–34). In this hierarchical psychic and moral universe the higher beings have power and mastery over the lower ones, but at the same time the former will be held responsible for their influence on the latter (MODD, pp. 70–71).

The existence of a hierarchical psychic and moral universe is connected with MacColl's baffling doctrine of the transmigration of the soul. Although souls have a permanent existence, they do not always have the same life. MacColl claims that "All forms of life, or at least of *conscious* life, life capable of feeling pain and pleasure, are in a state of transition, and are destined at death to pass into a higher life, with higher pleasures, higher pains. That higher life, too, will end and will be succeeded by a still higher, and so on for ever" (CP, p. 387). In that sense souls are not static entities; they have an evolution and develop progressively. Souls move dynamically through the psychic and moral hierarchy upwards, *ad infinitum*: " ... we may suppose these successive metamorphoses to go on forever into higher and higher spheres of existence, experience, and ever-increasing knowledge. The material body and other successive instruments of education, material or immaterial, would thus successively rise and pass, be born and die, while the Soul, the only permanent substance, would remain" (MODD, p. 67). MacColl admits that " ... this is a mere hypothesis built upon rather insufficient data" (MODD, p. 67). But as regards the lack of sufficient evidence the mere hypothesis of metempsychosis is on a par with other scientific hypotheses, such as that of the hypothetical "ether", acceptance of which was widespread in MacColl's day. Moreover, the idea of reincarnation is neither logically incoherent in itself nor logically inconsistent with the known scientific facts (MODD, p. 68). Furthermore, it cannot be excluded that science will have to leave " ... untouched the ground for which its methods and instruments are as yet, and may be for ever, unsuited" (CP, p. 396).

MacColl does not make it exactly clear what the scope and mechanism of the soul's transmigration are supposed to be. Do the infrahuman animal souls of "the lion and the lamb" (MODD, p. 84), for example, reincarnate in *homo sapiens* bodies and become human souls? What the transmigration mechanisms are and how they work is not specified. However, two necessary conditions for the upward progression are that on each level of the hierarchy, evil and good (pain and pleasure) coexist, and that the struggle for existence continues: "For the struggle is necessary for the development, and the struggle would be impossible without the incentives of pain and pleasure—pain at the failure, pleasure at the success. The idea of a perpetual, never-ending happiness in a future life, without struggle and without pain, is an unwholesome dream, beautiful but paralysing, like that of the opium-eater" (CP, pp. 387–88). Furthermore, "moral conduct" is the mechanism for speeding up the soul's upward progression: "Every duty rightly performed or attempted advances automatically, and every duty neglected automatically retards, the upward progress of the Soul. In this way virtue ultimately brings its own reward, and vice its own punishment" (MODD, p. 81). I doubt that even this rough sketch of the soul's transmigration can be made coherent, let alone a more detailed one. One cannot doubt, however, MacColl's moral motives behind the doctrine of the soul's transmigration, for it provides the possibility of a long process of self-perfection. This perfectionism is one of the most hopeful elements in theism: "He [God] allows men to commit errors, and he allows them to commit crimes ... in order that the discomforts and sufferings which those errors and crimes sooner or later entail, here or hereafter, may in the long-run purify their souls and accelerate their progress upwards" (MODD, p. 157).

Let me now examine hypothesis (3): the existence of One Supreme Being. The crowning piece of MacColl's Platonic metaphysics is the Deity: "And we cannot restrict our consideration to the class of sentient beings called the human. Below these are the infrahuman; above them are the superhuman; and over all we are almost compelled by instinct as well as logic consistency to infer the existence of a Supreme Being, who maintains and directs ... the evolution of the whole material and psychic universe ... " (MODD, p. 36). As to the nature of God, the qualities of omniscience and omnibenevolence can logically be ascribed to Him, but omnipotence only within limits, for "the laws of logical consistency cannot be altered even by omnipotence, and, for aught we know to the contrary, the highest happiness may be logically impossible without the preliminary pain and struggle" (CP, p. 388).

On the basis of this second logical impossibility—happiness without pain—MacColl rebuts the argument to atheism from the reality of evil. In light of the psychic hierarchy and the never-ending quest for moral perfection, this idea of pain and evil as necessary for upward progression is the kernel of his theodicy (MODD, pp. 37–42, 83–84). The seeming inconsistency between an omniscient, all-powerful and perfectly good God and the reality of physical as well as moral evil disappears as soon as one realizes that "evil and suffering exist because, for each sentient unit above or below the human, experience of evil and suffering, within limits, in the present stage of his development, is necessary for his progress upwards and for his future capacity to perform his allotted part in carrying out God's mysterious purpose in the infinite succession of lives to follow" (MODD, p. 83). If there were still drawbacks based upon the unacceptability of evil, then MacColl would close the discussion by rhetorically asking: "What is the suffering of this fleeting terrestrial life, however terrible it may feel to the sufferer at the time, in comparison with the sum total of the joys and sorrows of the Soul after an infinity of years in its evolutionary progress from life to life, from higher to higher?" (MODD, p. 78).

Two other closely related qualities must also be attributed to God. In the light of MacColl's divine-law conception of ethics, God is the source of moral laws: "By express definition we call what the Supreme Ruler approves *right*, and what he disapproves *wrong* ... " (MODD, p. 76). Moreover, He is also the source of natural laws which "*must be in conformity with the will of the Supreme Ruler of the psychic and physical universe*" (MODD, p. 105). Besides physical laws of nature, such as the law of gravitation, there are also psychic laws of nature; for example, " ... the act of prayer has set in motion a psychic law designed for the benefit of the soul, as the instinctive taking of wholesome food sets in motion a physiological law designed for the health of the body" (MODD, p. 97). How then do human beings know the moral laws and the laws of nature? In accordance with MacColl's scientific methodology, the epistemology of these laws cannot be based upon miraculous divine revelation, as laid down in the Holy Bible or other religious authorities. Man's knowledge of the laws must, therefore, be founded on the scientific study of *the book of nature* " ... by observation, experiment, and slow, inductive reasoning" (MODD, p. 77). The divine cosmic language of all natural phenomena—physical and psychic, external and internal—reveals God's will and purpose as regards the human race.

MacColl's theism is, as he himself admits, " ... built upon assumptions, some of which cannot easily be harmonised with the tenets of any existing theistic religion" (CP, p. 386). The most important reason, in my opinion, to call his metaphysical system *deistic* instead of theistic lies in his adherence to " ... the doctrine that God leaves his laws to carry out his purpose automatically without any direct interference with their working ... " (MODD, p. 96). That is to say, "God's will generally works itself out through what we call natural laws, and, to all appearance, automatically. Immediate, direct, or miraculous interference with these laws, though there is no logical reason against it, nor any proof that it ever occurs, must be so exceedingly rare that we have no right ever to expect it" (MODD, p. 79). But although there is no direct or exceptional intervention of God's will in the law-governed course of nature, God's will still remains, of course, the primary cause of the natural laws themselves and, furthermore, of all the natural events to which these laws give rise (MODD, p. 91). In discussing his non-standard theism, MacColl leaves untouched "the question so often discussed whether the Supreme Being is a 'personal', an 'impersonal', or an 'immanent' God ... because it is scarcely possible to enter upon such a discussion without losing oneself hopelessly in a maze of verbal and metaphysical ambiguities" (MODD, p. vi).

One important consequence of MacColl's deism—the belief in an absent and noninterfering God—is the non-existence of real miracles as orthodoxy construes them. MacColl accepts the existence of "miracles" only in a very attenuated sense, as events that strike us with wonder, and of which science can give instances in plenty, for example, telephone or wireless telegraphy. In this sense, miracles are not violations of the laws of nature, but seemingly unintelligible and perplexing phenomena at a certain time, of which a fully natural explanation in accord with the laws of nature is possible at a later time. According to MacColl, there is no essential or logical difference between the "miracles of science" and the "miracles of religion" as recorded in the Bible. It is true that the former are explicable, whereas the latter are inexplicable in the present state of human knowledge. But this explanatory difference is purely epistemological. If the relevant knowledge of the laws of nature were available, then the miracles of religion would be equally explicable: " ... the miracles recorded in the special writings which they hold sacred might be explained also if we knew all the laws of nature ... " (MODD, p. 94). However, if under such ideal epistemic conditions an alleged religious miracle or dogma were still to remain inexplicable, then MacColl would advise extreme caution and even *agnosticism* about the credibility of such miracles and dogmas.

Although MacColl starts from "assumptions, some of which cannot easily be harmonised with the tenets of any existing theistic religion" (CP, p. 386), and although he never says to which denomination he belongs, his attempt to rationalize religious doctrine and his motivation to safeguard ethical integrity make plausible, I think, the speculation that he was a *unitarian dissenter* and an *ecumenist.* MacColl's deism and his attendant rejection of miracles (as well as dogmas) do not harmonize with the doctrines of the established Christian church. And given his general tendency to purge religion of superstition, he also would reject the doctrine of the Trinity. Moreover, the ideal of moral self-perfection, which forms a central part of MacColl's hypothesis of a dynamic psychic hierarchy, is essential to free churches, especially to methodism. Furthermore, MacColl explicitly mentions Jesus' Sermon on the Mount—Matthew, chapters 5 to 7—(MODD, p. 161), which is the main guiding passage for radical, free churches and ecumenical movements. MacColl's rationalistic religion, which "will satisfy both the logical demands of the intellect and the yearning aspirations of the human heart" (CP, p. 386), transcends the bounds of any particular denomination. MacColl's advice to be agnostic bespeaks an attitude of religious moderation and tolerance: "the fact that anything approaching irrefutable evidence is unattainable should make theists of all denominations, Catholics and Protestants, Jews and Mahomedans, humble, cautious, and mutually forbearing" (MODD, p. 95). Now, whether MacColl also officially belonged to one or other dissenting denomination in the Scotland of his days (e.g. Free Church, United Presbyterian), or whether he kept his dissenting convictions private and outwardly stayed within the national established church, is of course another matter and subject for further investigation. And whether MacColl also practised in one or other Protestant church in Catholic France, where he lived for the most part of his life, I do not know.

MacColl was a metaphysical realist, and were he still alive today his realistic belief in God would be uncluttered by anti-realistic Wittgensteinianism, voluntaristic fideism or obscure Whiteheadian process theology, let alone postmodern deconstructionism. His rational justification of his conviction that there objectively is a God is by a teleological proof of God's existence. MacColl's argument from "the machines constructed by nature—the living animals which reproduce their kind . . . " (MODD, p. 128) for divine Design is modeled after that of William Paley. In his classical treatise *Natural Theology* (1802), Paley argued from the purposeful natural, living organisms—or parts of them, such as the eye—to the existence of a designing God by analogy to the necessary inference from a watch or a telescope to the existence of a designing instrument maker. In the same vein, MacColl's teleological argument sets up an analogy between an appeal to human designing intelligence to explain "calico weaving in a cotton factory" and an appeal to superhuman designing intelligence to explain reproductive living organisms. MacColl's defence of theism by a design argument is to be situated in the context of post-Darwinian, Victorian Great Britain. According to the atheistic or agnostic evolutionist, a theistic explanation of the data is not required, because purely random mutations and the process of natural selection do the explaining just as well. Apparent purpose in nature is not contrived by divine Design, but is simply the outcome of pure chance. According to MacColl, however, "*Blind chance* (as in natural evolution) is the apparent automatic cause; but the real though invisible cause (also as in natural evolution) is *intelligent and foreseeing design*" (CP, p. 385). I will now go into his reasons for opposing Darwinian evolutionary explanation.

