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ARE OTHER LOGICS POSSIBLE? MACCOLL'S LOGIC AND SOME ENGLISH REACTIONS, 1905–1912

By the mid-1900s Hugh MacColl had published enough in the way of papers and notes on logic that a more connected presentation could be made. Accordingly, he reworked several items into a book published in 1906 and entitled *Symbolic Logic and its Applications*. In this paper I comment on some main features of the book and note reactions to it at the time, especially by Russell. Consequences of his ignorance of contemporary literature are also considered. The account begins with his little-known book on algebra.

1. Prologue: MacColl's Algebraic Background

In his professional life MacColl worked as a school-teacher in Boulogne-sur-Mer in France. This work led to his first book, 100 pages of *Algebraic Exercises and Problems, With Elliptical Solutions* by "Hugh McColl, *Late Mathematical Master at the Collège Communal*" in the town (MacColl 1870). The costs were seemingly borne by the author; the publisher was the London house of Longmans, Green, who gave it publicity in the 31 August 1870 issue of their trade journal *Notes on books*, with a short text which presumably he wrote himself (Appendix A).[1] A ponderous sub-title explained that the exercises were "Framed so as to combine constant practice in the simple reasoning usually required in the solution of problems, with constant practice in the elementary rules ... and the mechanical operations of algebra generally". The word "elliptical" in his title did not signal a return to geometry, but described his model answers for several questions, which the boys were supposed to copy out and then fill in the ellipsis dots with suitable intermediate calculations. Of the exercises themselves,

[1] A copy of *Notes on books*, and MacColl's correspondence, is held in the University of Reading Library Archives, Longmans and Chatto and Windus Archives (hereafter cited as "RULC").

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around 250, many were applied, though often notionally so; for example, given that someone is now $ax/(a-x)$ years old, how old was he $2ax^2/(a^2-x^2)$ years ago? An appendix contained a few simple methods, on one uniform plan, for "resolving algebraical equations" (more sub-title) by methods of indeterminate coefficients.

Given MacColl's later concern with logic, which may have already been active, the link of algebra with algorithm, and thus with logic via "reasoning", is noteworthy, especially as he published at the time when *geometry* was in a state of educational ferment in his home country, with the foundation of the Association for the Improvement of Geometrical Teaching in 1871 (Price 1994, ch. 2). But his book was well enough received for a "new edition" to appear in 1877, with the same text but a much shorter sub-title; the preface was also slightly changed. The mastership was no longer mentioned; instead, MacColl recorded his "B.A. London University".[2] The pages were apparently made up from quires stored after the original printing in 1870, because the publisher's ledger records sale under that form only: 1,000 copies had then been printed, and in 1881, when records cease, 488 copies remained. By 1886 there were still 457 unbound copies left, and it was proposed that MacColl should take back all but 12 of them; he agreed on 29 August, though grudgingly, and presumably the book then disappeared.[3]

By the time of its second edition, MacColl was in his 40th year, in a leisurely and unambitious academic career; but some main ideas in logic had come to him, as described elsewhere in this volume. I turn now to his second phase, during the 1900s.

2. On MacColl's Kinds of Proposition

Like his *Exercises*, MacColl's *Symbolic logic and its applications* was published on commission for the author by Longmans, Green, in a print-run of 1,000 copies (MacColl 1906); in the first two years, fewer

[2]This self-description on the new title page was not correct. His degree was awarded by the University of London (UoL); "London University" was the name of the private company launched in the mid-1820s, and renamed "University College London" in 1836 when the UoL was established as a degree-examining and -awarding body. The distinction between internal and external students came with the Parliamentary UoL Act of 1898; it came up in 1901 in MacColl's correspondence with Russell, when he misunderstood another aspect of the reforms (MacColl 1901a, p. 4; 1901c, pp. 13–14).

