Recent developments in formal pragmatics

Part 1/3: Optimality Theory

Forum for Theoretical Linguistics, University of Oslo, September 2019
Kjell Johan Sæbø

BiOT emerged at the turn of the millennium as a fusion of Radical Pragmatics and Optimality Theoretic Semantics (Blutner 2000). (Benz and Mattausch 2011: 1)

→ Radical Pragmatics (Levinson 2000, Atlas 2005, building on earlier work): Pragmatics loops back into semantics; implicatures influence content

→ Optimality theoretic semantics: Choices of interpretations are governed by a competition among alternative candidate interpretations

Blutner (1998, 2000) extended this original version by taking also alternative forms into account that the speaker could have used, but did not. (van Rooij and Franke 2015)

What is optimal is not just interpretations with respect to forms, but rather form-interpretation pairs.

---

Figure 1: The roots of pragmasemantics

---

1 Strong optimality and scalar implicatures

Consider the query (1) and the three possible responses in (2):

(1) How often are you satisfied with the quality of the sex?
(2) a. Sometimes.
    b. Often.
    c. Always.

These three responses form an entailment scale, a so-called Horn scale.

Under a definition of optimality like (3) and the four assumptions 1.–4., the pairing of the content sometimes, not often with the form sometimes and the pairing of the content often, not always with the form often are optimal.

(3) Optimality of \(<f, c>\)

A form - content pair \(<f, c>\) is optimal iff for any \(<f', c>\) or \(<f, c'>\),

\[ P(c/[f]) \geq P(c/[f']) \quad \text{and} \quad P(c/[f]) \geq P(c'/[f]) \]

\(P(c/[f])\) is the probability of the truth of content \(c\) given \(f\)’s literal content.

<table>
<thead>
<tr>
<th>(P(x/[y]))</th>
<th>(\exists, &lt; n)</th>
<th>(&gt; n, \neg \forall)</th>
<th>(\forall)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>sometimes</strong></td>
<td>(\Rightarrow \frac{1}{3})</td>
<td>(\frac{1}{3})</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td><strong>often</strong></td>
<td>0</td>
<td>(\Rightarrow \frac{1}{2})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td><strong>always</strong></td>
<td>0</td>
<td>0</td>
<td>(\Rightarrow 1)</td>
</tr>
</tbody>
</table>

Table 1: Scalar implicatures as optimal interpretations

The non-zero non-optimal form-content pairs are blocked.

1. Stronger scalemates are relevant
2. Sender authority: common ground that S knows

---

3. No **bias**: prior probabilities are evenly distributed

4. Same **cost**: no significant differences in complexity among scalemates

The first two assumptions are preconditions for the definition (3) to apply; in a *wh* question context like (1), both are given.2

The last two assumptions can be lifted to provide more complex measures, to which we will have occasion to return in due course.


2 Case study: simple versus complex reflexives

A theory based on Bergeton (2004) and Eckardt (2001) can explain the low acceptability of *self* in (4), but not the necessity of *self* in (5):

(4) Narcissus speaker seg (# selv).
Narcissus reflects SEG (# SELF)

(5) Narcissus beundrer seg # (selv).
Narcissus admires SEG # (SELF)

The keys to explaining both are, first, the focus structures in (6) and (7), second, OT pragmatics applied to the two alternatives in (7) (Sæbø 2009):

(6) a. Narcissus [speaker seg]F.
   Narcissus mirrors SEG
b. # Narcissus [speaker]F [seg selv]F.
   Narcissus mirrors SEG SELF

(7) a. # Narcissus [beundrer seg]F.
   Narcissus admires SEG
   Narcissus mirrors SEG SELF

2Here are two cases where relevance and sender authority are not (yet) given:

(i) “Is this tequila distilled twice, as required by the Mexican government?” “Yes.”
(ii) At this point, we can already say that half of the cats found their way home.

3 M-implicatures and weak optimality

The infelicity of (6b) can be explained with its focus presupposition (8):

(8) **Focus presupposition of Narcissus speaker seg selv**

There are propositions φ such that ∃P ~ speaker and ∃y ~ Narcissus such that φ = P(y)(Narcissus), and
there are propositions ψ such that ∃Q ~ speaker and ∃z ~ Narcissus such that φ = Q(z)(Narcissus).

This would be satisfied if there were a prior probability that someone speaker someone else or if there were to be alternatives to seg selv in the discourse. That is difficult, but for beundrer the presupposition is easily satisfied.