III. Serious Metaphysics and Darwin's Dangerous Idea

While sketching the three main tenets of MacColl's Platonic dualism, I did not critically engage with the many different hypotheses and doctrines that compose this grand metaphysical scheme. An important preliminary question to ask before starting to question the validity of MacColl's *Weltanschauung* in the context of late 20th century philosophy seems to me to be this: *Can it be taken seriously any longer?* Perhaps MacColl was an innovative logician, but he surely was an old-fashioned "Victorian" metaphysician. If MacColl's Platonic dualism cannot be taken seriously any more, then it is perhaps not worthwhile to engage with it critically. Now "serious" metaphysics is, at least in the context of contemporary analytical philosophy, materialistic and atheistic. A radical application of the scientific methodology in philosophy has led to the naturalization of metaphysics, especially in the philosophy of mind and action. Contrary to MacColl's view that science, transcendental metaphysics and religion harmonize, orthodox contemporary philosophy adopts a harsh conflict-view which proclaims the elimination of all transcendental and theistic explanatory hypotheses.

Some unorthodox contemporary philosophers, however, still take the realization of a metaphysical project such as that of MacColl's dead seriously. The best example in contemporary philosophy is, I think, Richard Swinburne. Apart from the doctrine of a psychic hierarchy, Swinburne vigorously defends mind-body dualism and the existence of (the Christian) God on the basis of pure reason and scientific methodology.[8] Moreover, although substance dualism only occupies a marginal position in naturalized philosophy of mind[9], the debate on the coherence of theism and the existence of God within the context of metaphysical realism is still prominent in contemporary philosophy of religion.[10] So, whether MacColl's metaphysical foundations of divine command ethics rest upon good or bad arguments, there certainly is still some support for something like MacColl's dualism and theism in contemporary philosophy. A thoroughgoing evaluation of MacColl's metaphysical project and its execution would have to take account of

[8] For his defence of substance dualism, see Swinburne 1997, pp. 145–99, 298–312. For a good summary of his defence of God's existence, see Swinburne 1996.

[9] Besides Swinburne's defence, see Foster 1991.

[10] See, for example, Smart and Haldane 1996.

these contemporary supporting arguments as well. Such a detailed examination is, however, beyond the scope of the present paper.

Contemporary mainstream English-language philosophy is, to a large extent, naturalized. In addition to physical "Big Bang" cosmology and the scientific understanding of the mind, there is one other central instrument in this process of naturalization, namely evolutionary biology. In conclusion, I highlight the reasons why MacColl resists as well as attacks the alternative Darwinian evolutionary explanation of the central mysteries of life—an explanation which enormously excited the intellectual community in his day.[11] MacColl's most important opponent in the debate on Darwinism is a famous predecessor of contemporary serious metaphysics based on evolutionary biology, namely Ernst Heinrich Haeckel (1834–1919). Haeckel believed that his evolutionary monism—his so-called "law of substance"—could resolve all seven riddles of the universe which Emil Du Bois-Reymond had enumerated in his 1880 address to the Berlin Academy of Sciences: the nature of matter and force, the origin of motion, the origin of life, the order in nature, the origin of simple sensation and consciousness, rational thought and speech, and freedom of the will. MacColl often ridicules Haeckel's view in the sharpest words possible:

> So this is the creed of the great apostle of 'Monism' as modern atheists have chosen to call their new religion! The Ultimate Cause, the real creator and sustainer of all phenomena, mental and physical, the phenomenon of human intelligence included, is no intelligent Deity or Deities. It is nothing at all analogous to, much less surpassing, the intelligence of man. It is simply an immense attenuated, yet all-powerful, eternally vibrating jelly! And this strange jelly-god is endowed with sensation and will, "though necessarily of the lowest grade"! Alas! Alas! what wild nonsense some eminent specialists can write when they venture beyond the narrow limits of their own familiar domain! (CP, pp. 390–91)

MacColl does not object to Darwin's ideas of variational evolution, natural selection and adaptation, but only to the "dangerous" idea of an all-embracing and all-explaining principle of evolution. That is to say, he does not attack evolutionary theory as such and even admits its truth within limits, but he protests against "the outrageous pretentions of those who would explain every phenomenon in the universe, and even the universe itself, on evolutionary principles ... " (MODD, pp. 124–25). For example, Haeckel and also Herbert Spencer (1820–1903) belong to the latter category of exaggerations. So, "as one important factor in the gradual changes which animals undergo

[11] For a good introduction to Darwin and contemporary evolutionary biology, see Mayr 1991.

in successive generations, this Darwinian evolution seems reasonable enough; but to speak of it as the sole cause of, or even as the most important factor in, the development of animals, or of such marvellous organs as the human eye or the human brain, from some primary inorganic non-sentient matter, betrays an extraordinary ignorance of the first principles of probability" (MODD, pp. 100–101). In a generalized Darwinian theory today, not only the natural, but also the social and moral worlds of human beings are explained on evolutionary and sociobiological principles.[12] MacColl would have been horrified and deeply shocked by these contemporary applications of Darwinism to social and moral life.

MacColl's reaction to the theory of evolution is not as radical as, for example, that of the creationists who dogmatically hold that evolution explains nothing whatever. His reaction is more moderate in that he acknowledges that " ... within due limits, and taken in conjunction with other and far more important factors, it helps to obtain clearer notions of the progressive *steps* in plant and animal development—so far as their *material* structure is concerned" (MODD, p. 125). It is true that evolution alone does not explain anything; variational evolution, natural selection and adaptation cannot be the whole story. Although evolution certainly does not explain everything, it explains something. According to MacColl, evolution explains bodily development and material structure, yet it affords no explanation whatsoever of the directing and designing forces " ... which constrain the original cells and seeds to take those steps—which constrain some to develop into oaks, others into cabbages, some into fishes or reptiles, others into dogs, horses, cows, or human beings ... " (MODD, p. 125). Furthermore, evolution offers no explanation whatsoever of " ... the origin and development of *mind*—of feeling, consciousness, thought, etc. ... " (MODD, p. 133). In addition, MacColl evidently rejects an evolutionary explanation of the emergence of life "from some primary inorganic non-sentient matter ... " (MODD, p. 101).

I take it that MacColl is claiming that there are three important "missing links" or "gaps" in the evolutionary story: the transition from the non-living to the living (*the emergence of life*), the transition from basic forms of life to reproductive species (*the origin of reproduction or the problem of speciation*), and the transition from the mindless to the minded (*the origin of the mind*). The second not so well-known gap comes to the following. Cumulative selection over successive generations—the evolutionary mechanism of development and

[12] For such a sociobiological account, see Wilson 1978.

speciation—*presupposes* some mechanism of reproduction in the ancestry. But it is hard to see how these reproductive mechanisms as adaptive features can themselves then be the product of cumulative selection *without a further cause.* Now MacColl's metaphysical scheme fills in these lacunas with the "God of the gaps". Although he does not give any detailed explanation of the three pivotal transitions, he suggests that God is the best explanatory hypothesis to resolve these big riddles of the universe. In comparison with Haeckel's "embryology of the soul" (MODD, pp. 134–37) and his general explanatory hypothesis of the "vibrating ether" MacColl's theistic hypothesis is, I think, the better one in terms of simplicity and even credibility. Haeckel's extreme Darwinism is, however, too speculative and shot through with anthropomorphic projection. Sober contemporary evolutionary theory and its sophisticated applications in serious metaphysics and the naturalized philosophy of mind is, of course, a much more challenging alternative to theistic explanation.[13] Yet it remains to be seen whether such a universal Darwinism will eventually be able to fill in the remaining gaps and to dissipate the central mysteries of nature.

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[13]For a fascinating book on sophisticated Darwinian thinking in biology and philosophy, see Dennett 1995.

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HUGH MACCOLL—VICTORIAN

In addition to his work on logic and *Man's Origin, Destiny, and Duty*, MacColl also published two novels, *Mr Stranger's Sealed Packet* (1889) and *Ednor Whitlock* (1891), both of which give the impression that MacColl, in spite of his innovative work in logic, had conservative attitudes and opinions when it came to central questions of the day such as faith and doubt, the role of women, etc. In the following the background against which MacColl's fictional work as well as his defence of Christianity must be understood, is explored, and his fictional works are related to this background. MacColl's conception of the issues and the arguments of the time as well as his attitudes to these issues will be compared to those of other "eminent Victorians", in particular those of men of letters and writers of fiction. The value of such an analysis is not only that it fills out our picture of MacColl, but that it gives insight into what attitudes an intelligent and enlightened Victorian intellectual, whose specialization in logic would well equip him for clear thought, could have of attitudes to and opinions on important questions of the day.

I.

Hugh MacColl was born in 1837, the year of Queen Victoria's accession to the throne of England. He died in 1909, eight years after the Queen's reign had ended. In that year he published *Man's Origin, Destiny, and Duty* (Williams and Norgate, London, 1909), a defence of a theistic position on the Christian faith against the onslaught of materialist science and evolutionary biology. It is a work deeply concerned with problems of faith and doubt, religion and science; problems that we recognize as centrally Victorian problems. In addition to *Man's Origin, Destiny, and Duty* and his work on logic, MacColl also published two novels, *Mr. Stranger's Sealed Packet* (Chatto & Windus, London, 1889) and *Ednor Whitlock* (Chatto & Windus, London, 1891).

Nordic Journal of Philosophical Logic, Vol. 3, No. 1, pp. 197–229.

Though the two novels are different, they share a number of motifs and concerns, and, particularly in *Ednor Whitlock*, MacColl anticipates a number of the arguments he was later to spell out in *Man's Origin, Destiny, and Duty.*

Neither *Mr. Stranger's Sealed Packet* nor *Ednor Whitlock* are great works of literature. Their value lies elsewhere. They are novels not written out of any absorbing literary ambition, but because the author had enthusiasms and concerns he wanted to share with a greater public and, where necessary, he wanted to educate that public. The novels thus provide interesting insight into MacColl's concerns and opinions outside his special field. Such information is always welcome to the biographer of any eminent specialist. More important is that MacColl's concerns and opinions were those of an intelligent and enlightened Victorian intellectual. Intellectual history tends to concentrate on those who rebelled or refused to conform, and there is often an assumption that enlightened people of any period would hold views that the 20th century liberal academic could recognise as enlightened. Indeed, much left-wing literature on the role of the "intellectual" tends to include an anti-establishment, critical attitude as part of the definition of an intellectual. That Hugh MacColl should turn out to have typically conservative Victorian attitudes to and hold typically conservative Victorian views on important intellectual questions, may help us to realise how deeply these attitudes and views were entrenched in the outlook of even well informed and intelligent Victorians whom the 20th century liberal academic would expect to hold more "enlightened" views.

II.

Before proceeding to discuss MacColl's attitude and views in relation to those of his time, a brief look at the more technical aspects of his novels will reveal that in some respects MacColl was surprisingly modern. Both novels are the product of the new way of publishing fiction that spread in the 1880s and 1890s and that gave birth to the one-volume, well-focused novel that became the standard novelistic mode in the 20th century. One of the most profound changes in England towards the end of the last century was the emergence of the generation that had been educated under Forster's Education Act, which had been passed by Parliament in 1870.[1] The main effect of

[1] In 1870 W. E. Forster piloted the Elementary Education Act through parliament. It was a result of the obvious failure of the voluntary schools of the religious organisations to provide elementary education for all. The act did not do away with the voluntary schools, but empowered the government to "fill the gaps." In

this act was a rapid increase in literacy that created a vast new mass readership.[2] The emergence of such a numerous readership had two consequences. It represented a large new market for authors and publishers of fiction, and a market that could not be supplied through the circulating libraries. Because of its very size, it was also a readership that offered large economic rewards for those who could capture their attention. Secondly, it was a readership that exercised no discrimination in aesthetic matters, and that was essentially uneducated in the traditional sense. In other words, both the economics and the aesthetics of book publication changed. The cheap, one-volume novel took over from the Victorian three-decker novel, and a new kind of fiction

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the districts where no voluntary schools existed, it set up local School Boards to be responsible for education, with power to provide and maintain elementary education out of public funds. The act did not in itself make school attendance compulsory, but it did empower the School Boards to make attendance compulsory within their districts. At that time it would have been impossible to enforce compulsory attendance everywhere, since there were simply not enough places for all children of the relevant age group. To this day there is no national legal requirement in England that a child should attend school, though there is a requirement, laid down in 1880, that parents and local authorities be responsible for children between stated ages being efficiently educated in accordance with the requirements laid down by law.