[3]Information taken from RULC, Longmans Commission Ledgers 15, 16 and 19 (ledger 19 containing MacColl's letter).

than 200 copies were sold.[4] *Notes on books* again brought publicity, in March (see Appendix B); as stated there and in the Preface (dated August 1905), the text was an assemblage of many of his articles over the decade and even a few bits from earlier ones, all shaped into a coherent text. In the first of its two parts, MacColl laid out (in 105 pages) the principles of his logic, followed by a catalogue of logical operations. Then he treated various standard topics, such as the valid syllogistic modes (where he had some cavils with traditional readings), enthymemes and inference; he also solved some unidentified "recent examination questions". The last chapter began the more mathematical concerns, such as mathematical induction and definitions of infinitude. Some aspects of this part are considered here and in the next two sections; the last one will also take the second part of the book, in which he handled a "Calculus of limits".

The account of the system was both prosodic and algebraic, and the chosen symbolism hinders understanding of the words. Notable from his first phase of the late 1870s for putting forward the proposition rather than the class or the terms as the basic logical unit, he symbolised the form of subject A and predicate B as "A^B"; but he used the same symbol structure to notate the kind of proposition and the corresponding 'attribute', such as 'certainty' with "ϵ" and 'is *certain*' with "${}^{\epsilon}$". He wrote not only "C^{ϵ}" as 'proposition C is certain', but also "ϵ^{ϵ}" for 'a certainty is certain'. He is often regarded as using "ϵ" ambiguously; but I see *a clear distinction*, though somewhat spoilt by using the same letter in two different symbols. It seems to be based upon the unstated principle $C^{\sharp} = (C = \sharp)$, where "$\sharp$" and "${}^{\sharp}$" each runs through its quintet of cases and (his) symbol "=" denotes the equivalence between two propositions. One signal consequence was that he could define each logical connective for propositions (pp. 7–9). But he could have presented the distinction more clearly; the attributes for the main five kinds were given on pp. 6–7, but only three of the kinds themselves were given (on p. 9), after he had stated a string of symbolised propositions.

More doubt surrounds the kinds 'certainty' and its opposite, 'impossibility'. They are associated with the respective probability values 1 and 0 for propositions of these kinds (p. 7). However, and ironically in a logician who had developed his theory from a basis in probability theory, only implication holds here: if C is certain or impossible, then its probability value is 1 or 0, but not necessarily vice versa.

[4]RULC, Longmans Commission Ledger 22. Appendix B comes also from this source.

The kind ‘variables’, symbolised “θ”, is also conceptually unclear. It corresponds naturally to the attribute “is *variable*”, occupying the middle ground between certainty and impossibility, as the kind ‘possible but uncertain’, with some associated probability value within $(0, 1)$ (pp. 6–7).[5] As always with his propositions, the kind has to be understood in terms of the form of words rather than of its reference; for example (his own), ‘Mrs Brown is not at home’ is a variable according to her status at the (implicit) time of consideration: “To say that [it] is a *different proposition* when it is *false* from what it is when it is *true*, is like saying that Mrs. Brown is a *different person* when she is in, from what she is when she is *out*” (p. 19). He did not consider the case of a proposition which would be orthodoxly true or false, but maybe of unknown value (for example, “Mrs Brown was born in this house”). Otherwise, his position is quite clear; but it clashed with the prevailing philosophy of propositions.

In his review of the book in *Mind*, which in several respects was quite positive, Russell (1907) picked on this feature, regarding the kind as specifying a propositional function; for him a proposition was always true or false, although we may not know which one. In October 1905 he gave a lecture to the Oxford Philosophical Club on “necessity and possibility” (Russell 1905); it is possible that his recent correspondence with MacColl, quite intense that year, had motivated the study. At all events he considered various senses of modalities, including appraisal, as logical or epistemological. He appraised uses in various current figures: Meinong (psychological), Bradley (confusing the necessity of a proposition from that of implication), Bosanquet (confusing hypothesis with disjunction), Moore (failed effort to establish logical priority among propositions) and MacColl (confusing proposition with propositional function). Ironically, his conclusion was illogical:

> I conclude that, so far as it appears, there is not one fundamental logical notion of necessity, nor consequently of possibility. If this conclusion is valid, the subject of modality ought to be banished from logic, since propositions are simply true or false, and there is no such comparative and superlative of truth as is implied by the notions of contingency and necessity.

