Branching Possibility and the Potentialist Translation

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Oslo Potentialism Workshop, 23/09/2020

Topic

I aim to raise some points for discussion regarding the idea of branching possibilities in mathematics and the potentialist translation.

Potentialist System

Following Hamkins and Linnebo (2019), we let a *potentialist system* be a collection $\mathcal{W}$ of structures, or worlds, in a common language $\mathcal{L}$ with a reflexive and transitive accessibility relation which is inflationary in the domains of the worlds.

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The axiomatic system S4 is valid at every world in any potentialist system. Similarly for the converse Barcan formula.

Varieties of Potentialism

If $\mathcal{W}$ is directed, S4.2 is valid:

.2 $\Diamond\Box\phi \rightarrow \Box\Diamond\phi$.

If $\mathcal{W}$ is linearly ordered, S4.3 is valid:

.3 $(\Diamond\phi \wedge \Diamond\psi) \rightarrow \Diamond(\phi \wedge \Diamond\psi) \vee \Diamond(\psi \wedge \Diamond\phi)$

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If $\mathcal{W}$ is linearly ordered, S4.3 is valid:

.3 $(\Diamond\phi \land \Diamond\psi) \to \Diamond(\phi \land \Diamond\psi) \lor \Diamond(\psi \land \Diamond\phi)$

Hamkins and Linnebo (2019) show how to study the exact validities of various potentialist systems. If the modal validities of $\mathcal{W}$ are exactly S4.3, then the possibilities of $\mathcal{W}$ are characterised by a *linear inevitability*. If it is S4.2, we have *directed convergence*. And if it is S4, we have *branching possibility*.

Convergence versus branching

In the work of Linnebo (2013), Hamkins and Linnebo (2019) and Brauer (2020), it has become clear that the choice between a convergent versus branching form of potentialism is quite substantial.

In particular, the issue is whether the so-called potentialist translation works so that we can prove what Linnebo (2013) calls the mirroring theorem or what Hamkins and Linnebo (2019) calls Theorem 1. In short, below S4.2 those theorems break down.

The Potentialist Translation

The idea behind the potentialist translation is as follows: for every ϕ in $\mathcal{L}$ we obtain the potentialist translation $\phi^{\Diamond}$ by replacing $\exists x$ with $\Diamond\exists x$ and $\forall x$ with $\Box\forall x$.

Hamkins and Linnebo (2019) prove the following:

Theorem (1.)

If $\mathcal{W}$ provides a potentialist account of a structure M, then truth in M is equivalent to potentialist truth at the worlds of $\mathcal{W}$. Namely, for any $\mathcal{L}$-formula ϕ and any $a_0, ..., a_n \in M$, we have:

$$M \models \phi(a_0, ..., a_n) \leftrightarrow W \models_{\mathcal{W}} \phi^{\Diamond}(a_0, ..., a_n),$$

for any $W \in \mathcal{W}$ in which the individuals $a_0, ..., a_n$ exist.

Linnebo (2013) proves a related mirroring theorem in terms of provability.

As we remarked, we need at least S4.2 to prove this theorem. If we move to S4, the theorem breaks down.

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If one thought that potentialists are in need of Theorem 1, this would mean that branching potentialism is disqualified from the outset.

The first point I would like to raise for discussion is what assumptions underlie the need for Theorem 1. and whether there are other approaches to potentialism where it is not needed.

The Purpose of Mirroring

One of the main ideas behind Theorem 1. is that it allows the potentialist to be non-revisionary with respect to ordinary, non-modal mathematics. The translation allows the actualist and the potentialist to agree on what is true in some sense.

Furthermore, it vindicates the idea that although ordinary mathematics is not explicitly modal, it can be seen as implicitly so via the translation and Theorem 1.

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Furthermore, it vindicates the idea that although ordinary mathematics is not explicitly modal, it can be seen as implicitly so via the translation and Theorem 1.

This makes sense to the potentialist who sees the modal resources as a way of explicating more clearly the metaphysics of the structures studied in ordinary mathematical discourse, without invoking any substantial mathematical disagreement.

Branching potentialism

But could there be an alternative approach to potentialism? One where the translation and Theorem 1 is not needed?

My main reason for rasing these questions is to understand the prospects for a substantial branching potentialism. In particular in relation to set theory.

Giving up the translation

For example, so-called multiverse views in set theory are often described as being about a vast variety of different set-theoretic *possibilities*. So it would be natural to try characterising the multiverse in a potentialist setting.

However, on a radical enough view one would like to allow branching in the multiverse. So Theorem 1. would not apply in such a case. I would like to suggest that this is as expected.

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As the validities of a radical multiverse will not agree with the actualist on truth (in V), it seems that the multiversist potentialist would not want to translate back to the non-modal language for the purpose of characterising the multiverse. For such a purpose, it might perfectly allowed to work simply in the modal language itself. In fact, it seems that the radical multiversist/potentialist would *want* Theorem 1 to fail.

Implicit actualism?

The second point I would like to raise for discussion, is the charge from Hamkins (2018) that potentialist systems for which Theorem 1. hold are implicitly actualist.

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for any $W \in \mathcal{W}$ in which the individuals $a_0, ..., a_n$ exist.

Of course, the potentialist does not have the structure M in their ontology, much as the actualist does not have various worlds and the accessibility relation in theirs, but the charge is that Theorem 1. shows that M has at least an implicit existence to the relevant potentialist.

Implicit/Explicit

The charge might be mitigated by pointing out that Theorem 1. goes both ways. So, if the potentialist is implicitly actualist, then the actualist is implicitly potentialist.

According to this picture, it seems neither the potentialist nor the actualist is privileged, at least from the perspective of Theorem 1, in providing an understanding of some piece of mathematics, as the one view is implicit in the other.

What would be, if anything, at stake in the debate between actualists and potentialists of this sort?