[2]I adopt here the conventional view summarised by Walter Allen in *The English Novel* (Penguin Books, Harmondsworth, 1958) pp. 260–62:

> Forster's Education Acts of 1870 provided compulsory primary education for all, and the result, over the years, was an enormous increase in the reading public. But the gap between the best education and the worst was so great that the highbrow–lowbrow dichotomy with which we are now wearisomely familiar was inevitable. (p. 260)

It is repeated in other surveys and introductory books such as Michael Wheeler, *English Fiction in the Victorian Period 1830–1890* (Longman, London, 1985), pp. 155–56, and John Carey, *The Intellectual and the Masses. Pride and Prejudice among the Literary Intelligentsia, 1880–1939* (Faber and Faber, London, 1992), pp. 5–7:

> The "Revolt of the Masses" which these cultural celebrities deplored was shaped by different factors in each European country. In England, the educational legislation of the last decades of the nineteenth century, which introduced universal elementary education, was crucial. The difference between the nineteenth-century mob and the twentieth-century mass is literacy. For the first time, a huge literate public had come into being, and consequently every aspect of the production and dissemination of the printed text became subject to revolution. (p. 5)

An alternative view is presented by Patricia Anderson in *The Printed Image and the Transformation of Popular Culture 1790–1860* (Clarendon Press, Oxford, 1991). Anderson argues that the transformation of popular culture into mass culture started in the early 1830s and had been more or less completed by 1860. However, though Anderson's evidence establishes that there was a considerable literate public in existence before Forster's Education Act, it does not challenge the conventional view that there was a *dramatic expansion* of this audience in the late 1870s and 1880s.

appeared that had no aesthetic pretensions, but simply aimed at providing undemanding entertainment for the new mass readership. But if the new mass audience held out the promise of economic reward for those who wanted to pander to their simple tastes, it also offered an opportunity to the writer who wanted to educate, inform and persuade the large number of people who would not naturally participate in the discussion of, nor know much about, the great questions of the day. Both MacColl's novels belong to this new kind of relatively short one volume novel, and both have a strong didactic element, explaining issues of science and religion to those semi-educated people who would have no deeper understanding of them as well as arguing strongly and simply for certain points of view on these matters.

MacColl, at least in *Mr. Stranger's Sealed Packet*, is modern also in some of the narrative techniques he employs and in his choice of subjects. In *Mr. Stranger's Sealed Packet* there is a development of the device of embedded narration. The novel has a frame-story with a first-person narrator who receives a sealed packet from Mr. Stranger containing a manuscript which he is authorised to publish if Mr. Stranger himself fails to reappear within a period of five years. The main narrative is to be found in this manuscript. This device is not itself new. It goes back to the very beginning of the novel in England in the 18th century when it was common to provide a mock preface by "the editor" of the story which the novel presented. The usual function of these prefaces was to confer credibility to the story by providing it with a genealogy. However, the framing device goes much further in *Mr. Stranger's Sealed Packet*, where the reader is presented with a narrator who is a colleague of Mr. Stranger in the school where he works as a science master. The narrator, Percy Jones, is the English master, and in addition to him, the reader is presented with the mathematical master, Richard Johnson, and the classical master, John Greywood. They constitute both an audience for the story of the voyage to Mars when it is finally published, as well as a panel of judges on Mr. Stranger's character and the credibility of his story. By virtue of their different academic backgrounds they represent different perspectives and thus have different views to offer on the plausibility and possibility of Mr. Stranger's story. The credibility of Mr. Stranger's story is thus made a theme of the novel in itself. However, MacColl makes a much more naïve use of the device of embedded narrative than Joseph Conrad would do only a few years later in his Marlow stories, where, in particular in *Lord Jim* (1900) and *Heart of Darkness* (1902), the very possibility of arriving at truth through narrative is put in question through this narrative technique. Indeed, one might argue that MacColl does not even utilise the potential of the device to the extent that some of his predecessors, like Emily Brontë, did.[3] However, this may be because MacColl had a much simpler goal: to instruct a broadly ignorant audience in the possibilities opened up by science. For this purpose an audience of schoolmasters with different backgrounds would be well suited to offer comments on Stranger's discoveries and the use to which he put them.

MacColl's use of a device such as the embedded narrative indicates a certain level of interest in and consciousness of the problems of literary craftsmanship. Such an interest is, perhaps, no more than one should expect from someone who for "twelve or thirteen years ... devoted [his] leisure hours to general literature."[4]

III.

In *Mr. Stranger's Sealed Packet*, MacColl was also something of a pioneer in his choice of subject. It was the third novel in English about Mars to be published. In 1880 Percy Gregg published *Across the Zodiac*, and in 1887 Hudor Genone published yet another novel with Mars as its subject, *Bellona's Bridegroom: A Romance*. MacColl's *Mr. Stranger's Sealed Packet* then followed in 1889, and before the turn of the century, another eleven novels about Mars were published in English.[5] Among these was the most famous novel ever to be published about Mars, H. G. Wells *The War of the Worlds* (1898). Mars in

[3]In *Wuthering Heights* (1847) the narrator, Lockwood, retells a story, told him by the sceptical and down-to-earth Nelly Dean, which he does not really understand. Lockwood is the sophisticated city man to whom the Yorkshire Moors, where the elements play freely, are unfamiliar, and who therefore finds it difficult to grasp the elemental aspect of the love between Catherine Earnshaw and Heathcliff.

[4]Letter to Bertrand Russell, 17 May 1909.

[5]Camille Flammarion, *Uranie* (1890); Mortimer Leggett, *A Dream of a Modest Prophet* (1890); Robert Cromie, *A Plunge into Space* (1891); Alice Jones, Ilgenfritz and Ella Marchant, *Unveiling a Parallel: A Romance* (1893); Gustavus W. Pope, *Romances of the Planets, No. 1: Journey to Mars, the Wonderful World: Its Beauty and Splendor: Its Mighty Races and Kingdoms: Its Final Doom* (1894); James Cowan, *Daybreak: a Romance of an Old World* (1896); George DuMaurier, *The Martian* (1897); Kurd Lasswitz, *Two Planets* (1897); H. G. Wells, *The War of the Worlds* (1898); Ellsworth Douglas, *Pharaoh's Broker: Being the Very Remarkable Experiences in Another World of Isior Werner. Written by Himself* (1899). The frequency and number of novels about Mars increases further in the first decades of the 20th Century. The above listed books are to be found on the BookBrowser site on the Internet at http://www.bookbrowser.com/TitleTopic/mars.html This bibliography lists novels about Mars chronologically as well as by author. An even fuller bibliography comprising all fictional stories about Mars has been compiled by Gene Alloway, Senior Associate Librarian, University of Michigan (NSF/NASA/ARPA Digital Library Project), available on the Internet (http://www-personal.engin.umich.edu/~cerebus/mars/index.html#bibs).

those days was a topic made popular by new discoveries about the Red Planet. In 1877, the American astronomer, Asaph Hall (1829–1907), had discovered the two moons of Mars, Deimos and Phobos, and calculated their orbits. More importantly for imaginative literature, in the same year the Italian astronomer and senator Giovanni Virginio Schiaparelli (1835–1910) reported to have observed groups of straight lines on Mars. Schiaparelli called the peculiar markings he observed *canali.* The word, erroneously translated into English as "canals" instead of "channels," led to widespread speculation whether the "canals" were constructed by intelligent beings, and thus touched off much controversy about the possible existence of life on that planet. Oddly enough MacColl does not mention these discoveries. In his voyage to Mars, Stranger observes the two satellites of Mars and even alights on one of them, but there is no reference to their recent discovery by Hall. And MacColl does not mention and makes no use of the "canal" theory. However, the novel does make full use of known scientific theories and facts to make Stranger's story as plausible as possible. Stranger himself is an example of a new breed, the professional scientists, who all through the latter half of the 19th century were hard at work establishing their position as a professional class.[6] Stranger, when his father dies, withdraws from his public school, Classicton,[7] to carry out his father's wish that he should dedicate himself exclusively to science:

> Classics were to be completely thrown aside, and I was to devote myself wholly to science, and especially to mathematics, astronomy, chemistry, electricity, and practical mechanical engineering—a sufficiently wide curriculum. (pp. 22–23)

[6]The term "scientist" was not coined until the mid 1830s. The *Oxford English Dictionary* (2nd ed., Oxford University Press, Oxford, 1992) quotes an article from the *Quarterly Review* in 1834, where the problem of what to call men doing science is discussed:

> Science ... loses all traces of unity. A curious illustration of this result may be observed in the want of any name by which we can designate the students of the knowledge of the material world collectively. We are informed that this difficulty was felt very oppressively by the members of the British Association for the Advancement of Science, at their meetings ... in the last three summers. ... Philosophers was felt to be too wide and too lofty a term, ... ; *savans* was rather assuming, ... ; some ingenious gentleman proposed that, by analogy with artist, they might form scientist, and added that there could be no scruple in making free with this termination when we have such words as *sciolist*, *economist*, and *atheist*—but this was not generally palatable. (1834 *Q. Rev.* LI. 59)

On the lack of career possibilities for British scientists even in the last half of the 19th century see David Knight, *The Age of Science. The Scientific World-view in the Nineteenth Century* (Blackwell, Oxford, 1986), pp. 132–33.

This is to enable him to finish the development of the theories and discoveries that his father has made and to build a machine that can take him into space. MacColl goes into some detail about these theories and discoveries, in the same way as he later gives detailed descriptions of what Earth and Mars look like from space. Stranger's spaceship can produce artificial gravitation, and this gives MacColl the opportunity to make some effective points about the relative nature of spatial position and of the terms "up" and "down." In cases like this MacColl's didactic purpose becomes irritating. He repeats and reinforces the explanations of apparently paradoxical observations that his interplanetary voyagers make, to an extent where it becomes annoying to the reader.

However, when it comes to describing the spaceship itself and, later, the planet Mars and its inhabitants, MacColl falls back on the world he knows. This, as has been observed by Arthur Danto, is a general feature of science fiction. "Nothing so much belongs to its own time," says Danto,

> as an age's glimpse into the future: Buck Rogers carries the decorative idioms of the 1930s into the twenty-first century, and *now* looks at home with Rockefeller Center and the Cord automobile; the science fiction novels of the 1950s project the sexual moralities of the Eisenhower era, along with the dry martini, into distant eons, and the technical clothing worn by its spacemen belong to that era's haberdashery.[8]

It is certainly true that MacColl's description of Mr. Stranger's spaceship and the society he meets on Mars is deeply Victorian. The spaceship has the form of a cigar, the form of the projectiles of the day as well as of the airships that were then being built in Germany and France. It is controlled by handles and wheels, just as a train engine in those days would have been, except that the wheels and handles are made of ivory.[9] Mars itself is like Earth, with a few elements

[7]This unsubtle reference to the domination of classical languages in the curricula of the Public Schools and their neglect of scientific subjects indicates that MacColl had in mind a readership that needed this sort of blatant hint.

[8]Arthur C. Danto, "The End of Art", in Arthur C. Danto, *The Philosophical Disenfranchisement of Art* (Columbia University Press, New York, 1986), p. 83.