However, just because there is no definitive version of the theory, why should one abandon all of them? At that time the status of the axioms of choice was similarly unsettled, but nobody wished to abandon the entire concern. Again, to announce bivalency categorically begged

[5] Sadly, MacColl stipulated that these probability values could only be rational numbers. This restricted the scope of this theory in ways of which he may not have been aware.

the question which MacColl wished to address. Finally, the association of certainty and impossibility with comparative and superlative truth-values does not characterise MacColl's position. In a final irony, from the late 1910s Russell was to adopt MacColl's sense of necessity and possibility, though explicitly attributed to propositional functions (Russell 1919, p. 163). However, it never became a main part of either his logic or of his epistemology.[6]

3. MacColl on Realities and "Null Classes"

Since MacColl considered predicates, he had to handle collections of objects which satisfied them, and he made a distinction between individuals with "a *real existence*" and those without it. Construing classes in the usual part-whole sense, he considered "pure" classes composed of individuals of one or another type, and "mixed" ones containing both. No general criterion of distinction was given, but by way of example he assigned to the first kind "*horse, town, triangle, virtue, vice*" (MacColl 1906, p. 42). However, these examples were of concepts, not objects at all. Further, while the first three candidates seem unexceptionable, the latter two are surprising; the reason given was that "the statement 'Virtue exists' or 'Vice exists' really asserts that virtuous persons, or vicious persons, exist; a statement which every one would accept as true". What, however, are the grounds for this reduction? Are philosophies which wish to reify virtue and vice as concepts (or individuals) in their own right categorically rejected, and on what grounds?

In addition, individuals of the second kind were contained in "the class 0" as "unrealities" to which "necessarily belong such as *centaur, mermaid, round square, flat sphere*" (pp. 42–43; curiously, this novelist did not propose the example of fictional characters). He stressed that his "null class" differed from the usual definition as containing no members, and "*contained in every class*, real or unreal; whereas I consider it to be *excluded from every class*" (p. 77). Apart from the unclear distinction, it is most unfortunate that he gave classes of this kind a name which was already widely used in quite different senses in set theory, part-whole theory, algebraic and mathematical logic. In his review Russell did not respond well to this proposal. However, his own use of "existence" is confusingly multiple. In his writings I find the following senses, for individuals x (I) and for classes u (C):

[6] Dejnožka (1990) claims that Russell developed theories of modality; however, while the texts cited are instructive and more numerous than one might guess, the claim is well tempered by Magnell (1991).

Case	Notation(s)	Sense of existence
I1	$\exists x$ $(\exists x)$	As in existential quantification
I2	$E!(\imath \mathrm{x})(\phi x)$	Of a referent of a denoting phrase
C1	$\exists u$ $(\exists u)$	As in existential quantification
C2	Eu	Abstractable from a propositional function
C3	$\exists`u$ $\exists u$ $\exists!u$	Non-emptiness of a class

These senses can conflict; for example, the empty class exists (or may do so) à la C1 and C2, but not C3, while conversely the class which generates Russell's paradox exists only in C3. His own criticism of MacColl was based on restricting existence to the Peanesque sense C3, which is contentious.

4. MacColl on Infinitudes

In a footnote to his calculus of limits MacColl indicated that "the symbol 0, representing *zero*, denotes … that particular non-existence through which a variable passes when it changes from a positive infinitesimal to a negative infinitesimal" (1906, p. 106). He was referring to another eccentricity, in which he stated that "a *negative infinitesimal* denotes any positive quantity or ratio *too small numerically in any recognised notation*" (p. 104), adding similar "definitions" for the negative counterpart, and also for the positive and negative infinite with the property of "too large" instead of "too small" (pp. 108–110). Thus, just when Russell (and Gottlob Frege before him) had sorted out the tri-distinction between zero, the empty set and nothing, MacColl now specified zero as a limiting value of sequences of infinitesimals. He also distinguished his infinities from "non-existences" such as 1/0 and 3/0, which were unrealities belonging to the (or a) null class 0.

These formulations again show MacColl's view of a proposition as a form of words, and with the word "recognised" the same reduction to personal or social assessment that we saw above over virtue and vice. Thus his position was consistently maintained, but it hardly brings conviction. In his last letter to Russell, written on 18 December 1909 just before his death, he fell into impredicativity as well as error when claiming that, M^M, where M is a million, "would be inexpressible, *and therefore infinite*", in Roman numerals! (MacColl 1909a, p. 9). His philosophy of propositions risks conflating words with their references; one may define (or specify) infinitude finitely, as Georg Cantor and Richard Dedekind, among others, had long done, in literature of which he was unaware.