But the infelicity of (7a) cannot be explained with its focus presupposition:

(9) **Focus presupposition of Narcissus beundrer seg**

There are propositions φ such that ∃P ~ beundrer seg such that φ = P(Narcissus).

This is easily satisfied, regardless of alternatives to seg.

But BiOT can predict that the optimal interpretation of (7a) includes the anti-presupposition that the focus presupposition of (7b) is not justified –

(10) **Focus implicature of Narcissus beundrer seg** (loosely)

There is no prior probable or salient alternative to beundrer, or there is no prior probable or salient alternative to seg.

In other words, the verb should be sufficiently predictable from the reflexive, or vice versa, which is not the case when the verb is ‘admire’.

3 M-implicatures and weak optimality

The ‘Division of Pragmatic Labor’ (Horn 2004: 16): “as a result of general pragmatic interactions, unmarked expressions are generally used to convey unmarked messages and marked expressions are generally used to convey marked messages” (Davis and Potts 2010: 42).

3.1 The simple and the stereotypical

In the world of Dostoevsky’s Besy ‘Demons’, (11) is false while (12) is true.
(11) Stavrogin confessed that he had killed Matryoshka.

(12) Stavrogin confessed that he had caused Matryoshka’s death.

Assumptions 3 (no bias) and 4 (same cost) in section 1 become relevant here:

- unmarked expressions have a lower cost than marked expressions, and
- unmarked interpretations have higher prior probabilities than marked interpretations.

The association unmarked form – unmarked content turns out to be optimal under the definition (5) if only cost is taken into account as a negative factor, but the association marked form – marked content turns out not to be:

\[ P(x/\lbrack y \rbrack) - c(y) \]

<table>
<thead>
<tr>
<th>$cause to die$</th>
<th>indirect causation</th>
<th>direct causation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.2</td>
<td>.4</td>
</tr>
</tbody>
</table>

| $kill$        | .4                 | $\Rightarrow .6$ |

Table 2: Division of pragmatic labor

So far, the prediction is that $cause to die$ is blocked tout court.

Definition (5) is replaced by these two ($$ is here the cost function):

(13) **Strong optimality of** $< f, c >$

A pair $< f, c >$ is strongly optimal iff for any $< f', c >$ or $< f', c' >$,

\[ P(c/\lbrack f \rbrack) - $$f) \geq P(c/\lbrack f' \rbrack) - $$f' \land P(c/\lbrack f' \rbrack) - $$f \geq P(c/\lbrack f' \rbrack) - $$f' \]

(14) **Weak optimality of** $< f, c >$

A pair $< f, c >$ is weakly optimal iff no $< f', c >$ or $< f', c' >$ such that

\[ P(c/\lbrack f' \rbrack) - $$f < P(c/\lbrack f \rbrack) - $$f \lor P(c/\lbrack f \rbrack) - $$f < P(c/\lbrack f' \rbrack) - $$f' \]

Then, the left upper cell in Table 2 turns out to be weakly optimal, since neither the horizontal nor the vertical competitor is strongly optimal.

3.2 The brief and the vague

**Observation** → Round number words tend to have round interpretations (Krifka 2002)

Thus (15a) can be true at the same time as (15b) is false but (15c) is true:

(15) a. The distance between Tromsø and Vadsø is 400 km.
    b. The distance between Tromsø and Vadsø is 409 km.
    c. The distance between Tromsø and Vadsø is 418 km.

This can be explained in Directional Optimality Theory if it is assumed that

- 409 and 418 are more costly expressions than 400, and
- approximate interpretations are preferred over precise ones.

Again, the pairing of the worse form with the worse content comes out as weakly optimal – it is deblocked because its two competitors are blocked:

\[
\begin{array}{|c|c|c|}
\hline
& \pm .5 & \pm 20 \\
409 & \Rightarrow .2 & .4 \\
400 & .4 & \Rightarrow .6 \\
\hline
\end{array}
\]

Table 3: Brief and vague or else elaborate and precise

Note, though, that Krifka (2007) substitutes a Game theoretic account.

4 Outlook

For two reasons, BiOT has faded into the background of pragmatic theory.

4.1 Assimilation to Game Theory

Over the first decade of the new millennium, BiOT was gradually superseded by Game Theory as the dominant framework of formal pragmatics (indeed, Dekker and van Rooij (2000) called BiOT “an application of game theory”).