[9]It is worthwhile continuing Danto's remarks on visions of the future here:

> Robida [in *Le vingtième siècle* (1882)] imagined there would be restaurants in the sky to which customers would come in airborne vehicles. But the boldly anticipated eating places are put together of ornamental ironworks of the sort we associate with Les Halles and the Gare St. Lazare, and look a lot like the steamboats that floated the Mississippi at that time, in proportion and in decorative fretwork. They are patronized by gentlemen in top hats and ladies in bustles, served by waiters wearing long aprons from the Belle Epoque, and they arrive in balloons Montgolfier would recognize. (Ibid. p. 82)

added from recent discoveries of remains of prehistoric animals, so that Mars at times becomes a kind of Jurassic Park which Mr. Stranger describes for the reader's entertainment. Similarly, the scenery is that of Earth, but with a few changes of colour to explain why Mars is the Red Planet. It is also more majestic and more beautiful than even MacColl's native Scotland, though one suspects that Switzerland or Norway could probably hold their own with Mars in so far as scenery is concerned:

> It was a glorious spectacle. A majestic ocean lay before me, rolling its heavy swell against the rocky bases of a long, sweeping range of precipitous mountains underneath me. This range was broken and indented in many places by deep ravines, down which foaming torrents rushed headlong, forming numberless cascades and waterfalls, the confused noise of which was almost deafening. The sea ran in among the clefts and fissures of the rocky shore in long and narrow streaks—in some places cutting whole portions off and forming them into islands. (p. 51)

The Martians turn out to be human and not merely humanoid, having been transferred to Mars in a prehistoric catastrophe when Mars came so close to the Earth that its gravitational pull transferred a large number of people to its surface. Their only biological difference from Stranger is that their skin colour is bluish, and that they have large hazel eyes. Both features are caused by the food they eat, and are acquired by Stranger when he makes a longer stay on the planet. The Martians dress uniformly in what looks essentially like a Victorian bathing suit:

> They all, men, women, and children wore dresses of the same uniform pattern—a single garment, like a bathing dress, which covered the whole body with the upper portions of the arms and legs, exposing the head and neck, the hands and arms to a little above the elbow, and the feet and legs to a little below the knee. Though the sexes were not distinguished by any difference in the pattern of their dresses, they were strongly distinguished by their difference of colour: the dresses of the men and boys were uniformly and without exception red; that of the women and girls uniformly and without exception green. Both sexes had short, black, curly hair. (pp. 71–72)

They have all the normal human reactions, indeed, to a great extent they feel, act, and speak like Victorians. Stranger is taken into a family, having rescued one of the children from certain death, and, of course, falls in love with the daughter of that family, Ree. She turns out to be all you could expect of a Victorian woman. She is docile, obedient to her parents and to her husband, when she becomes Stranger's wife. She is emotional, her "bosom heaves" on several occasions when she becomes agitated. But she also has that inner emotional strength and wisdom that made a woman the angel of the house and the guardian of the central values of home and hearth: love, compassion, faithfulness. She thus comes very close to being the ideal woman described by Ruskin in "Of Queens' Garden."[10]

The adventure story involving travel and exploration was a traditional genre often employed to present the reader to a Utopian society that could be used as a point of reference for criticism of one's own contemporary society. There is a clear Utopian character to the Martian society that MacColl describes. It is a rational society and rationality produces uniformity. There is no foolish vanity of dress. Everyone dresses alike. Le Corbusier would have felt at home in the architectural environment. The buildings are functional and uniform in design as well as in the building-material, a marble-like substance that is neither "wood, stone, nor metal." The interiors are unadorned. The food is simple but healthy and produced by a chemical process from substances extracted from the air. There is no illness on Mars. And when Stranger brings his fiancée back to Earth simply for a sightseeing trip, she dies because she has no resistance to the bacteria there.[11] The absence of illness is paralleled by an absence of conflict in the society of the Grensum, which is the name of the people Stranger is first introduced to. A council of elders settles all controversial issues. There seems to be an influence of MacColl's Scottish background here: the society is a Presbyterian one with a strong puritan element.[12]

Though the society is technologically advanced in that it has self-propelling carriages, electric light and phonographic machines that can

[10][Woman] must be enduringly, incorruptibly good; instinctively, infallibly wise—wise, not for self-development, but for self-renunciation: wise, not that she may set herself above her husband, but that she may never fail from his side: wise, not with the narrowness of insolent and loveless pride, but with the passionate gentleness of an infinitely variable, because infinitely applicable, modesty of service—the true changefulness of woman. (John Ruskin, *Sesame and Lilies*, II, "Of Queens' Garden", 1871 (*World's Classics*, London, 1916), pp. 99–100)

[11]MacColl is apparently not the first to make use of the death of a Martian through Terran bacterial exposure. In Percy Gregg's two-volume *Across the Zodiac* (1880), "Gregg's hero travels to Mars in a vehicle powered by an antigravity device ... and encounters Martians so completely human that he takes several of them as wives. *One of them, alas, dies of an Earthy disease against which she has no resistance*" (Arthur C. Clarke, "Introduction" to the Everyman edition (J. M. Dent, London, 1993) of H. G. Wells, *The War of the Worlds*, italics in the text). H. G. Wells, of course made very effective use of this device in *The War of the Worlds*.

[12]The MacColls were not Presbyterians, but members of the Episcopal Church of Scotland. Hugh MacColl's mother, however, was a Gaelic-speaking Presbyterian who joined the Episcopal Church on her marriage. (See Michael Astroh, Ivor Grattan-Guinness, and Stephen Read, "A Biographical Note on Hugh MacColl," *History and Philosophy of Logic* (forthcoming).)

register speech as writing, it lacks totally the technology of war. There are no guns and only some very simple defensive weapons. This is a disadvantage as there is an external enemy, a barbarian people called the Dergdunin. The war with the barbarians gives Stranger the opportunity to play the hero, as the spaceship is also equipped for defence and attack. MacColl characterises this conflict in very simple, not to say simplistic, moral terms. The barbarian-civilised distinction is absolute. There is no moral nuance. The barbarians are bad and deserve everything they get.

The same simplistic moralism and moral self-righteousness manifest themselves in the attitude Stranger displays to animals. Without a tinge of irony MacColl describes how Stranger becomes morally enraged by the aggressive animals he meets on his travels. The animals, like the barbarians, are brutes not only in the naturalistic sense, but in the moral sense. On one occasion he shoots and kills a tiger-like animal that is pursuing a small rabbit. As Stranger sees it, this is protecting the weak from the strong. Neither he, nor his companions, his fiancée, her brother and her mother, are able to adopt the wider perspective in which killing the tiger is essentially no different from the tiger killing the rabbit. Both in the attitude to animals and in the attitude to the barbarians it is possible to see Victorian moral smugness and an attitude of moral self-righteousness, an unquestioning belief in the superiority of one's own values that was a necessary precondition for the imperial venture: Stranger goes to Mars to colonise.

Though MacColl to a certain extent presents the Martian society as a Utopian one where Stranger decides he wants to spend the rest of his life, MacColl makes little of use of this Utopia to comment on contemporary British society. As I have tried to indicate in the above discussion, MacColl was too deeply committed to the moral and social values of his own society to criticise it effectively. This certainly says something about MacColl's conservatism, in particular if we remind ourselves that 19th century Britain was excruciatingly self-critical. By the time MacColl wrote *Mr. Stranger's Sealed Packet*, he had available to him the whole crop of social and cultural criticism that had been steadily and vociferously produced throughout the century by Carlyle, Arnold, Mill, Ruskin, and Morris (who, in the year after the publication of *Mr. Stranger's Sealed Packet*, himself published his utopian novel, *News from Nowhere*).

IV.

Mr. Stranger's Sealed Packet gives a clear impression that MacColl was knowledgeable about science and scientific debates as well as an enthusiast for science. The novel has a scientific ethos. The characters move in a world governed by scientific laws which man and the Martians utilise for their benefit. Where possible, MacColl explains the scientific laws at work, and, as noted above, he repeatedly comes back to certain concepts like relative movement and explores the apparent paradoxical phenomena that arise because man tends to experience movement and position as absolute. There is, furthermore, no disapproval in his treatment of Stranger as the modern, professional scientist who must give up all humanistic studies to dedicate his life to science. The endorsement of science is not, however, unproblematic for MacColl. For the Victorian Period saw science develop to a stage where it came into conflict, or, as some saw it, apparent conflict, with the revealed truths of the Christian religion.

It is this conflict between science and religion and the related problems of faith, doubt, and unbelief that MacColl seeks to tackle in his second novel, *Ednor Whitlock*. Ednor Whitlock, the son of a clergyman, is a young lad of nineteen who, when he one day seeks shelter from the rain in the local library, by chance picks up an issue of the *Westminster Review*.[13] His eyes fall accidentally on an article arguing the untenability of the Christian belief in the resurrection of Christ. He becomes absorbed in the argument, and his faith is shaken to the extent that he decides against taking Holy Orders, something that he up to that moment had been planning on doing after completing his degree at Cambridge. Shortly after this, Ednor's father and mother die of typhus and Ednor and his sister, Ethel, two years his senior, are left to face the world alone in a struggle for existence.

Ednor obtains a position as a teacher in the family of the Reverend George Milford who succeeds Ednor's father as Rector in the parish of Wishport. Ethel also takes up teaching, accepting a post as an

[13]In 1851 John Chapman became the editor and proprietor of the *Westminster Review*. He was also a publisher, "mainly of books which were theologically heretical, and, I am sorry to say, did not pay" (William Hale White, *The Early Life of Mark Rutherford* (London, 1913), p. 82). Alvar Ellegård classifies the *Westminster Review* as politically radical, highbrow, and neutral in religious questions. He also gives it a full score on all occasions when it discussed Darwin's theories, for support both of the Theory of Evolution in its general application and for the theory as applied to man. See Alvar Ellegård, *Darwin and the General Reader. The Reception of Darwin's Theory of Evolution in the British Periodical Press, 1859–1872* (Almqvist & Wiksell, Gothenburg, 1958) p. 384.

English teacher in a private school run by a Mademoiselle Lacour in the French town of Blouville. When the Reverend Milford's son wins a scholarship to a public school, Ednor too seeks and gets a position at a school in Blouville as a mathematics tutor, maths being his special subject. The school is run by an Englishman, Charles Hubert Kent, M.A., of Trinity College, Cambridge (p. 90). Mr. Kent is known as the "Blouville crammer":

> He prepared young men for the English competitive examinations, especially the examinations for the Army and Indian Civil Service. He had selected Blouville for his scholastic establishment in order to give his pupils greater facilities for learning the French language, for which a good many marks were allotted in the said examinations. (p. 52)

In Mr. Kent's house Ednor meets and falls in love with Laura Kent, Mr. and Mrs. Kent's only child, who is also a day pupil at Mademoiselle Lacour's establishment. In spite of a fleeting attraction to Amy Milford, the Reverend Milford's beautiful daughter, who also enters Mademoiselle Lacour's school, Laura Kent becomes the object of his mature love. He does well in Mr. Kent's school and is accepted as a suitor for Laura by her parents, provided he shows that he can support her in a decent manner. In order to do this, he enters, at the end of the book, for an external degree at University College, London and achieves an honours degree there:

> As soon as Ednor had taken his degree, Mr. and Mrs. Kent allowed his probationary engagement with their daughter to terminate in the way which all had hoped, namely, in the intimate, lifelong union of marriage. (p. 340)

Part of the narrative focuses on Ethel's life and work in Mademoiselle Lacour's school. Ethel is contrasted with both Mademoiselle Lacour herself and the German teacher, Fräulein Hartmann. The contrast between the English Ethel and the very French Mademoiselle Lacour is a source of amusement and light comedy, but MacColl gives an unflattering portrait of Fräulein Hartmann. It is clear that the many years that he had already taught in a Boulogne school when he wrote the novel, had led him to share French prejudices against the Germans. However, Fräulein Hartmann is half-English as well as half-German, though for most of the novel she hides her English ancestry. She is, as are most of the other characters, a foil to Ednor and Ethel, but she is developed much further. She is the only character in the book in addition to Ednor who is described relatively fully and who is given a role to play in a subplot that is used to define her. She has a doubtful past, she is an evil force in the present, but then she is reformed through love and reaches a new faith by feeling rather than, like Ednor, by reason.