Finally, links with theology need to be noted. MacColl was a Christian believer; he wrote on religion in the *The Hibbert Journal*, a "quarterly review of religion, theology and philosophy" founded in 1902 with the funds established 50 years earlier by the Victorian philanthropist Robert Hibbert (1770–1849), attacking arguments of chance against divine design (1907a). But he did not make theological links to mathematics via his infinitude, a view which had gained some currency at the time, stemming in part from Cantor himself (Dauben 1979, esp. ch. 6). When the American mathematician and believer C. J. Keyser (1862–1947) wrote a rambling piece in the journal concerning "the message of modern mathematics to theology" (1909), MacColl (1909b) expressed general sympathy in his reply, but rehearsed his own theory of infinitude and "unreal ratios" such as 2/0, with no appeal to the Maker. There is a curious parallel with Russell, who in 1904 had opposed Keyser's claim in the journal, argued partly out of theology, that an axiom of infinity was needed in set theory; in the end Russell also adopted one (Grattan-Guinness 1977, pp. 24, 127).

Some of MacColl's remarks on limits occurred in the second part of the book, in which he presented in 35 pages a "Calculus of limits" in the differential and integral calculus as the principal of his applications. The basis was an analogy between truth-values in compound propositions and sign laws in algebra; for example, True and False is False in logic, Positive times Negative is Negative in algebra.[7] He used superscripts again, with "^{P}" and "^{N}" to define appropriate propositions, such as

$$(1) \qquad (x-3)^P = (x>3), \text{ and } (x-3)^N = (x<3)$$

(p. 107, with "=" serving as equality by definition—in addition to its use as arithmetical equality and equivalence between propositions). The ensuing theory advanced to an algebraic method of expressing the change of limits in multiple integrals, where inequalities such as in (1) were used to state that a variable lay between given values. The procedures were quite ingenious, but hardly an application to which mathematicians would rush; some of the cases, such as finding the roots of a quadratic equation (pp. 112–113), can be effected rather more quickly by the usual means.

[7] Indeed, truth tables were introduced with the symbols + and − in the doctoral dissertation of 1920 of Emil Post (a student of Keyser, incidentally).

5. Appraisals by Shearman and Jourdain

Although Russell was the most important and penetrating of commentators on MacColl, he was not the only one; here we note two other compatriots. The first was A. T. Shearman (1866–1937), also a graduate of the University of London (in 1888, when studying at University College Aberystwyth in Wales), and later lecturer in philosophy at University College London.[8] He addressed MacColl's work in a lecture on current symbolic logics to the Aristotelian Society in London (Shearman 1905), and especially in a book of his own published in 1906. This was a pleasant though not profound historico-philosophical survey of these logics, to which I largely confine this summary.

Shearman's knowledge of MacColl's work was based on the recent papers. The main discussion came in a section of 23 pages; in contrast to his praise of MacColl's "very ingenious system" in his 1905 paper, here he focused upon aspects "wherein I think he falls into error" (Shearman 1906a, p. 149). They included MacColl's (apparent) conflation of propositions with propositional functions; the dependence of modalities upon thinkers, for example, with an attendant confusion of "events with statements"; and the theory of unrealities, with attached issues of existence, where he took Russell's side. MacColl commented on this book, and on a short criticism (Shearman 1906b) in *Mind* of his views on existence, in his reply (1907b) to Russell's review. In addition to (rightly) defending his use of symbols, he firmly upheld the independence of modalities from thinkers, giving as an example of a certainty the mathematical theorem that $3 \cdot 141 < \pi < 3 \cdot 142$. A short non-discussion between the two logicians followed in later volumes of *Mind*. Both Shearman and MacColl were reviewed anonymously, and coolly, in *Nature* (Anonymous 1906).