→ Topic for the third installment (GT and the Rational Speech Act model)!
4.2 A challenge: embedded implicatures

Embeddings (Chierchia 2004) → A local approach is better than a global one

(16) Usually you may only take an apple. So, if you may take an apple or take a pear, you should bloody well be pleased. (Kamp 1973: 279)

Blutner’s response (2006: 11):

I will argue that both approaches can coexist in optimality theoretic pragmatics: a global theory describes the principal forces that direct communication – it has a diachronic dimension . . . ; a local theory describes the actual synchronic dimension – it explains how online, incremental interpretation of complex sentences is possible. The connection . . . results from assuming that the results of global optimization fossilize into a local mechanism . . . (my emphasis)

→ Topic for the second, next installment (The grammatical theory)!

References


Recent developments in formal pragmatics

Part 2/3: The Grammatical Theory

Forum for Theoretical Linguistics, University of Oslo, September 2019
Kjell Johan Sæbø

The grammatical theory was developed by Chierchia (2006) and Fox (2007). It attributes implicature computation to a silent grammatical operator... (Sauerland 2012)

Implied implicatures sometimes need to be computed in embedded positions; conversational implicatures can be generated by sub-parts of sentences (Fox and Spector 2018: 2; Potts 2012)

Exh is free to apply to embedded propositions (Sauerland 2012)

1 Key evidence

...the facts suggest that SIs are not pragmatic in nature but arise, instead, as a consequence of semantic or syntactic mechanisms, ... (Chierchia, Fox and Spector 2012: 2316)

The evidence in favor of embedded exhaustification comes in two main forms, both of which involve cases where an argument proposition arguably includes a scalar implicature:¹

- the proposition is the argument of a modal function, such as an attitude or a generic or conditional operation,
- the proposition is a disjunct.

¹“CFS’s argument is mainly based on intrusive implicatures: cases where the implicature seems to be incorporated into the argument to a truth-functional operator in order to maintain consistency.” (Potts 2013: 26)

1.1 Attitudes, conditionals, generics

Consider (1)–(3):

(1) “Hva synes Atle om ‘Le Bureau’?” “Han synes den er bra.”
what thinks Atle of Le Bureau he thinks it is BRA
→ Atle thinks ‘Le Bureau’ is not very good

(2) (Debate: Should marijuana be legalized?) Sure, there will always still be some around, but surely some is better than a lot?
∀ surely some, not a lot is better than a lot

(3) Don’t use up all your carrots. It recharges quicker if you’ve used some of them, than if you’ve used all of them.
∀ It recharges quicker if you’ve used some but not all of them

Indeed, (2) and (3) are arguably contradictory if some is read at face value.

1.2 Disjunctions

Consider (4) and (5):

(4) The monogenesis hypothesis posits that a single language... was ancestral to most or all of the Atlantic creoles.

(5) you need to specify the direction by a preposition or a prefix or both

Unless most is read as ‘most, not all’, and or is read as exclusive, these two cases should fall victim to Hurford’s constraint (Hurford 1974):

The joining of two sentences by or is unacceptable if one sentence entails the other; otherwise the use of or is acceptable.

But since they are acceptable, (6b)/(7b) is evidently not understood to be entailed by (6a)/(7a):

(6) a. A single language was ancestral to all of the Atlantic creoles.
b. A single language was ancestral to most of the Atlantic creoles.
(→ it was not ancestral to all of them)

(7) a. you specify the direction by a preposition and a prefix
b. you specify the direction by a preposition or a prefix
(→ you do not specify it by both)
2 The canonical theory

The reviewed evidence provides motivation for an exhaustification operator.

Under the Neo-Gricean Theory, scalar implicatures are computed on the basis of principles that regulate the choice of communicative acts, and therefore do not apply to sub-constituents of a sentence. By contrast, under the Grammatical Theory there is – within grammar – an implicature-computing operator, and, if no . . . , there should be no ban on embedding this operator . . . (Fox and Spector 2018: 2)

Here is the canonical definition of the exhaustification operator:

(8) Exhaustification operator Exh

\[ \text{Exh}_{\text{Alt}}(p) \equiv p \land \forall q \in \text{Alt} : [p \Rightarrow q] \Rightarrow \neg q \]

Here Alt is the set of alternatives to the sentence p.  