V.

As a novel of faith and doubt *Ednor Whitlock* joins a number of other novels that were written in the latter part of the century such as Samuel Butler's *The Way of All Flesh* (written 1873–1884, published 1903); William Hale White's Mark Rutherford trilogy, *The Autobiography of Mark Rutherford* (1881), *Mark Rutherford's Deliverance* (1885), and *The Revolution in Tanner's Lane* (1887); Mrs. Humphrey Ward's *Robert Elsmere* (1888); and Edmund Gosse's autobiographical *Father and Son* (1907). These novels are either spiritual autobiographies or stories about conflict between two generations. In the first category the conflict between faith and doubt takes place within the individual conscience and the reader is presented with the spiritual struggle and the suffering connected with this struggle. It is a testimony to the religious vigour of the age that both the struggle and suffering are intense and of long duration, and that considerable imaginative effort is expended on portraying the conflict. The outcome is neither predetermined nor uniform. In literature, as in life, some succeeded in reaffirming an (often modified) Christian faith, while others became agnostics (a term invented by Thomas Huxley)[14] or even atheists.

The conflict appears not only in late Victorian novels. The best known literary example of this struggle is Tennyson's *In Memoriam A. H. H.* written in the period from 1833 to 1850, long before the publication of Darwin's *The Origin of Species.*[15] Tennyson is also the standard literary example of one who won through to a reaffirmation of his belief in

> That God, which ever lives and loves,
> One God, one law, one element,
> And one far-off divine event,
> To which the whole creation moves.
>
> (*In Memoriam A. H. H.*, Epilogue, pp. 141–44)

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[14] "Suggested by Prof. Huxley at a party held prior to the formation of the now defunct Metaphysical Society, at Mr. James Knowles's house on Clapham Common, one evening in 1869, in my hearing. He took it from St. Paul's mention of the altar to 'the Unknown God'. R. H. Hutton, in a letter of 13 March 1881." (Exerpted from the *O.E.D.*, entry A under *Agnostic.*)

[15] The state of public knowledge of and interest in theories of evolution prior to the publication of *The Origin* is perhaps best illustrated by the popular and anonymously published *Vestiges of the Natural History of Creation* (1844):

> Its message was that the transformation of species was not merely a fact but a law. Chambers drew on developments in astronomy, evidence from fossils and comparative embryology, the popular science of phrenology (which promised character

And if Tennyson is the standard literary example of one who was able to reaffirm his faith, Matthew Arnold in “Dover Beach” (1867) is the standard literary example of one who came to see the world as emptied of divine purpose:

> Ah, love, let us be true
> To one another! for the world, which seems
> To lie before us like a land of dreams,
> So various, so beautiful, so new,
> Hath really neither joy, nor love, nor light,
> Nor certitude, nor peace, nor help for pain;
> And we are here as on a darkling plain
> Swept with confused alarms of struggle and flight,
> Where ignorant armies clash by night.

The outcome of the struggle between faith and doubt was also socially important, though much more so early in the century than towards the end. A rejection of Christian faith involved the rejection not only of a world *view* but of the *world* in which the protagonist had grown up: of parents, brothers and sisters, friends, neighbours etc. Thus it also involved the loss of place, of what had been one's home on this earth. Mark Rutherford not only rejects the doctrines of a narrow Calvinistic dissent, but also the world that goes with it, a world that is portrayed in some detail in the first chapter of *The Autobiography of Mark Rutherford*. This is the world of what Matthew Arnold called the “Dissidence of Dissent and the Protestantism of the Protestant Religion”, “a life of jealousy of the Establishment, disputes, tea-meetings, openings of chapels, sermons . . . .”[16] However, as “unlovely”,

determination from the topography of bumps on the head), even experiments purporting to show the in vitro production of microorganisms—all to make the point that the scientific elite who were denying organic evolution were missing the wood for the trees. All that was required for the emergence of new species was an abnormally long period of gestation in the development of an embryo. Widely considered a recipe for disaster, his book sold like hotcakes. (John Hedley Brooke, *Science and Religion. Some Historical Perspectives* (Cambridge University Press, Cambridge, 1991), p. 222)

The book was written by the Edinburgh publisher Robert Chambers and went through eleven editions of about 24,000 copies up to 1860. See Alvar Ellegård, op. cit., p. 11. Ellegård also has an interesting comparison between the impact on the wider public made by Chambers' *Vestiges* and Darwin's *The Origin* (ibid., p. 333). A twelfth edition of *Vestiges* was published in 1884. See also David Knight, *The Age of Science*, pp. 50–51.

[16] Matthew Arnold, *Culture and Anarchy, The Complete Prose Works of Matthew Arnold, Vol. V*, ed. R. H. Super (University of Michigan Press, Ann Arbor, 1965), pp. 102–103.

"unattractive", "incomplete", and "narrow" as life in this world may have been,[17] it is the world to which Rutherford had belonged from childhood. Rejecting it, he experiences a loss of community. Resigning his post as a Dissenting minister, he loses his career and his direction and becomes a wanderer physically as well as spiritually. Rutherford's loss of career is representative. Until late in the century loss of faith also had implications for careers and career prospects. Intellectuals who were atheists or agnostics could not be employed by the ancient universities, nor by the major employer of intellectuals in 19th-century England, the Church of England.

In the other type of novel about faith and doubt the struggle is externalised. It becomes a struggle between the religious orthodoxy of the older generation and the rejection of that orthodoxy by the younger generation. This struggle is against both parental and, in particular, patriarchal authority, as well as a revolt against the unthinking way in which the orthodoxy is practised and imposed on the younger generation by authoritarian means. The struggle is as intense as is the internal struggle, and at times tragic since it divides the new generation from its roots. However, when the conflict is a generational one, the loss of faith does not necessarily lead to the dark mood of despair hinted at in Arnold's poem and exemplified above all by Mark Rutherford's mood towards the end of *The Autobiography of Mark Rutherford*. Nor does the loss of faith lead, as it does with the protagonist of *Robert Elsmere*, to a desperate search for a new faith to replace the old. On the contrary, the rejection of orthodoxy is often felt as a liberation that goes together with the liberation from parental tyranny. Ernest Pontifex (in *The Way of All Flesh*) and Edmund Gosse (in *Father and Son*) represent a new secularised man for whom religious orthodoxy has ceased to be important. When Ernest Pontifex publishes his first book, its aim is to demonstrate an openness of mind that is precluded by any form of orthodoxy:

> The writer urged that we become persecutors as a matter of course as soon as we begin to feel very strongly upon any subject; we ought not therefore to do this; we ought not to feel very strongly even upon that institution which was dearer to the writer than any other—the Church of England. We should be churchmen, but somewhat lukewarm churchmen, inasmuch as those who care very much about either religion or irreligion are seldom observed to be very well bred or agreeable people.[18]

[17] Ibid., p. 103.

[18] Samuel Butler, *The Way of All Flesh* (Penguin Books, Harmondsworth, 1966), p. 415. The book that Ernest Pontifex publishes, is

> . . . a series of semi-theological, semi-social essays, purporting to have been written

And when Edmund Gosse sums up his experience of being brought up by orthodox parents, the emphasis is on the narrowness of mind, the joylessness, and the "spirit of condemnation" of orthodox religion:

> Let me speak plainly. After my long experience, after my patience and forbearance, I have surely the right to protest against the untruth (would that I could apply to it any other word!) that evangelical religion, or any religion in a violent form, is a wholesome or valuable or desirable adjunct to human life. It divides heart from heart. It sets up a vain, chimerical ideal, in the barren pursuit of which all the tender, indulgent affections, all the genial play of life, all the exquisite pleasures and soft resignations of the body, all that enlarges and calms the soul are exchanged for what is harsh and void and negative. It encourages a stern and ignorant spirit of condemnation; it throws altogether out of gear the healthy movement of the conscience; it invents virtues which are sterile and cruel; it invents sins which are no sins at all, but which darken the heaven of innocent joy with futile clouds of remorse. There is something horrible, if we will bring ourselves to face it, in the fanaticism that can do nothing with this pathetic and fugitive existence of ours but treat it as if it were the uncomfortable antechamber to a palace which no one has explored and of the plan of which we know absolutely nothing.[19]

VI.

Ednor Whitlock has elements of both spiritual biography and the novel of generational struggle. The protagonist is a young man whose faith is undermined by exposure to new scientific ideas and to the historical criticism of the *Bible*, and the novel chronicles his spiritual crisis and the journey towards a reaffirmed but modified Christian faith. The spiritual crisis is compounded by Ednor's realisation of the devastating effect his apostasy will have on his parents and his sister, and by the practical consequence that he cannot now take orders in the Church of England as planned. Like Mark Rutherford, whose loss of faith leaves "a God shaped hole in [his] heart,"[20] Ednor considers his loss of faith a

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> by six or seven different people, and viewing the same class of subjects from different standpoints. . . . Ernest had wickedly given a few touches to at least two of the essays which suggested vaguely that they had been written by a bishop. The essays were all of them in support of the Church of England, and appeared both by implied internal suggestions, and their *prima facie* purport to be the work of some half-dozen men of experience and high position who had determined to face the difficult questions of the day no less boldly from within the bosom of the Church than the Church's enemies had faced them from without her pale. (Ibid., pp. 413–14)

There is thus also a distance from the issues discussed in Ernest Pontifex's book that is built into the literary form itself. Ernest Pontifex is above all a man of letters.

[19] Edmund Gosse, *Father and Son* (1907) (Alan Sutton, Gloucester, 1984), p. 197.

[20] The expression is used by William S. Peterson in his introduction to the *World's Classics* edition of the book, p. ix.

personal catastrophe and not a release from tyranny; and like Robert Elsmere, "the religious man,"[21] he struggles to regain his faith. Like the protagonists of both types of novels, he must leave his parental home and the traditional and familiar surroundings in which he has grown up, though the reason is the death of his parents rather than his loss of faith.

Ednor, like Ernest Pontifex and Edmund Gosse, is the son of orthodox parents:

> From his childhood he had breathed a religious atmosphere. His father was a clergyman of the strictest orthodoxy, as well as zealous, energetic, and sincerely pious; his mother shared her husband's views and feelings; and he himself, with the full approval of both parents, intended to take Holy Orders. (p. 2)

And, like them, he is of the young generation that grew to maturity in the years after the publication of Darwin's *The Origin*, and of *Essays and Reviews* (1860) which brought to the attention of a wider English public the conclusions and implications of the so-called "German Higher Criticism."[22]

However, the interest of *Ednor Whitlock* is in the ways in which it differs from the better known novels of faith and doubt. Though Ednor is brought up by orthodox parents, he does not rebel. He is not oppressed by orthodox religion like the protagonists of the novels of generational conflict. Indeed, MacColl removes the whole problem of generational conflict by letting Ednor's parents die at the very beginning of the book. And though Ednor comes to doubt the basic doctrines of Christianity, he retains, in his ways of thinking, his morals, and his behaviour the characteristics of the Puritan believer. It is not merely, as he assures his sister, that

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[21] Elsmere is contrasted with the atheist Squire Roger Wendover, steeped in the latest German textual scholarship and theories, who presents to Elsmere the arguments against miracles in general and in particular the arguments against the resurrection of Christ. Wendover, however, is "constitutionally" different from Elsmere in not having a religious nature:

> Had he ever yet grasped the meaning of religion *to the religious man! God* and *faith*—what have these venerable ideas ever mattered to him personally, except as the subjects of the most ingenious analysis, the most delicate historical inductions? Not only sceptical to the core, but constitutionally indifferent, the squire had always found enough to make life amply worth living in the mere dissection of other men's beliefs. (Mrs. Humphrey Ward, *Robert Elsmere* (1888) (*World's Classics*, Oxford, 1987), p. 373)

[22] David Friedrich Strauss' *Das Leben Jesu* (*The Life of Jesus Critically Examined*) had been translated from the German by George Eliot in 1846.

whatever might be his uncertainties with reference to some articles of faith, his moral sense was not affected thereby. The path of duty, he said, was still clear and distinct, and he would always endeavour to tread in it. (p. 49)

In this respect Ednor resembles a large group of distinguished Victorians who all had ceased to believe in the metaphysical claims made by the *Bible*, but who retained the conviction of the truth of Christian morality. It is rather that Ednor is modest and sober to an extent that would please the most committed Puritan:

"You are not a teetotaler, I suppose?"