The second commentator was Philip Jourdain (1879–1919), a former student of Russell at Cambridge who devoted much of his career to set theory (not very impressive) and to its history and that of mathematical and algebraic logics (much better). In particular, he wrote a suite of articles on the work of six logicians, publishing them as a three-piece paper in a mathematical journal. The second contained 18 pages on MacColl (Jourdain 1912, pp. 219–236); typically dense with references to many of MacColl's publications from the start, it gave a fair summary of MacColl's theory and its applications. He stressed the role of probability theory (without describing it in detail), and MacColl's priority for asserting the primacy of the proposition.

[8] By a bequest to University College London a trio of "Shearman lectures" on logic and philosophy is held from time to time, of (in my experience) variable merit and relevance to the intended topics.

However, Jourdain's criticisms were quite strong, especially on the non-discussion of correspondence and the non-admittance of propositional functions; he mainly followed Russell's views and quoted some of them. Upon receiving Jourdain's manuscript of the article in September 1909 Russell was "glad to find you so much in agreement with me as regards the points about which MacColl and I have differed". Three years earlier he had judged it "amazing how MacColl's reply [to his review] ignores all the points that I have raised" (Grattan-Guinness 1977, pp. 119, 101). MacColl himself received the manuscript, apparently in March 1909,[9] and Jourdain quoted him in the published version on three points: a detail on symbolism (Jourdain 1912, p. 221); an explanation of real and unreal classes (p. 232); and a general note at the end, where with his usual courtesy he thanked Jourdain for the attention but regretted that he had "sided with the symbolists" on many issues, and stated some of the exercises in logic and uses of the calculus of limits where he felt his own approach to be currently the best (p. 236).

6. MacColl's Historical Place

As the Jourdain article showed, MacColl was working in an environment largely unsympathetic to or uninterested in his main concerns. But MacColl's concessions on some quite elementary features did not help to fight disinterest or resistance. In particular, in his book he distinguished between a statement as any kind of utterance, including "a shake of the head, the sound of a signal gun", and so on, and a proposition as the special case of declarative sentence in subject-predicate form (p. 2); however, in his reply (1907b) to Russell's review he asserted that "a proposition is simply a conventional arrangement of words or symbols employed to convey information or express a judgement", which is surely the previous construal of "statement". Some technical terms were poorly chosen; for example, to use the phrase "null class" in such a non-standard way is surpassed in misfortune only by his ridiculous "definitions" of infinitude, both put forward at a time when Georg Cantor's theory of sets and transfinite numbers had become so important in foundational studies in both mathematics and logics. These cases lay in the applications of the logic rather than at its centre, but they must have discouraged enquiry into the logic itself.

[9] No correspondence with MacColl is held in the surviving fragments of Jourdain's *Nachlass* in the Institut Mittag-Leffler, Djursholm, Sweden. There is also none with John Venn at Gonville and Caius College, Cambridge.

On notations, apart from the ambiguity over the Greek letters on the line and in superscript, MacColl's are among the most forgettable of my acquaintance. His use of normal symbols such as "0" and "=" followed the tradition of multiple interpretation in algebraic logic which stemmed from Boole; but ambiguities arise, especially in the work of Ernst Schröder, and MacColl also manifests them. The tradition of mathematical logic from Frege and Peano onwards had tried to avoid such oversell, and this avoidance marks one of the significant differences between mathematical and algebraic logics (Grattan-Guinness 1975). Another useful influence from algebraic logic was duality; while less marked than in Schröder, it was evident in his five kinds of propositions, with impossibility and certainty as poles on either side of truth and falsehood, and variability in the (unclear) middle.

Perhaps because of his self-education, MacColl comes across as rather half trained in mathematics; the applications of his system there are not exciting. Further, in his last letter to Russell, summarising an argument about limits in a paper published posthumously in *Mind* (MacColl 1910, art. 5), he wondered why the notation "dy/dx" could not be read as a ratio of infinitesimals (MacColl 1909a, p. 5), apparently unaware that this was *exactly* how Leibniz introduced it (with his own sense of infinitesimal) and his successors used it (Bos 1974). Among other cases, in the general journal *The Athenaeum* (MacColl 1904)[10] he held out against non-Euclidean geometries due to their supposed assumption that parallel lines meet, and so placed them among his unrealities; Russell's reply (1904) reads somewhat like a tutorial note.