The distribution of Exh is constrained by an Economy Condition, informally:

(9) Economy Condition on Exh

An occurrence of Exh in a sentence S is not licensed if eliminating this occurrence leads to a sentence S' which entails or is equivalent to S. *An occurrence of Exh is licensed only if it leads to strengthening.*

(Nicolae 2017: 7)

– In other words, Exh should not be semantically vacuous. Consequently, it should be illicit in the scope of a scale-reversing operator unless this operator is itself embedded under a non-upward-entailing operator (Crnić 2012: 548).

\[ \Rightarrow \text{(So GT is not all grammatical and local but partly pragmatic and global (Potts 2013))} \]

(10) and (11) are cases in point:

(10) I do not often agree with John Kenneth Galbraith, but . . .
\[ \neg \Rightarrow \text{it is not the case that I often, not always agree with him} \]

(11) Hver gang jeg dater en mann som er litt alrøt blir jeg . . .
\[ \neg \Rightarrow \text{every time I date a man that is bit alright become I . . .} \]
\[ \neg \Rightarrow \text{every time I date a man who is only just better than average,} \]

Strengthening under negation or in \( \forall \) restrictors leads to overall weakening.

Another context where Exh should not be licensed is when a sentence with a low scalar item is conjoined with a sentence with the negation of a higher scalar item, as in (12):

(12) Most lay eggs, but not all do.
\[ \neg \Rightarrow \text{most but not all lay eggs but not all do – or perhaps better:} \]
\[ \# \text{Exh(most lay eggs) but not all do} \]

As conceded by Chierchia, Fox and Spector (2012: 2317),

there is nothing that forces the presence of the operator O in a sentence containing a scalar item. Optionality is thus predicted, and the correlation with various contextual considerations can be captured under the standard assumption…that such considerations enter into the choice between competing representations (those that contain the operator and those that do not).

However, there are still problems, or there is still a need for refinements.

3 Problems or refinements

The grammatical theory faces challenges, some to do with overgeneration, calling for a stronger Economy Condition, some to do with undergeneration, calling for a weaker condition. Also, some key evidence has been challenged.

3.1 Singh’s asymmetry and incrementality

Singh (2008) observed that although a case like (13a) gets around Hurford’s constraint (see (4) and (5)), it becomes infelicitous if the order is reversed.²

(13) a. . . that means either 14 of the 15 Lauryn songs got at least one ten . . . or all of them did.

b. # . . . that means either all of the 15 Lauryn songs got at least one ten . . . or 14 of them did.

²Actually, the contrast could be less clear here than in Singh’s example (i)/(ii):

(i) John ate some of the cookies or he ate all of them

(ii) # John ate all of the cookies or he ate some of them
Here is an example where the b version is authentic:\(^3\)

\[(14)\]
\begin{enumerate}
\item a. … and Robert or John or both of them lifted William’s hand out of bed and guided the same to the will and...
\item b. #… and Robert and John or one of them lifted William’s hand out of bed and guided the same to the will and...
\end{enumerate}

Singh proposes to account for this contrast by imposing an asymmetric redundancy constraint on disjunctions, but Fox and Spector (2018) propose to account for it by an incremental economy condition on \(Exh\), (15).

\[(15)\]
**Economy Condition on \(exh\)**

An occurrence of \(exh\) in a sentence \(S\) is not licensed if this occurrence … is incrementally vacuous in \(S\).

An occurrence of \(exh\) which takes \(A\) as argument is *incrementally* vacuous in a sentence \(S\) if it is globally vacuous for every continuation of \(S\) at point \(A\).

An occurrence of \(exh\) is *globally* vacuous in a sentence \(S\) if eliminating it does not change truth conditions.

Note that the ‘good’ Hurford disjunctions, like \(\ldots some\ldots or all\ldots\), in fact violate (Hurford’s constraint or) (9), \(exh(some\ldots)\) being globally vacuous, but not (15), while the ‘bad’ ones, like \(\ldots all\ldots or some\ldots\), do violate (15), because at point \(A = some\ldots\), the truth conditions for \(S\) are the same with as without \(exh(\cdot)\) no matter how \(S\) continues.

This way to account for Singh’s asymmetry makes the correct prediction that when the two disjuncts involve non-adjacent scale items (‘Distant Entailing Disjuncts’, Fox and Spector (2018); ‘Non-convex disjunctions’, Bergen, Levy and Goodman (2016)), the constraint loosens:

\[(16)\] The rest of the cast… sounds either super fantastic or okay.

\(^3\)Hannah Barker, *Family and Business during the Industrial Revolution*, Oxford 2017 (www.oapen.org/download/?type=document&docid=1001049); Singh’s example is:

(iii) (John or Mary) or both came to the party

(iv) #(John and Mary) or (John or Mary) came to the party

Indeed, *all or some* does occur (about \(\frac{1}{8}\) as often as the other way around), and arguably non-adjacency is one reason.

