Substitution Puzzles and Substitutional Semantics

In sentences like, for example, (1) ‘Clark Kent went into the phone booth, and Superman came out’ and (2) ‘N.N. knows that Mark Twain is a writer’ a substitution of ‘Superman’ by ‘Clark Kent’ and of ‘Mark Twain’ by ‘Samuel Clemens’, respectively, does not appear to be warranted, even though the relevant pairs of names are supposed to be co-referring. Typically such substitution puzzles are addressed from the perspective of a referential theory of predication within the framework of a denotational semantics. In my talk I’d like to present an approach to the substitution puzzles about cases like (1) and (2) within the non-referential framework of associative substitutional semantics.

The substitutional language to be employed for this purpose extends the first-order language discussed in [1] by supplementing the base language $L_0$ (of the extended language $L$ which contains the apparatus for substitutional quantification) with a predicate for substitutional identity $\equiv$. We let $Atm$ be the set of pure atomic sentences of $L_0$. Moreover, we define the sets $Atm(\alpha)$ and $Atm(\varphi^n)$ as follows: $Atm(\alpha) = \{ A \in Atm: A \text{ contains at least one occurrence of the nominal constant } \alpha \}$; and $Atm(\varphi^n) = \{ A \in Atm: A \text{ contains an occurrence of the predicate } \varphi^n \}$.

An associative substitutional model is a triple $I = \langle C, P, v \rangle$, where $C$ is a non-empty substitution class of nominal constants of $L_0$ and $P$ is the set of pure predicates of $L_0$. (Note that $P$ does not contain $\equiv$.) The assignment $v$ is defined as follows: $v : C \to \varphi(Atm)$ such that $v(\alpha) \subseteq Atm(\alpha)$ and $v : P \to \varphi(Atm)$ such that $v(\varphi^n) \subseteq Atm(\varphi^n)$. (It will be noted that, unlike in first-order denotational semantics, two distinct constants (or predicates) can never receive the same semantic values.) The truth conditions for the sentences of $L$ in a model $I = \langle C, P, v \rangle$ are given by the following clauses (here we state only the clauses for predication and for substitutional identity): 1. $I \models \varphi^n \alpha_1 \ldots \alpha_n$ iff $\varphi^n \alpha_1 \ldots \alpha_n \in v(\alpha_1) \cap \ldots \cap v(\alpha_n) \cap v(\varphi^n)$; otherwise $I \not\models \varphi^n \alpha_1 \ldots \alpha_n$. 2. $I \models \alpha_1 \equiv \alpha_2$ iff for all sentences $B_1$ and $B_2$ in $Atm$ where $B_2$ is like $B_1$ except for containing occurrences of the nominal constant $\alpha_2$ at one or all places where $B_1$ contains the constant $\alpha_1$: $I \models B_1$ if $I \models B_2$.

Associative models are admissible just in case they satisfy certain constraints imposed upon $v$. (We must omit them in this abstract.) Roughly, these constraints are governed by nominal definitions for the constants in $C$ and by meaning postulates for the predicates in $P$ as determined by the relevant piece of fiction or discourse. In effect, the semantic value of a constant contains all the information which is compatible with the nominal definition for that constant, whereas the semantic value of a predicate contains all the information associated with that predicate in the relevant piece of
fiction or discourse. We call the semantic values of nominal constants and pure predicates their sense-extensions. Constrained associative models are intended to represent the level of sense of a language (roughly, the totality of the sense-extensions of its constants and predicates) rather than its level of reference (which may be taken to be represented by denotational first-order models).

In order to obtain restricted associative models, $\mathcal{I}_a = \langle C, P, v_a \rangle$, we relativize the valuation $v$ to an agent $a$. Intuitively, $v_a(\alpha)$ represents the portion of the sense-extension of the name $\alpha$, i.e. $v(\alpha)$, that is accessible to the agent $a$ in virtue of the part of the nominal definition of $\alpha$ that is accessible to $a$. Similar remarks apply to $v_a(\varphi^n)$. Accordingly, restricted models may then be taken to represent the portion of the level of sense that is accessible to an agent.

Now, consider for case (1) the following model $\mathcal{I}_a = \langle C, P, v_a \rangle$, where $C = \{c, s\}$, $P = \{I, O\}$, $v_a(c) = \{Ic\}$, $v_a(s) = \{Os\}$, $v_a(I) = \{Ic\}$ and $v_a(O) = \{Os\}$. According to clause 1 (for restricted models) and the obvious clause for conjunction, we get $\mathcal{I}_a \models Ic \land Os$ but $\mathcal{I}_a \not\models Ic \land Oc$, $\mathcal{I}_a \not\models Is \land Os$, and $\mathcal{I}_a \not\models Is \land Oc$. We also have $\mathcal{I}_a \not\models c = s$. (In unrestricted models all sentences considered will come out true.)

To treat cases like (2) we extend the language with an operator for knowledge, $K_a$. A restricted modal model will be a tuple $\mathcal{M}_a = \langle S, R, C, c, P, v_a \rangle$, where $S$ is a non-empty set of indices $s, t, \ldots$; $R \subseteq S \times S$; $C, P$, are as before; $c : S \rightarrow \wp(C)$ with $c(s)$ being the substitution class for some $s \in S$ and $C = \bigcup_{s \in S} c(s)$; and $v_a : C \times S \rightarrow \wp(Atm)$ such that $v_a(\alpha, s) \subseteq Atm(\alpha)$ and $v_a : P \times S \rightarrow \wp(Atm)$ such that $v_a(\varphi^n, s) \subseteq Atm(\varphi^n)$. The clause for the $K_a$-operator is $\mathcal{M}_a \models_s K_a(A)$ iff for all $t \in S$, if $sRt$ then $\mathcal{M}_a \models_t A$. The other clauses are adapted in the natural way. For case (2) consider the model $\mathcal{M}_a = \langle S, R, C, c, P, v_a \rangle$, where $S = \{s, t\}$, $R = \{(s, t), (s, s), (t, t)\}$, $c(s) = c(t) = \{m, a\}$, $P = \{M, W\}$, $v_a(m, s) = v_a(m, t) = \{Mm, Wm\}$, $v_a(a, s) = \{Ma\}$, $v_a(a, t) = \{Ma, Wa\}$ $v_a(M, s) = v_a(M, t) = \{Mm, Ma\}$, $v_a(W, s) = \{Wm\}$, and $v_a(W, t) = \{Wm, Wa\}$. We have $\mathcal{M}_a \models_s K_a(Wm)$, that is the truth of (2), but $\mathcal{M}_a \not\models_s K_a(Wa)$ and $\mathcal{M}_a \not\models_s m = a$.

We take associative models and the level of sense to be relevant for the evaluation of sentences that involve non-denoting terms and/or intensional operators. And we regard denotational models as being appropriate for the referring and non-intensional portion of language. It will be noted that due to the lack of referents philosophical problems concerning fictional objects or transworld identity do not arise.