In addition, MacColl's failure to learn German or Italian, mentioned in his letters to Russell (MacColl (1901b, p. 2), MacColl (1909a, p. 2) and MacColl (1901c, pp. 10–11)), prevented him from knowing some major sources and thus from contributing effectively. His inability to read *technical* Italian when he lived so long in France seems pathetic. He did not even draw on pertinent writings in French, such as papers by Peano, Schröder and others in the proceedings of the 1900 Congress of Philosophy in Paris, to which he himself contributed his longest paper (MacColl 1901d) summarising many features of both logic and his applications. That paper was the best contact he made with the mathematical community this century; it was enhanced by an accurate summary published in an American review article of mathematical lectures given at the Congress (Lovett 1901, pp. 166–168). However, he knew

[10]From 1871 to 1900 *The Athenaeum* had been edited by one Norman MacColl (1843–1904), otherwise a Spanish scholar; although also Scottish-born, he seems to have been no relation to Hugh. His successor as editor was Vernon Randall.

little of the Anglo-Saxon representatives of algebraic logic; he used C. S. Peirce's symbol for implication in a passage of his book where he also noted Schröder's (1906, pp. 78–80), but he did not discuss or use their systems. He also ignored both their logic of relations and an alternative version produced by Russell in 1901 for mathematical logic, and did not develop one of his own, although it was recognised as a major component of both traditions in logic.

For these reasons MacColl got the good idea of modal logic off to a bad start, and has never been given his due in the history of the subject. Thus it is not surprising that the bibliographical sleuth Giuseppe Peano omitted him from the lists of recent writings in his compendium *Formulario mathematico*, even in its final edition of 1908 (where, curiously, Frege is also missing). When C. I. Lewis came to restart modal logic in the early 1910s, especially in various papers in *Journal of Philosophy* and *Mind*, MacColl was almost entirely ignored. In a book of 1918 he found MacColl's systems only "suggest somewhat" his own (1918, p. 108), and omitted them even from the long opening historical chapter. His main inspiration, and negative one, had come from *Principia mathematica*. In a later discussion of Lewis's logic, W. T. Parry went to great pains to contrast it with MacColl's (Parry 1968, pp. 21–24).

7. The Fight for Logical Pluralism

In any case, the philosophical climate was hostile. For example, Russell prepared the manuscript of his lecture (1905) on necessity and possibility to the extent of furnishing most of the needed references; but he never bothered to finish it off for publication. Again, in a review of MacColl's book in *The Philosophical Review*, the American philosopher J. G. Hibben dismissed the three extra kinds of proposition as "a needless complication", and found the section on the calculus of limits to be the most original part (Hibben 1907). Although he writes in a conciliatory way, MacColl may have been arguing for his system as *the* correct logic while Russell and others were opposing him with the traditional bivalency; in this section I shall take MacColl as a source for logical pluralism, in which the traditional form is not the only possibility (as it were).

The struggle for logical pluralism has been long and hard; in this regard I end with an anecdote. In 1972 McMaster University held a conference to celebrate the centenary of Russell's birth, and also the establishment of the Russell Archives. One of the speakers was Nicholas Rescher, who gave a talk on "neglected aspects of Russell's work on logic"; from my memory of the occasion, much of the content appeared also in a later paper on "Russell and modal logic", published as Rescher 1974 in a volume on modality and reprinted five years later in a tributary book on Russell. Not much in the way of tribute was provided, however; apparently, Russell had found MacColl to be "so much old-fashioned fairy tale nonsense", thereby exercising "a baneful influence" on the acceptance of modal logics.

This historical appraisal seems to be rather implausible; if MacColl was old-fashioned, then who were his predecessors? Rescher presented Russell's position on modalities only from his book of 1900 on Leibniz, written before his discovery of Peano or acquaintance with mathematical logic, and thus a quite different figure from the one who confronted MacColl from 1904 onwards.[11] When the lecture finished I began the discussion period with (what I hoped was seen as) an *historical* appraisal of Russell's situation when he came across MacColl, as follows. Thanks to Peano he had been able to envision a comprehensive foundation for (much) mathematics in the propositional and predicate calculi with quantification, in which Cantor's set theory played a central role. Thus, when he found MacColl's alternative approach to which he not only felt unsympathetic but which in any case was fraught with unclarities, weak definitions and unwelcome uses of technical terms, his reaction was understandably cool.