"No; but I seldom touch spirits."

"Quite right; I am glad to hear it. Your predecessor, though an excellent teacher, gave me some trouble in that way." (p. 91)

He blushes when he must encounter the other sex, displaying the sort of modesty that would win him favour with anxious parents of good-looking daughters:

The lad blushed as he walked up in answer to the summons, partly from pleasure at the thought of being near Miss Kent, partly also from his natural timidity in the presence of the other sex. (p. 141)

Not only is Ednor naturally timid, but, MacColl seems to imply, his timidity is natural in the sense of being right. He is contrasted with Reginald Pulting, the blackguard of the novel, whose lack of modesty indicates a morally corrupt nature. Ednor, of course, meets with the approval of the parent generation, whereas Pulting meets with disapproval. Rather than rebelling against the orthodox, older generation, Ednor conforms to their attitudes, to their patterns of behaviour, and ways of thinking.

Conforming in the way he does, Ednor comes across to the reader as judgmental and self-righteous in his attitude to other people. When he observes Reginald Pulting dancing with Laura Kent at the end-of-term party at Mademoiselle Lacour's *pensionnat*, he reacts with concern, a concern that is expressed in the most conventional terms:

Short as had been his time at Trinity House, it had been long enough to convince him that Reginald Pulting was not a desirable acquaintance for any girl. (p. 140)

His reaction is identical with that of the proper and pious older generation:

But other eyes besides those of Ednor's were on the pair, and thoughts not very dissimilar were passing at the same time through the minds of Mr. Kent, Mrs. Kent, and Mademoiselle Lacour. (p. 140)

This judgmental attitude also manifests itself on occasions when Ednor considers other people's opinions on questions of religious faith:

"You still admit the force of the argument which I employed at our last discussion?"

"Fully, sir; and what is more, I have had recourse to it myself in a discussion which I had, not very long ago, with some of Mr. Kent's pupils."

"Who were upholding atheism?"

"Yes."

"Alas! And are those mischievous errors so widely disseminated even among the young?"

"I am afraid so—at least, among young men. Those were not exactly boys; they are about my own age."

"And how did you put the argument, Whitlock? And how did they take it?"

"I laid it before them in the way in which I heard it from you, sir, as nearly as I could. In fact, I related to them the circumstances of that evening's debate. Two of the fellows seemed struck and acknowledged the force of the reasoning. The other two evidently did not wish to be convinced, and as they could not deny the significance and relevancy of the randomly-evolved figures, they denied their possibility."

"Their acquaintance with mathematics must have been rather elementary."

"It was, and not very accurate even within the limits attained. Yet they had all the stock arguments of the atheist by heart." (p. 201)

Ednor, communicating the experience of a discussion with Reginald Pulting and his three friends to the Reverend Milford, casts doubts on the honesty and questions the intellectual ability of those who cannot be brought to agree with the points he is making. As one reads MacColl's presentation of Ednor, one is reminded of Edmund Gosse's words about orthodox religion encouraging

a stern and ignorant spirit of condemnation; it throws altogether out of gear the healthy movement of the conscience; it invents virtues which are sterile and cruel; it invents sins which are no sins at all, but which darken the heaven of innocent joy with futile clouds of remorse.[23]

Technically, *Ednor Whitlock* is simpler than *Mr. Stranger's Sealed Packet*. There is no ironic distance in the former. MacColl uses a third person, omniscient narrator who does nothing to put Ednor's attitudes and behaviour in perspective. On the contrary, Ednor is being presented as an ideal young man: intelligent, principled, modest, sober, honest, handsome, "plucky," compassionate, kind, a good brother, and generally a loving and loveable man. His foil, Reginald Pulting, is

[23] Loc. cit., p. 212.

dishonest, deceitful, bad-tempered, brutal (he threatens to beat his own sister), vain, proud, a womaniser, and generally a cad. The contrast is stark and it is meant to be. The different moral qualities of the two MacColl explains with reference to their different religious upbringing:

> The two had some points in common; both were well endowed physically—well-built, healthy, and good-looking; and both were fond of cricket and athletics generally; but in the more important elements of character they were as wide apart as the poles. Their previous education as well as nature had something to do with this. Ednor's past history and training we have already described. He had been, as we have said, brought up in a pure, religious atmosphere, and by parents who sincerely strove to make their daily practice conform to their convictions. Pulting, on the contrary, had been brought up in an atmosphere of hypocrisy. His father was one of those clergymen—few, let us hope—who do not believe themselves in the Gospel which they preach to others. His son was not slow to discover this, and became at an early age a scoffing atheist and cynic. (p. 166)

It comes as no surprise when his sister, Ada Pulting, in conversation with her brother reveals that her father is "often unkind to mother and does other things which are not right" (p. 248). The absence of Christian faith leads, in the Pultings, to immorality. As one of Pulting's friends puts it:

> "Now, thanks to modern science, that frightful inquisitive spectre [i.e. God] has been laid, and I can munch the forbidden fruit in comfort and security." (p. 172)

The Pultings and Reginald Pulting's friends are used by MacColl to make two related points about the connection between Christianity and morality. The first is that Christian moral values cannot be justified if their metaphysical foundations are removed, and that therefore moral standards will dissolve. The argument is explicitly made by Pulting who welcomes the outcome:

> "In the pre-scientific age, when people believed in the Bible, the matter was very simple: to do *right* was to do God's will; to do *wrong* was to disobey Him. No definition could be clearer or neater. But to us enlightened moderns, to whom the God of Christianity is as mythical as the Jupiter of ancient paganism, the term *God's will* conveys no meaning. It seems to me that *right* should for the future, denote mere obedience to the laws of one's country, and *wrong* any violation of those laws." (p. 170)

Moral standards, however, are not identical with laws, laws being a codification of public opinion which is the ultimate authority on what is right and wrong:

> "There is no such thing, then, as *moral* right and wrong as distinguished from *legal* right and wrong?"

"I don't say that; but I do say that the morality or immorality of any action is decided by the vague unwritten code of public opinion, which is still more shifting than the written law." (p. 171)

Public opinion can impose no moral obligation on anyone, nor can it be a substitute for an all-seeing and all-directing God when it comes to instilling conscience in men:

"But there is this vast difference, Pulting, that, in the former case [when morality was assumed to be God's law], the conscience was rendered much more sensitive by the belief that God saw him; whereas, in the latter [when morality is considered to be merely what public opinion holds to be right], its sensibility may be completely destroyed by the belief that his actions are known to himself alone."

"Well, and what if it be so? Does not all that tell in favour of the new morality—at any rate, as regards the happiness of the individual?" (pp. 171–72)

As well as using them to present the argument that Christian moral values cannot be justified if their metaphysical foundations are removed, and that therefore moral standards need not be respected, MacColl uses Pulting, his family, and friends to illustrate the point which the argument makes. Their immoral behaviour is the result of their loss of faith in the metaphysical foundations of Christianity. They find no reason to conform to moral standards other than fear of being exposed should they be discovered.

In his portrayal of Pulting, his family and friends, MacColl gives expression to the majority view, widespread even in the late Victorian period, that unbelief causes immorality, and is therefore damaging to the social fabric. Religious evil was linked to moral and, consequently, to social evil:

The connection was a natural one. If religious belief was affected, the social fabric itself would disintegrate. A writer in the low-brow and somewhat goody *Family Herald* made this point quite bluntly: "Only let our scientific friends show the people, who are quick to learn, that there was no Adam ... that nothing certain is known, and then that chaos which set in during the lower Empire of Rome will set in here; we shall have no laws, no worship, and no property, since our human laws are based upon the Divine." That was written in 1861; ten years later the journal was still of the same opinion: "Society must fall to pieces if Darwinism be true." That the *Times*, in its review of *Descent*, gave prominence to this sort of argument only confirms how widespread was the attitude which gave rise to it. "A man incurs grave responsibility who, with the authority of a well-earned reputation, advances at such a time the disintegrating speculations of this book. He ought to be capable of supporting them by the most conclusive evidence of facts. To put them forward on such incomplete evidence, such cursory investigation, such hypothetical arguments as we have exposed, is more than unscientific—it is reckless."[24]

However, setting up Pulting as the target for condemnation, MacColl vulgarises and simplifies this view. Pulting's argument is crude: in the absence of reasons for believing in an all-seeing, all-directing, and vengeful God, there are no reasons for acting morally. With Pulting and his friends it is the absence of belief in certain punishment that opens the way for acting without reference to moral standards. Other and more thoughtful versions of the view placed much greater emphasis on the weakening of the ideal of Christ as an example, which it feared would be the consequence of a weakening of the belief in Christ as God. The argument was that if men came to see themselves as of the order of beasts rather than as made in God's image, they would no longer be able to adopt the noble motives which were so necessary for living "noble and virtuous lives":

> It is impossible to over-estimate the magnitude of the issue. If our humanity be merely the natural product of the modified faculties of the brutes, most earnest-minded men will be compelled to give up those motives by which they have attempted to live noble and virtuous lives, as founded on a mistake ... our moral sense will turn out to be a mere developed instinct ... and the revelation of God to us, and the hope of a future life, pleasurable daydreams invented for the good of society. If these views be true, a revolution in thought is imminent, which will shake society to its very foundations by destroying the sanctity of the conscience and the religious sense.[25]

In aligning himself with the majority view of the relationship between religion and morality, and a particularly crude version of it at that, MacColl is far from those many liberal Victorian intellectuals who for Christianity substituted a Religion of Humanity which would strengthen the moral bond between man and man rather than weaken it. In this *Ednor Whitlock* differs from *Robert Elsmere*, which tells

> The story of how an ex-vicar, who ceased to call himself Christian, and a devout wife whose faith was unshaken, suffered in their marriage for a time but at last came through to trust in each other again, by divorcing their moral unity from their religious opinions. The wife, wrote Mrs. Ward when the conflict was over (iii, 322), had "undergone that dissociation of the moral judgment from a special series of religious formulae which is the crucial, the epoch-making fact of our day."[26]

[24] Ellegård, op. cit., pp. 100–101.

[25] From the review of the *Descent of Man* in the *Edinburgh Review* 134 (1871), pp. 195–96; quoted in Ellegård, op. cit., p. 100.

[26] Owen Chadwick, *The Victorian Church. Part Two. 1860–1901* (2nd ed., Adam & Charles Black, London, 1972), pp. 120–21.