### 3.2 Contrast focus and exhaustification under negation

Recall that \(Exh\) is ruled out directly under negation, as long as this is not in turn in a downward entailing context:

\[(17)\]
\begin{enumerate}
\item a. #The Kannada word *mara* does not mean ‘tree’ or ‘wood’.
    It means both.
\item b. The Kannada word *mara* does not mean ‘tree’ or ‘wood’.
    It means both.
\end{enumerate}

Unless the scalar item has narrow, contrastive focus:\(^4\)

\[(18)\] The Implicature Focus Generalization

Implicatures can be embedded under a downward entailing (DE) operator only if the (relevant) scalar term bears pitch accent.

For and Spector set out to show how this generalization can be made to follow from a refined version of their economy condition – the idea is that embedded implicatures are in principle possible in every context, but require a pitch accent on the relevant scalar item in DE contexts because the narrow focus serves to restrict the set of alternatives \(Alt\) for \(Exh\) as a focus sensitive operator.

This seems a good idea, but the details are complicated.

Interestingly, even positive polarity items are licensed in such contexts:

\[(19)\] we won’t be together *some* of the time, we’ll be together *constantly*!

As it appears, a PPI can be ‘shielded’ by \(Exh\) (Szabolcsi 2004).

\(^4\)Actually, the contrast could be less clear here than in the Fox and Spector example:

(i) #John didn’t do the reading or the homework. He did both.

(ii) John didn’t do the reading OR the homework. He did both.
3.3 Challenges to key evidence

Potts (2013) argues that the empirical foundation for Hurford’s constraint is not firm. He has collected 161 counterexamples here: http://goo.gl/VAGqnB. Let me add a particular class: scalar items modified by \textit{at least}.

\begin{equation}
\text{(20)}\quad \text{For example, all new cars launched by Volvo from 2019 will be at least partially or completely battery-powered, . . .}
\end{equation}

Potts concludes, in fact, that HC does not exist and hence cannot furnish an argument for embedded implicatures.

Russell (2006) offers counterarguments to data like (1)-(3) being evidence for a grammatical theory. See also Simons (2010).

One other source of concern is data like the following:

\begin{equation}
\text{(21)}\quad \text{This is what everybody wants, but \#(only) some people manage to do.}
\end{equation}

\begin{equation}
\text{(22)}\quad \text{I think the ‘Golden Ratio’ is something that you always strive for but \#(only) hit some of the time.}
\end{equation}

The fact that \textit{only} is necessary in these contexts is a bit mysterious if \textsc{Exh} applies freely and has the effect of adding ‘only’. Is the addition not-at-issue?

3.4 Outlook

Recent work in Game theoretic pragmatics – e.g., Potts, Lassiter, Levy and Frank (2016) – aims at accounting for embedded exhaustification without an embedded exhaustifier, in a not solely grammatical theory that incorporates neo-Gricean hypotheses about lexical alternatives. This work thus contributes to a synthesis of grammatical and probabilistic views on pragmatic inference.

\rightarrow \text{Next installment!}

References

Recent developments in formal pragmatics

Part 3/3: Game Theory

Forum for Theoretical Linguistics, University of Oslo, September 2019
Kjell Johan Sæbø

Note two facts about · Bidirectional Optimality Theory on the one hand and · the Grammatical theory on the other:

- In BiOT, something is going on behind the scenes; what one sees is a stable state in the addressee’s perspective, and it does not really show how implicatures can be amplified or dampened when the agents reason about each other (Potts 2013).

- In GT, only Quantity implicatures, scalar implicatures in a wide sense, can be derived – it offers no way to derive, say, markedness implicatures. Besides, it has no systematic way to model the sensitivity of implicatures to factors like message cost and a state’s prior probability.

A theory of signaling games, whether in terms of iterated best response or lexical uncertainty, is more explicit and more comprehensive.

1 Interlocutors as functions from each other to functions from <message, state> pairs to reals

Let us go through the paradigm scalar implicature case from the first, BiOT installment, to see how a Game theoretic model makes a difference, where production and interpretation are modeled as a recursive process where the listener and speaker reason about each other reasoning about each other. (Potts 2013)

Let us begin with very simple definitions of the initial listener, the speaker and the listener, adapted from Potts (2013):¹

(1) Initial listener

\[ L_0(<m, \sigma>) = \frac{I(\sigma \in [m])}{|\{m\}|} \sum_{\sigma'} I(\sigma \in [m]) \]

(2) Speaker

\[ S(L)(<m, \sigma>) = \frac{S(<m, \sigma>)}{\sum_{m'} S(<m', \sigma>)} \]

(3) Listener

\[ L(S)(<m, \sigma>) = \frac{L(<m, \sigma>)}{\sum_{\sigma'} L(<m, \sigma'>)} \]