Rescher began a reply, but was interrupted from the audience by Max Black, who denounced categorically non-classical logics of all kinds (that is, without argument). His advocacy of logical monism was immediately acclaimed by several other participants, so that the historical discussion which I had tried to launch degenerated into a philosophical attack on logical pluralism. However, I maintain it, and give credit MacColl for pioneering it in modern times; there is indeed more than contextual difference between the truths of "$7 > 4$" and of the residential properties of Mrs. Brown.[12] But this reaction in the 1970s shows how hard and durable was the resistance.

At some time, perhaps when his manuscripts were being organised for sale in the late 1960s, Russell annotated some of his collections of letters with remarks on the correspondent, often quite warm. For MacColl he merely put "a mathematical logician with whom I disagreed". However, he kept all of the letters that he had received from MacColl,

[11] For criticisms of the paper, see Dejnožka 1990, pp. 406–412.

[12] At this time Henri Poincaré (1905, p. 827) also read propositional functions this way, as sometimes true and sometimes false; but I take him as obtusely making a (pathetic) criticism of the mathematical logic which he despised but did not trouble properly to learn.

unlike other collections of which he kept little (for example, and more typically, about 15% of the letters from Jourdain). Presumably he sensed some merit in MacColl's proposals for others to find. At last we can realise his archival investment and give credit to a pioneer modal logician and maybe logical pluralist whose ideas have their historical place and perhaps still some future of their own.

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Appendix A

Notes on Books, August 31 1870:

Algebraic Exercises and Problems, with Elliptical solutions; framed so as to combine Constant Practice in the Simple Reasoning usually required in the Solution of Problems with Constant Practice in the Elementary Rules, the Simplification of Fractions and other expressions, and in the Mechanical Operations of Algebra generally. With an Appendix containing a few Simple Methods, on one uniform plan, for Resolving Algebraical Expressions into their Elementary Factors. By HUGH MCCOLL, late Mathematical Master at the Collège Communal, Boulogne-sur-Mer. 12 mo. pp. 108, price 3*s*.6*d*. cloth. [*June* 16, 1870.

THIS little book, intended for beginners in Algebra, is not meant to supersede any of the ordinary systematic treatises on the subject.

Its main object is to train beginners in the application of the elementary operations of algebra to the solutions of problems. The elliptical solutions at the end of the exercises have been written with this view. The pupil is supposed to copy these on paper, filling up the blanks as he goes along. Each link in a chain of reasoning is thus brought before him in logical succession; but he is expected to join the links and complete the chain himself. If he does all this successfully and without the assistance of his teacher, the latter may feel satisfied that his pupil has understood both the problem proposed and its solution; while, on the other hand, if the pupil meets with any difficulty the teacher can at once see where the difficulty arises, and remove it. This secures economy of labour on the part both of teacher and pupil. The latter, instead of wasting time and energy in attempting to solve by his own unaided powers those problems which upon a fair trial he finds too difficult for him, is enabled to concentrate his attention successively upon single points, and so conquer his difficulties in detail; while the former is spared the trouble of explaining difficulties which his pupil might not perhaps find very formidable if left to himself, and may reserve his assistance for those cases which really require it.

Many of the examples are so framed as to make hardly any demand upon the reasoning powers, while at the same time they afford good practice in the four elementary rules, the simplification of fractions and other expressions, and in the mechanical operations of algebra generally. The pupil will find the answers to these at the end of the exercises, but, generally speaking, no solutions.

Appendix B

Notes on Books, March 1906:

Symbolic Logic and its Applications. By Hugh MacColl, B.A. (London). 8vo. pp. xii + 142, price 4*s.* 6*d.* net.

The system of symbolic logic explained in this volume is founded on a series of papers contributed by the author to various magazines, English and French, including *Mind*, the *Athenæum*, the *Educational Times*, and the *Proceedings of the London Mathematical Society.* The author hopes that in this, its final form, his system will be found so simple that schoolboys of ordinary intelligence can master its fundamental principles and apply them to the solution of problems both in logic and in algebra. Solutions of questions set at recent university examinations in logic are given as illustrations. The last two chapters treat of probability and the limits of multiple integrals.

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