MacColl is far from recognising this "crucial" and "epoch-making fact." He does nothing to indicate that one needs to question Pulting's crude argument on grounds other than that it has the false premise that one can no longer assume the existence of "an all-seeing, all-directing God" (pp. 173–74). Ednor is really answering an argument that not many Victorian agnostics and atheists would subscribe to. Of course, Ednor addresses himself to the basic premise concerning the proof for God's existence, but even should he succeed in establishing this premise, the argument would remain as crude as ever: that the only reason for acting morally is fear of punishment after death. There is nothing here about noble motives or the example of Christ.

The second point about the connection between Christianity and morality that MacColl makes through the contrast between Ednor and Reginald Pulting, is that right moral sense is dependent upon an orthodox Christian upbringing. Pulting's moral character is destroyed because he "had been brought up in an atmosphere of hypocrisy." Ednor retains his moral sense even when he comes to question "some articles of faith," because he had been "brought up in a pure, religious atmosphere and by parents who sincerely strove to make their daily practice conform to their convictions." In addition, MacColl adds an interesting detail to Pulting's education:

> We said that Reginald Pulting at an early age had become an atheist and a scoffer at all religion. His two years' residence in Germany, where he met with many congenial spirits, both older and younger, did not alter his views in this respect, nor teach him more modesty in expressing them. (p. 167)

The German background is significant because among orthodox and less enlightened representatives of various British denominations, Germany and German universities, being the source of the "German Higher Criticism," were seen as the source of what they considered the anti-Christian attacks on the basic doctrines of the faith. When Mark Rutherford describes his education at the Dissenting College to which he is sent to be educated for the ministry, he remarks that "the word 'German' was a term of reproach signifying something very awful, although nobody knew exactly what it was" (p. 16). For MacColl, as for the dissenters at Rutherford's college, the German connection has clear, negative connotations. In MacColl's case these negative connotations would also have been due to his strong distaste for the scientific rationalism that became so popular in Germany as a result of Ernst Haeckel's (1834–1919) efforts.[27] Haeckel appears as the main target of attack in *Man's Origin, Destiny, and Duty*, where MacColl develops

[27] Chapter 10 of *Man's Origin, Destiny, and Duty* has the title "The Fallacies

more fully the arguments for theism and a future life that are presented in *Ednor Whitlock* by the Reverend Milford.

The German motif reappears in connection with the other depraved character in the novel, Fräulein Hartman. Fräulein Hartman is half-English and half-German, and though her German parentage is not used to explain her lack of moral sense, MacColl equips her with some unattractive qualities that are clearly connected with her German ancestry. She is a strict disciplinarian, has an inordinate degree of national pride, and is uncompromising and unyielding. Because she is German, she is the object of some animosity on the part of the French characters in the novel, an animosity that is shared by the narrator of the story. The rhetorical effect is to bring about an association between being German and being an immoral atheist, by combining the two qualities in one and the same character.

In attaching strongly negative connotations to the German background of both Pulting and Fräulein Hartman, *Ednor Whitlock* differs radically from the better known novels of faith and doubt where Germany and German scholarship is seen as the source of new knowledge and new insights.[28] When Mark Rutherford remarks about the President of the Dissenting College to which he belongs that he "knew nothing at all of German literature" (p. 16), this disqualifies him in Rutherford's eyes from dealing meaningfully with "unbelief." In *Robert Elsmere* the atheist Wendover may be an unpleasant and unhappy char-

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of Haeckel." Brooke, op. cit., p. 300 sums up Haeckel's influence in Germany as follows:

> It was [in Germany] that Haeckel had turned Darwin's science into a popular movement with its own world view—a substitute religion with its own catechism of nature worship. Of the major European countries, Germany had seen the greatest surge in mass literacy, creating the conditions for Darwinism to engage a wider public. An expanding market for popular science created opportunities that the churches seemingly overlooked but which were seized by the advocates of scientific rationalism. Prominent among them were Friedrich Ratzel, Carl Vogt, Ludwig Büchner, Arnold Dodel, Edward Aveling, and Wilhelm Bölsche—each of whom added their volumes to those of Haeckel and, in their different ways, peddled the notion that Christianity was defunct, evolution the victor. Scientific progress had not only rendered special creation obsolete. It had made it inconceivable.

For further comments on Haeckel by Brooke, see pp. 288–89.

[28] In two letters to Betrand Russell, MacColl expresses regret that German research is inaccessible to him because he does not know any German: "I understand that Schroeder has written a big work on the subject [the Logic of Relations], but it is in German, a language of which I know nothing" (June 28, 1901); " … unfortunately all German works are debarred to me because I do not know the language, so that I know nothing of Cantor's and Dedekind's views on infinity." (December 18, 1909).

acter, but the German scholarship that he represents is recognised as having the most advanced thoughts of the day in questions relating to the understanding of the *Bible*. It must be taken on board and assimilated. It cannot be dismissed or ignored.

In Fräulein Hartman MacColl creates a far more interesting character than Reginald Pulting, who is merely a cardboard villain. Fräulein Hartman, too, is used to illustrate the influence of upbringing on moral attitudes, and her moral character is essentially similar to that of Pulting:

> Gertrude Hartman was also spoilt by her parents in another sense, and a worse sense. Her moral instincts were perverted from her very infancy. It is questionable whether she ever really understood the difference between right and wrong. She heard those words often enough, and occasionally employed them herself, but from the example, if not from the precept, of her parents she attached a rather heterodox meaning to them. To do wrong was to offend the prejudices of Mrs. Grundy; to do right was to keep in Mrs. Grundy's good graces. The path of duty seemed thus a very simple one, and not unreasonably hard to follow. Nothing was ever wrong so long as it could be kept concealed; but if ever it got to the ears of Mrs. Grundy and happened to be on that lady's black list, it became wrong directly. (p. 106–107)

However, in the case of Fräulein Hartman, MacColl gives a much more substantial presentation of her background, her parents, their fate, and the influence this has on her moral character than he does in the case of Pulting. Fräulein Hartman is as immoral as Pulting, but her lack of moral fibre is better motivated in the novel. MacColl also gives a full account of her early years and provides her with some redeeming features that are finally instrumental in saving her for faith. MacColl, however, cannot resist the temptation to endow her too richly with negative qualities, making her deceitful, hypocritical, resentful, unforgiving, and bad-tempered. Because she is not Christian, either by conviction or upbringing, she is not allowed to possess Christian virtues.

In *Ednor Whitlock*, as in *Mr. Stranger's Sealed Packet*, MacColl works with a simplistic and rigid moral scheme. There is a moral outrage that comes out in his portrayal of the unchristian characters of the novel that is lacking in charity, understanding of or insight into other points of view. Moreover, the moral values that he propagates are those that to enlightened Victorians came to seem particularly oppressive. Ednor may condemn Pulting and MacColl may condemn Fräulein Hartman, but in the eyes of a liberal reader they themselves stand condemned by their condemnations.

VII.

In *Ednor Whitlock* the question at issue in the conflict between faith and doubt is whether evidence can be found for the existence of an all-seeing, all-directing God, for the belief in a future life, and for belief in Christ as man's saviour through his suffering and resurrection. The atheism of Pulting and Fräulein Hartman results from the failure of the revealed truths of religion to stand up to scientific examination. "Abler persons than I, and than you also," says Pulting to Ednor, "have shown irrefutably that atheism is a necessary consequence of Darwin's theory" (p. 174). Even after Fräulein Hartman becomes Madame Delanoy and a reformed character she cannot accept the truths of religion: "She yearned for the simple, unquestioning faith of her girlhood; but it would not come back; while her reason remained unconvinced, her will was powerless to recover it" (p. 324). Ednor, in his struggle to regain his faith, tries to find *reasons* for believing in a future life and the continued existence of the soul after death, in the miracles and in particular the resurrection of Christ. There is no question of making a leap of faith. A substantial part of the book is taken up by providing such reasons as Ednor is looking for. The role of providing these reasons is given to the Reverend George Milford. He aims to take Ednor through a three-stage argument:

> We must proceed slowly, step by step. First, theism; then, the doctrine of a future life; and finally, Christianity. The establishment of the first is a great step towards a proof of the second; the establishment of the second a great step towards a proof of the third. (p. 202)

What he has to offer is "a strong argument from purely scientific principles." He takes up again Paley's "argument from design" (see below), but with a particular twist. Through a mathematical example (all the good and honest characters of the book, Ednor included, are expert mathematicians) Milford purports to show that chance can be given a role to play within the limits of design:

> "We have a great number of points falling hap-hazard on the paper [with a two-axis co-ordinate system], but under the restrictions of certain pre-assigned laws and limits, which would make them fall more thickly in certain places, more scantily in others, and exclude them entirely from others. The result *might* be a very pretty geometrical figure, the boundary of which might be the curve $\psi(x, y) = 0$, and the distribution of whose shading would be determined by the functions f and ϕ, since by supposition $x = f(u)$ and $y = \phi(v)$."
>
> "Then you admit that the exact shape of the resulting geometrical figure could be accurately determined beforehand?"
>
> "I do."

"And the distribution of its shading—the exact spots where it would be shaded darkly through the superabundance of the randomly falling points, and the exact spots where it would be shaded lightly from their comparative scarcity—could these also be accurately foretold beforehand?"

"They could in simple cases; but in others, to forecast the exact points or curves of maximum and minimum shading might baffle the skill of the best mathematicians."

"But with sufficient mathematical knowledge and skill, it would be perfectly possible?"

"It would. But what on earth has all this to do with evolution or theism?" (pp. 70–71)

The point of the example is that though the points that fall on the paper fall randomly, they fall within a design that is totally predictable once one knows the laws that limit the distribution of the points. In this way evolution can appear to take place by chance, but the occurrence of chance events may be so limited by laws as to make the outcome of evolution totally predictable for a sufficiently sophisticated intelligence. This argument is presented to Ednor both verbally, as when he is permitted to listen to a debate between Rev. Milford, Mr. Morley, who is an agnostic, and Mr. Manning, who is an atheist, and in written form, as when he is given a manuscript by Rev. Milford dealing with these issues. MacColl spends a substantial part of the book on these arguments, and it is here that MacColl's strong didactic intention emerges. The arguments are addressed as much to the reader as to Ednor.

MacColl also tries to thematise the design that is formed by apparently chance occurrences in the fate of Ednor and, to a certain extent, the fate of Fräulein Hartman. Already in the first few pages of the book the two motifs of chance and struggle for existence are introduced: "It is a trite remark," the book opens, "that important crises in men's lives sometimes turn upon circumstances apparently trivial" (p. 1). Then the story begins with a number of such "trite," chance incidents. A sudden, heavy shower of rain forces Ednor to look for shelter. As he happens to be passing the Wishport reading room, he enters (for the first time):

More as an excuse for this intrusion (as he almost considered it) than from any real desire to read, he took possession of the nearest empty chair and the nearest disengaged volume. This happened to be the *Westminster Review*, a magazine of whose very existence he had been till then ignorant. Opening it at random, his eye rested on an article entitled, "The Evidences of Christ's Resurrection." (p. 2)

Before Ednor can bring himself to reveal to his parents his personal religious crisis and the consequent decision not to take orders, another chance event intervenes: a "typhus fever" strikes them down. Such chance events continue to influence Ednor's life, and lead him back towards a modified Christian faith. However, these incidents are too few and have too little effect on the fate of the characters to create a distinct impression of chance as a force in their lives. A useful contrast here is the use which Thomas Hardy makes of chance in *Tess of the D'Urbervilles*, written in the same year as *Ednor Whitlock*. In *Tess*, chance dominates Tess's life like an evil fate; chance always decides the direction which the next phase of her life is going to take. The accidents that decide Tess's life are never obtrusive, yet there is no denying that they form a pattern.