Now consider the recursive application of \( L \) to \( S \) to \( L \) to \( S \) to \( L_0 \):

<table>
<thead>
<tr>
<th>( L_0(&lt;y, x&gt;) )</th>
<th>( \exists, &lt;n )</th>
<th>( &gt; n, \neg \forall )</th>
<th>( \forall )</th>
</tr>
</thead>
<tbody>
<tr>
<td>sometimes</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>often</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>always</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: 3 messages, 3 states, initial listener at a loss

<table>
<thead>
<tr>
<th>( S(L_0)(&lt;y, x&gt;) )</th>
<th>( \exists, &lt;n )</th>
<th>( &gt; n, \neg \forall )</th>
<th>( \forall )</th>
</tr>
</thead>
<tbody>
<tr>
<td>sometimes</td>
<td>1</td>
<td>( \frac{2}{3} )</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>often</td>
<td>0</td>
<td>( \frac{2}{3} )</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>always</td>
<td>0</td>
<td>0</td>
<td>( \frac{6}{11} )</td>
</tr>
</tbody>
</table>

Table 2: 3 messages, 3 states, \( S(L_0) \)

¹Here we assume a flat prior over states and cost-free messages; \( I \) is a valuation function returning 1 if the argument is true, 0 otherwise; \( \sigma' \) ranges over the alternative states.
Recent developments in formal pragmatics 3

Game Theory

2 Varying costs and priors: two cases

Let us look at two difficult cases where adding costs and priors helps.

2.1 Q-implicatures and the symmetry problem

According to Bergen, Levy and Goodman (2016), the symmetry problem is “a problem with constructing the scales for the implicature computations: there are multiple consistent ways of constructing the scales, and different scales will give rise to different implicatures.”

Because the only formal requirement on a scale is that items higher on it be logically stronger than those lower on it, a possible scale for *some* is 

<“some”, “some but not all”>. If this scale is used, “some” will imply that “some but not all” is not true, i.e., that “all” is true.

These authors break this symmetry by assigning *some but not all* a much greater cost than *some*. Formally, we need to introduce the ‘initial speaker’ and the cost function \( C \), ranging over \([0, 1]\), in the definition of the speaker:

\[
S_0(<m, \sigma>) = \frac{(\mathcal{I}(\sigma \in [m]) \cap m : \sigma \in [m]) C(m)}{\sum_{m'}(\mathcal{I}(\sigma \in [m']) \cap m : \sigma \in [m']) C(m')}
\]

(4) Initial speaker

\[
S(<m, \sigma>) = \frac{L(<m, \sigma>) C(m)}{\sum_{m'} L(<m', \sigma>) C(m')}
\]

(5) Speaker

The listener’s confidence in the implicature grows logarithmically but does not reach certainty. It can, however, be affected, positively or negatively, by

- differing costs attached to the messages,
- differing prior probabilities attached to the states.

Table 3: 3 messages, 3 states, \(L(S(L_0))\)

<table>
<thead>
<tr>
<th>(L(S(L_0))(&lt;y,x&gt;))</th>
<th>(\exists, &lt;n)</th>
<th>(&gt; n, \neg \forall)</th>
<th>(\forall)</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>sometimes</em></td>
<td>(\frac{55}{87})</td>
<td>(\frac{22}{87})</td>
<td>(\frac{10}{87})</td>
</tr>
<tr>
<td><em>often</em></td>
<td>0</td>
<td>(\frac{11}{15})</td>
<td>(\frac{5}{16})</td>
</tr>
<tr>
<td><em>always</em></td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4: 3 messages, 3 states, \(S(L(S(L_0)))\)

<table>
<thead>
<tr>
<th>(S(L(S(L_0)))(&lt;y,x&gt;))</th>
<th>(\exists, &lt;n)</th>
<th>(&gt; n, \neg \forall)</th>
<th>(\forall)</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>sometimes</em></td>
<td>1</td>
<td>(\frac{352}{1309})</td>
<td>(\frac{160}{1987})</td>
</tr>
<tr>
<td><em>often</em></td>
<td>0</td>
<td>(\frac{957}{1309})</td>
<td>(\frac{435}{1987})</td>
</tr>
<tr>
<td><em>always</em></td>
<td>0</td>
<td>0</td>
<td>(\frac{800}{1987})</td>
</tr>
</tbody>
</table>

Table 5: 3 messages, 3 states, \(L(S(L(S(L_0))))\)

