Upperbounded no more

One of Krifka’s (1999) arguments against the generalised quantifier approach to modified numerals is that it fails to explain why modified numerals do not give rise to the same implicatures as bare numerals do. For instance, why does Cody brought more than four friends not implicate that it is not the case that (the speaker believes that) Cody brought more than five friends? In this paper, I argue that not all modified numerals are alike w.r.t. the implicatures they give rise to. In fact, I will make the following much more general claim:

★ Expressing the relation < or > yields no (acceptable) implicatures
★ Expressing the relation ≤ or ≥ does give rise to implicatures

The above generalisation follows directly from Fox and Hackl’s (2006) thesis that measurement scales are always dense.\(^1\) To illustrate this in the domain of numerals, say that X represents the set of friends Cody brought to my party. A meaning |X|>4 would give rise to the implicature that it is not the case that |X|>4+d for any d>0. (These are the logically stronger alternatives). So, for instance, |X|>4 implicates that ¬(|X|>4.1), ¬(|X|>4.01), ¬(|X|>4.00001). . . etc. The implicatures end up contradicting the assertion and, so, no implicature surfaces. Things are different for |X|≥4, which implicates ¬(|X|≥4+d) for any d>0. Together these entail |X| = 4.

At first sight, this seems a wrong prediction to make, because Cody brought at least four friends cannot end up meaning that Cody brought exactly four friends. However, at least is an unfortunate candidate for testing the difference between > and ≥. As Geurts and Nouwen (2007) discuss in detail, superlative modifiers have a fairly rich semantics: they are highly focus-sensitive and express modal attitudes. In other words, at least and at most are not the natural language counterparts of ≥ and ≤. Better candidates are no fewer and no more. Here, the data are more promising. Cody brought no fewer than four friends indeed seems to suggest that Cody brought four (and no more) friends. Intuitions might be a bit blurred in these cases too, for it seems unclear why the speaker would negate a positive comparison with respect to 4, just to express 4. (The reason might be to communicate his belief that four friends count as quite many, a clear inference to follow from such sentences.)

Stronger evidence for the two claims made above comes from the interaction of < and and ≤ and modality. Heim (2000) discusses the scope-taking potential of degree phrases and accounts for the ambiguity of (1) in terms of the relative scope of the maximality operator in the comparative and the modal.

(1) (Vic’s paper is 20pp long). Cody’s paper is allowed to be less long than that.
   a. ◇[max₃(Cody’s paper is d-long)<20pp] \(\checkmark\) modal>er
   b. max₃(◇[Cody’s paper is d-long])<20pp \(\checkmark\) -er>modal

In the following example, a negated comparative, we would expect three readings, given that it is reasonable to assume that no can take scope independently of the comparative. (Only three because the maximality operator cannot scope over the negation, since this gives rise to reference to undefined degrees.) (Phrases like no longer than . . . are probably differentials rather than straightforward negations of comparatives. In Dutch and German, such differentials do not exist, but are expressed using negated comparatives like niet langer dan/nicht länger als (not longer than). For ease of exposition, I treat the English cases similarly. Nothing hinges on this.)

(2) Cody’s paper is allowed to be no longer than 20pp.
   a. ◇[max₃(Cody’s paper is d-long)≤ 20pp] \(\checkmark\) modal:negation>er
   b. ¬◇[max₃(Cody’s paper is d-long)> 20pp] \# negation>modal>er
   c. max₃(◇[Cody’s paper is d-long])≤ 20pp \# negation>er>modal
   d. max₃(◇[Cody’s paper is d-long])= 20pp \(\checkmark\)

\(^1\) Fox and Hackl focus mostly on more than and > and do not discuss this consequence to their theory in this way.
(2) has two readings, one of which is (2-a). (2-b) and (2-c) are equivalent, but they do not represent the other possible reading for (2). (2-b) says that papers longer than twenty pages are not allowed. What (2-c) says is equivalent: the maximally allowed length of Cody’s paper is in the 0-20pp range. On the relevant reading, however, (2) says more than that: it entails that Cody’s paper is allowed to be 20pp long. That is, (2) can express the meaning in (2-d). Note that the absence of the (2-b)/(2-c)-readings cannot be explained from a general restriction on the relative scope of modals and negation. For instance, in Dutch, where a negated comparative gives rise to the same intuitions as with (2), (3) is three-way ambiguous:

(3)  Je mag niet alleen Cody uitnodigen. (You may not only Cody invite)

 a.   ⋇[¬only(λx.invite(you,x),Cody)]     \(\checkmark\) modal>negation>only
 b.   ¬⎟[only(λx.invite(you,x),Cody)]     \(\checkmark\) negation/modal>only
 c.   ¬[only(λx.⎟[invite(you,x)],Cody)] \(\checkmark\) negation>only>modal

The puzzle of where the reading in (2-d) comes from also appears when we turn to negative comparative quantifiers. It follows from the first sentence in (4-a), but not from that in (4-b) that the hearer is allowed twelve cookies.

(4)  a.  You are allowed no more than twelve cookies. # In fact, you’re allowed none!
 b.  You are not allowed more than twelve cookies. In fact, you’re allowed none!

In fact, all modified numerals expressing an upperbound (i.e. those corresponding to \(\leq\), and not to <) behave the same in this respect, as (5-a) shows. (The full paper will discuss the modifiers in (5-a) in somewhat more detail, and offer an explanation as to why in non-modal cases, they do not give rise to implicatures. This explanation partly follows Geurts and Nouwen 2007).

(5)  a.  Your paper is allowed to be [ at most / maximally / up to ] 10 pages long. # In fact, it should have exactly 5 pages.
 b.  Your paper is allowed to be less than 10 pages long. In fact, it should have exactly 5 pages.

The data above can be explained if we assume that \(\leq\)-expressions not only can but, in the general case, must be interpreted exhaustively. For instance, if we allow no in (4-a) to scope over the modal, we get the interpretation in (6-a). The logically stronger alternatives are in (6-b). Combining (6-a) with the negation of these results in (6-c).

(6)  a.   ¬⎟[max\(_d\)(you have d-many cookies)>12] = ⋇[max\(_d\)(you have d-many cookies)\(\leq\)12]
 b.   ⋇[max\(_d\)(you have d-many cookies)\(\leq\)12 – a] for a > 0
 c.   max\(_d\)(⎟[you have d-many cookies])=12

The sentential negation in (4-b) gives rise to a denial of the assertion of ⋇[max\(_d\)(you have d-many cookies)>12]. No implicatures are to be expected from this.

In sum, the paper argues that Fox and Hackl’s density thesis predicts that expressions corresponding to the \(\leq\) or \(\geq\)-relation will have strong double-bounded readings. This initially counter-intuitive prediction is shown to be correct in the case of no -er. Standard theories of comparison cannot account for the resulting strong readings.