The argument that is offered by Milford is a late version of natural theology: science and the *Bible* are in mutual support and not in mutual opposition. For various political, social, and geographical reasons the marriage of science and religion in natural theology lasted longer in Britain than in other European countries.[29] In 1802 William Paley, an Anglican minister, published what was to become one of the most popular works of philosophical theology in 19th century Britain, *Natural Theology*. In this work Paley set out in detail "the argument from design" as it had developed in the two previous centuries, when the complexity and order that science had progressively revealed in both the micro- and macro-cosmos was taken as a confirmation—indeed, as a positive proof—of the existence of God. The design argument had of course preceded the development of modern science, but the discoveries made by science in the 17th and 18th centuries seemed immeasurably to strengthen this argument.

The attractiveness of the argument from design for the Enlightenment mind was that it reduced the role of revelation as a basis for belief. Belief in God could be rationally founded, and this gave Christian apologists an important weapon in their argument against non-believers who might reject the validity of revealed religious truth. However, increasing the reliance on reason and evidence in questions of faith also carried the new risk that faith could be disproved by developments in science. This risk was particularly great if too heavy a burden was placed on the sciences, e.g. if they were asked to prove not only that God was an intelligent artificer, but also his omnipotence and omnibenevolence.

In Britain it was the "new geology" as developed by Sir Charles Lyell in *The Principles of Geology* (1830–1833) that first put pressure on natural theology. Assuming that only those forces which science

[29] See Brooke, op. cit., pp. 198–203, for a summary of these reasons.

could observe in operation today had also operated in the past, he found a way to calibrate the past. His estimates of the age of the various strata of fossils, and consequently of the age of the earth, ran into hundreds of millions of years, thus challenging the literalist interpretation of *Genesis* which was still widely accepted in the 1830s.[30] More important than this was the change in the way of thinking about geological change that Lyell introduced. According to Lyell, geological change was slow but perennial: it was always going on and it affected even what appeared to be the most permanent features of the geological environment. The stability of the world was being undermined; the earth had developed throughout these hundreds of millions of years and was still developing. And the geological development was not necessarily a result of tidy design, but could plausibly be seen as a result of an indifferent process of cause and effect. Sensitive minds picked up this implication of Lyell's way of thinking about geological change.

> The hills are shadows, and they flow
>   From form to form, and nothing stands;
>   They melt like mist, the solid lands,
> Like clouds they shape themselves and go.

wrote Tennyson towards the end (stanza CXXIII) of *In Memoriam* (1850), recognising not only perennial change, but also adopting the language of causal process: the hills "shape themselves and go." The adoption of the language of causal process is important here because it involves the adoption of a different perspective, a different discourse, than the argument from design. The pressure on natural theology culminated with Darwin's *Origin*, not because it disproved the argument from design, but because it provided a non-teleological vocabulary that was rich enough to explain in causal terms what had up to then only been explicable in a teleological vocabulary. It did not provide an alternative *competing* explanation to that of the argument from design. It

[30]By the mid-1840s, says Owen Chadwick, "Educated divines had already abandoned the more vulnerable places of the Mosaic story. By the fifties they were saying that for many years no man of sense had believed in a creation of the world during six days of twenty-four hours." (*The Victorian Church. Part One. 1829–1859* (3rd ed., SCM Press, London, 1987), p. 563.) However, if one is to believe the literary portraits of clergy and important lay members of various Christian denominations (in, e.g., Butler's *The Way of All Flesh*, Gosse's *Father and Son* and William Hale White's *The Autobiography of Mark Rutherford*), literalism remained important among the less sophisticated (though "educated") Christians well into the second part of the century.

adopted a different perspective that legitimised a different type of discourse. One may say that *The Origin* exposed the logic of the argument from design. "Design" is an intentional term and its very use embodies intentionalist assumptions. There is, strictly speaking, nothing in the physical shape and construction of objects that in itself marks them out as designed. Seeing an object as a design-product involves a choice of perspective and cannot be a deduction from observation. To use the argument from design to prove the existence of God is therefore to misunderstand the possibilities of the argument. There is no inferential route from an object to its designer. However, once God's existence is assumed, the natural world can reveal God's nature and purpose to us.

Even before the publication of Darwin's *Origin* natural theology was losing ground. "Paley's argument from design," says Owen Chadwick,

> became irrelevant to any late Victorian theology that mattered. The first shadow of the knowledge of God seemed to lie in the heart or the conscience, not in nature; and only after God was apprehended through feeling or through moral judgment did nature become evidently sacramental of his being.
>
> On this matter the great divide came after the Bampton lectures (1865) of J. B. Mozley entitled *On Miracles*. The book is the last statement, by a great English Protestant theologian, of a world of divinity which henceforth vanished except in the scholastic manuals. Mozley's fundamental axiom was the need to "prove" Christianity, as Paley once proved it; and the internal evidence of heart and conscience can supply no "proof" to the reason.[31]

By the time Frederick Temple, as Bishop of Exeter, came to give *his* Bampton Lectures in 1884, he was able to open his fourth lecture without much danger of being opposed by saying,

> Religion is rooted in our spiritual nature and its fundamental truths are as independent of experience for their hold on our consciences as the truths of mathematics for their hold on our reason.[32]

[31] Owen Chadwick, *The Victorian Church. Part Two. 1860–1901*, pp. 30–31.

[32] Fredrick Temple, *The Relations between Religion and Science. The Bampton Lectures 1884*; Lecture VI, "Apparent Collision between Religion and the Doctrine of Evolution"; reprinted in Tess Cosslett (ed.), *Science and Religion in the Nineteenth Century* (Cambridge University Press, Cambridge, 1984), p. 192. Page references are to this collection. Frederick Temple was one of the contributors to *Essays and Reviews.* He was consecrated Bishop of Exeter in 1869 against considerable opposition. However, there was no similar opposition when he became Archbishop of Canterbury in 1896. Chadwick sees Temple's "elevation to the most senior see" without significant protest as marking "the final acceptance of the doctrine of evolution among the divines, clergy and leading laity of the established church, at least as a doctrine permissible and respectable in an eminent clergyman." (*The Victorian Church*, p. 23.)

For Temple the argument from design could only compel assent from those who had already responded to the inner voice. But if the argument from design could not prove God's existence, neither could the negative version of it prove that there is no God:

> The argument is not strong enough to compel assent from those who have no ears for the inward spiritual voice, but it is abundantly sufficient to answer those who argue that there cannot be a Creator because they cannot trace His action.[33]

If one assumes the designer, then it is easy to identify the design in creation. Even the Theory of Evolution can then be seen as a partial explanation of what God's design is, and thus gives us a deeper insight into God's ways:

> And the scientific doctrine of Evolution, which at first seemed to take away the force of this argument [the argument from design], is found on examination to confirm it and expand it. The doctrine of Evolution shows that with whatever design the world was formed, that design was entertained at the very beginning and impressed on every particle of created matter, and that the appearances of failure are not only to be accounted for by the limitation of our knowledge, but also by the fact that we are contemplating the work before it has been completed.[34]

In 1891, when *Ednor Whitlock* was published, the theological function of the argument from design had long since changed from a demonstrative to a non-demonstrative one. What was required for the truly Christian was a leap of faith. Once that was taken, problems of doubt did not arise, only problems of interpretation of the word of God. "A strong argument from purely scientific principles" for the existence of God and the future life of the soul had no theological support any more. It may still have had appeal to a relatively numerous audience who would have liked to see the continued marriage between science and religion rather than their separation. However MacColl's argument also shows a high degree of idiosyncrasy.[35] Just how idiosyncratic it

[33] Temple, op. cit., p. 208.

[34] Ibid., p. 208.

[35] Some parts of MacColl's argument would seem more farfetched and idiosyncratic today than they would have done at the time. For example, MacColl argues for the separate existence of the soul and for the influence of "higher" spiritual beings on man by invoking the *established fact* of telepathy (*Ednor Whitlock*, p. 236; *Man's Origin, Destiny, and Duty*, pp. 13–14; pp. 28–29; pp. 32–33). Telepathic and other psychic phenomena were the objects of scientific investigation in the last two decades of the 19th century. The Society for Psychical Research was founded in 1882 and had as members and officers many well-reputed scientists and clergy. See David Knight, op. cit., pp. 195–97.

was can be seen from the summary of his argument that he presents in the preface to *Man's Origin, Destiny, and Duty*, which he published eighteen years after *Ednor Whitlock*:

Basing my arguments upon facts admitted by nearly all scientists, I have striven in this little volume to establish the following propositions:—

(1) That, as regards man and all sentient animals, the soul (which I simply define as *that which feels*) and the body are different entities.

(2) That the soul will survive the body and, by successive transformations, will continually develop upwards.

(3) That a psychic universe exists containing numberless ascending orders of intelligent beings above the human; though these are imperceptible to man's senses in the present stage of his development.

(4) That the whole physical and psychic universe is maintained and directed by one infinitely powerful (according to a clear and rigorous definition of the word *infinite*) and infinitely intelligent Being, whose will, as shown in the so-called "laws of nature," it is man's duty to study, and, within the limits of his faculties and knowledge, to obey.

The question so often discussed whether the Supreme Being is a "personal," an "impersonal," or an "immanent" God, I leave un-touched: firstly, because I consider it irrelevant to my argument; and, secondly, because it is scarcely possible to enter upon such a discussion without losing oneself hopelessly in a maze of verbal and metaphysical ambiguities. (pp. v–vi)

VIII.

MacColl was an educated man, though he did not have the benefit of the best education. He was a schoolteacher with a passion for logic and mathematics that required considerable intelligence and intellectual investment. Yet outside these areas his attitudes and views were both conservative and simplistic. Particularly striking is the simplistic moral scheme that he uses in the construction of both his novels as well as the lack of tolerance and understanding of other points of view. MacColl's conservatism in these areas may be due to the fact that he moved to France early in his life and spent a large part of it there in isolation from the developments that took place on the intellectual scene in Britain. However, it is quite clear that he kept himself informed about developments in science and in the struggle between science and religion. Moreover, for "twelve or thirteen years" he "devoted [his] leisure hours to general literature," and, if one is to judge from the topics and techniques of the two novels, was well enough informed about at least certain kinds of developments in imaginative literature.

In his moral, social and religious attitudes it is plausible to see MacColl as representative of a broad Victorian public that continued to exist and exercise considerable influence well into the 20th century. This public was to be found partly within the Church of England and partly among the Nonconformists. Indeed, there are strongly Puritan elements in the moral schemes of the novels that one associates in particular with the Nonconformists. "Along with improper sex," says Richard J. Helmstadter in an article on "The Nonconformist Conscience,"

> gambling and drink made up a trio of sins that the Nonconformist conscience found particularly irritating—a trio that Nonconformist leaders tried, sometimes successfully, to raise to the level of important public issues. "The three deadly enemies of England" were identified by Hugh Price Hughes in the *Methodist Times* (6 June 1895) as "drink, impurity and gambling."[36]

Pulting, of course, is a womaniser who also gambles and drinks. Drink had been the problem of Ednor's predecessor at Mr. Kent's school. And the attitude to women that manifests itself in both the novels (Laura Kent and Amy Milford are as docile, obedient, and compassionate as Ree) is also characteristic of a broad, educated public that was not particularly progressive in its view.

In his enthusiasm for science MacColl is more distinctly modern. The fact that he writes one of the first novels in English about Mars places him among those who were abreast of the developments of the time rather than in the dark. He is also eager to inform about science and to preach his own version of natural theology. With regard to the latter he is clearly out of touch with late Victorian theological thought, and he remains a diehard rationalist in his view of the relationship between science and religion. Above all, he is idiosyncratic in the length to which he is willing to take this argument. But then it is his enthusiasm for and belief in science that give some charm to what otherwise are two rather dry novels.

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[36] Richard J. Helmstadter, "The Nonconformist Conscience" in *Religion in Victorian Britain, Vol. IV: Interpretations* (Manchester University Press, Manchester, 1988), p. 65.

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