<table>
<thead>
<tr>
<th>(L(S(L(S(L_0))))(&lt;y,x&gt;))</th>
<th>(\exists, &lt;n)</th>
<th>(&gt; n, \neg \forall)</th>
<th>(\forall)</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>sometimes</em></td>
<td>(\frac{2600683}{5953237})</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td><em>often</em></td>
<td>0</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td><em>always</em></td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6: \(S_0, C(\text{some}) = C(\text{all}) = 1, C(\text{somenotall}) = 0.5\)
Consider now the recursive application of $L$ to $S$ to $L$ to $S_0$: 

<table>
<thead>
<tr>
<th>$L(S_0)(&lt;y,x&gt;)$</th>
<th>$\exists, \neg \forall$</th>
<th>$\forall$</th>
</tr>
</thead>
<tbody>
<tr>
<td>some</td>
<td>$\frac{1}{2}$ ($\frac{1}{2}$)</td>
<td>$\frac{2}{3}$ ($\frac{1}{2}$)</td>
</tr>
<tr>
<td>some, not all</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>all</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 7: $L(S_0), C(some) = C(all) = 1, C(somenotall) = .5 (0)$

<table>
<thead>
<tr>
<th>$S(L(S_0))(&lt;y,x&gt;)$</th>
<th>$\exists, \neg \forall$</th>
<th>$\forall$</th>
</tr>
</thead>
<tbody>
<tr>
<td>some</td>
<td>$\frac{8}{11}$ ($\frac{1}{2}$)</td>
<td>$\frac{3}{11}$ ($\frac{1}{2}$)</td>
</tr>
<tr>
<td>some, not all</td>
<td>$\frac{3}{11}$ ($\frac{2}{3}$)</td>
<td>0</td>
</tr>
<tr>
<td>all</td>
<td>0</td>
<td>$\frac{2}{11}$ ($\frac{3}{7}$)</td>
</tr>
</tbody>
</table>

Table 8: $S(L(S_0)), C(some) = C(all) = 1, C(somenotall) = .5 (0)$

<table>
<thead>
<tr>
<th>$L(S(L(S_0)))(&lt;y,x&gt;)$</th>
<th>$\exists, \neg \forall$</th>
<th>$\forall$</th>
</tr>
</thead>
<tbody>
<tr>
<td>some</td>
<td>$\frac{80}{111}$ ($\frac{1}{2}$)</td>
<td>$\frac{33}{111}$ ($\frac{1}{2}$)</td>
</tr>
<tr>
<td>some, not all</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>all</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 9: $L(S(L(S_0))), C(some) = C(all) = 1, C(somenotall) = .5 (0)$

We see that as long as the signalling cost of the message *some, not all* is not taken into account, the listener is just as likely to interpret the message *some* as ‘all’ as ‘some but not all’, but as soon as a .5 factor cost is calculated, *some* is more and more likely to be interpreted as ‘some but not all’ again.

2.2 I-implicatures and the tension with Quantity

In the framework of the **Rational Speech Act** model (Frank and Goodman 2012, Bergen, Levy and Goodman 2016), Poppels and Levy (2015) address how to balance the pressure to strengthen an expression to exclude

- a stronger alternative (Q-based implicatures)
- and the pressure to strengthen an expression to exclude
- atypical cases (R-based or Informativeness implicatures).

Case in point: the indefinite article can be strengthened in two ways:

- to something possessive or
- to something anti-possessive.

(6) lends itself to a Q-implicature while (7) lends itself to an R-implicature:

(6) He was in a bed. $\sim$ not his own bed
(7) I broke a toe yesterday. $\sim$ one of my own toes

In both cases, a competes with the same-cost, stronger alternative his/my – but in (7), this is counterbalanced by the own’s higher prior probability. Let us introduce the ‘initial listener’ $L_0$ and the prior probability function $P$, ranging over $[0,1]$, in the definition of $L$:

| $L_0(<m,\sigma>) = \frac{(I(\sigma \in [m]) / |m : \sigma \in [m]|) P(\sigma)}{\sum_{\sigma'}(I(\sigma' \in [m]) / |m : \sigma \in [m]|) P(\sigma')}$ |

(8) Initial listener

(9) Listener

$L(S)(<m,\sigma>) = \frac{S(<m,\sigma>) P(\sigma)}{\sum_{\sigma'} S(<m,\sigma'>) P(\sigma')}$

Assume that $P(other’s) = .2$ and $P(own) = .8$ if you break a toe, and consider the recursive application of $L$ to $S$ to $L_0$ on the next page.

Note that $<a toe, other’s>$ approaches .5 listener confidence, more and more slowly. It never reaches implicature level.
Recent developments in formal pragmatics 3

Game Theory

3 Lexical uncertainty

Scientists like Potts et al. (2016) and Bergen et al. (2016) assume that word meanings are not fixed across speakers and contexts: discourse participants do not share one lexicon but consider many lexica and synthesize them.

The guiding idea is that, in interaction, pragmatic agents reason about possible refinements of their lexical items, with the base lexical meaning serving as a kind of anchor to which each word’s interpretation is loosely tethered. (Potts et al. 2016)

The lexical uncertainty version of the rational speech model is essential for deriving M-implicatures (marked expression ↔ marked interpretation).

Slightly simplified, this is how:

1. The listener is uncertain what the cost-free form $m_1$ and the costly form $m_2$ mean, the likely $\sigma_1$ or the unlikely $\sigma_2$. She considers nine lexica:

<table>
<thead>
<tr>
<th>$L_0(&lt;y,x&gt;)$</th>
<th>OTHER’S</th>
<th>OWN</th>
</tr>
</thead>
<tbody>
<tr>
<td>a toe</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{4}{5}$</td>
</tr>
<tr>
<td>my toe</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 10: $L_0$, $P($other's$) = .2$ and $P($own$) = .8

<table>
<thead>
<tr>
<th>$S(L_0)(&lt;y,x&gt;)$</th>
<th>OTHER’S</th>
<th>OWN</th>
</tr>
</thead>
<tbody>
<tr>
<td>a toe</td>
<td>1</td>
<td>$\frac{4}{5}$</td>
</tr>
<tr>
<td>my toe</td>
<td>0</td>
<td>$\frac{5}{5}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$S(L(S(L_0)))(&lt;y,x&gt;)$</th>
<th>OTHER’S</th>
<th>OWN</th>
</tr>
</thead>
<tbody>
<tr>
<td>a toe</td>
<td>$\frac{9}{25}$</td>
<td>$\frac{16}{25}$</td>
</tr>
<tr>
<td>my toe</td>
<td>0</td>
<td>$\frac{25}{47}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L(S(L(S(L_0))))(y,x&gt;)$</th>
<th>OTHER’S</th>
<th>OWN</th>
</tr>
</thead>
<tbody>
<tr>
<td>a toe</td>
<td>$\frac{41}{105}$</td>
<td>$\frac{64}{105}$</td>
</tr>
<tr>
<td>my toe</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 11: $L(S(L(S(L_0))))$, $P($other's$) = .2$ and $P($own$) = .8

Then she calculates the utility values for each pairing over all these lexica, taking the prior probabilities into account, say, .8 for $\sigma_1$ and .2 for $\sigma_2$:
Subsequently, the speaker calculates the utility values over all these values, taking the costs into account, say, the factor $f$ for $m_2$, and in a next step, the listener averages over these values to compute her utility values:

<table>
<thead>
<tr>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>$\frac{4}{5}$</td>
<td>$\frac{1}{5}$</td>
<td>$m_1$</td>
<td>$\frac{4}{5}$</td>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td>$m_2$</td>
<td>$\frac{4}{5}$</td>
<td>$\frac{1}{5}$</td>
<td>$m_2$</td>
<td>$1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>$\sigma_2$</td>
<td>$\sigma_1$</td>
<td>$\sigma_2$</td>
<td>$\sigma_1$</td>
<td>$\sigma_2$</td>
</tr>
<tr>
<td>$m_1$</td>
<td>$\frac{4}{5}$</td>
<td>$\frac{1}{5}$</td>
<td>$m_1$</td>
<td>$0$</td>
<td>$1$</td>
</tr>
<tr>
<td>$m_2$</td>
<td>$0$</td>
<td>$1$</td>
<td>$m_2$</td>
<td>$\frac{4}{5}$</td>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>$\sigma_2$</td>
<td>$\sigma_1$</td>
<td>$\sigma_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_1$</td>
<td>$0$</td>
<td>$1$</td>
<td>$m_1$</td>
<td>$0$</td>
<td>$1$</td>
</tr>
<tr>
<td>$m_2$</td>
<td>$0$</td>
<td>$1$</td>
<td>$m_2$</td>
<td>$1$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

There is a certain association between costly form and unlikely content here, and this association can be strengthened by

- more response iterations,
- increasing the degree of “greedy rationality”, encoded in parameter $\lambda$,
- including the “null utterance” true in all states and very costly.

It is an open question, though, whether this is more explanatory than the notion of weak optimality introduced by Blutner (2000).

References


