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BOOLEAN ALGEBRAS AND NATURAL LANGUAGE: A MEASUREMENT THEORETIC APPROACH*

In the first section of this paper I discuss several accounts of the way Boolean algebra applies to the logical sentential connectives of natural languages. In the second section I present a superior account, modeled after the use of numbers in measurement.

I.

An algebraic structure is a set with one or more operations defined on it. For example, the natural numbers with addition and multiplication are such a structure. An algebraic structure A is a Boolean algebra if it has two binary operations ($*$ and $+$), one unary operation ($\sim$), and two designated elements (0 and 1) such that for any x, y and z from A (x, y and z are variables whose domain is the underlying set of the given structure):

(*i*) $x + (y + z) = (x + y) + z.$

(*ii*) $x * (y * z) = (x * y) * z.$

(*iii*) $x + y = y + x.$

(*iv*) $x * y = y * x.$

(*v*) $x * (y + z) = (x * y) + (x * z).$

(*vi*) $x + (y * z) = (x + y) * (x + z).$

(*vii*) $x + (x * y) = x.$

(*viii*) $x * (x + y) = x.$

(*ix*) $x + (\sim x) = 1.$

(*x*) $x * (\sim x) = 0.$

The Boolean axioms apply to several mathematical domains, e.g. in plane geometry and set theory; however, these axioms are also applicable to sentential logic: the Boolean axioms are well known to describe (approximately) the logical behavior of the English words ‘and’,

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'or', and 'not' as sentential connectives. Indeed, the Boolean operations are usually referred to as conjunction ($*$ in the axioms above), disjunction ($+$) and negation ($\sim$), terms that are associated with the above-mentioned English words. Also, the three English connectives are often called (e.g., by computer scientists) Boolean connectives.

But what, exactly, is the connection between Boolean algebra and the three English logical connectives? In what sense are the Boolean axioms (and the identities that follow from them) applicable to natural language? This question might seem trivial, but it is not; indeed, several conflicting answers to it can be found in the literature. I proceed to discuss several of these answers, and then to offer one that is superior.[1]

The first answer to be considered here is clearly a non-starter, but its presentation and rejection will be of use later. A common feature to the three Boolean functions and the three English connectives is that they are all operations. The Boolean conjunction, for example, takes any given pair of elements in a Boolean algebra and yields an element as an output. Similarly, the connective 'and' in English helps us create a new sentence from any given pair of sentences. Thus it might be suggested that the Boolean operations capture the properties of 'and', 'or' and 'not' as syntactical sentential operations.

However, this suggestion is clearly misguided. Apart from their being all operations, there is no further resemblance between the Boolean functions and the English logical connectives considered as *syntactical* operators. The latter are merely recursive syntactical mechanisms that help us churn out more and more sentences, exactly like many other such mechanisms in natural language. The Boolean operations, on the other hand, *are governed by axioms*: these axioms are what makes these operations Boolean. For example, any two elements of a Boolean algebra x and y satisfy the equation $x * y = y * x$; this is one of the axioms governing conjunction. Now is the same true of 'and' as a syntactical operator? Of course not. Consider, for example, these two sentences:

(i) John talks and Bill walks.

and

(ii) Bill walks and John talks.

[1] In what follows I consider 'and', 'or' and 'not' only as sentential connectives, putting aside for now all their other uses in language (e.g., to connect adjectives, adverbs, etc.). As usual in these cases, I assume that the remarks made here could be later extended to these further uses.

These two sentences are *not* one and the same sentence (i.e. they are not the same syntactic entity), contrary to what the Boolean equation says. Therefore the rules that govern the Boolean operations are not applicable to the English connectives as syntactical operations.

There is nothing Boolean, then, about the syntax of the English logical connectives. Rather, what the Boolean axioms apply to is not the syntax of these words, but their meaning. Speaking *loosely*, the Boolean axioms capture the meaning of the logical connectives, and therefore every Boolean identity represents a logical inference that we all agree to. Sentences (i) and (ii) above are indeed distinct sentences, but we are always happy to infer each of them from the other. The two sentences are logically equivalent, and this fact is captured by the Boolean identity.[2]

The claim made in the previous paragraph was stated in very vague terms, and there are several ways in which it can be made more explicit. According to one, every natural language sentence has related to it an abstract object which is its meaning. This object is usually called the *proposition* expressed by the sentence. Whenever a sentence is uttered by a speaker, the proposition expressed by the sentence is referred to, or invoked; successful communication consists in the hearer being able to grasp what this proposition is. Now according to this view the domain of propositions simply *is* a Boolean algebra: conjunction, disjunction and negation are operations in this domain, satisfying the Boolean axioms. For example, consider again the axiom $x*y = y*x$. According to the propositionalist the 'x' and 'y' are *variables* that pick propositions as their values, the symbol '$*$' stands for conjunction as an operation on the domain of propositions, and the axiom tells us something about that domain: whatever x and y may be, the proposition $x * y$ is the same as $y * x$.

To know the meaning of the English word 'and', this explanation goes on, is to know that it signifies the above-mentioned operation on propositions (i.e., conjunction). Thus a competent speaker of English knows that sentences (i) and (ii) above have the same proposition as their meaning, and this fact is captured by the Boolean axioms.

In the account given above, then, the application of Boolean algebra to natural language is made more explicit and clear through the intro-

[2]It should be noted that the claims made in this paragraph about sentences (i) and (ii) do not apply to all uses of 'and' as a sentential connective, without exception. For example, sometimes a sentence of the form 'p and q' implies that p and q are temporally ordered in a certain way, and in such cases we shall not be happy to infer 'q and p' from 'p and q'. I shall presuppose here that such problematic cases can be put aside for later treatment.

duction of a powerful tool, i.e., propositions. It is therefore no wonder that the main objections to this account are raised against the very idea of propositions, not against its further application. It is objected in Quine 1953b, for example, that synonymy is a relation that cannot be made clear sense of, and hence it is not clear how propositions should be individuated. (The individuation of propositions depends on our ability to say when two sentences express the same proposition, i.e., when they are synonymous.) Therefore, by Quine's maxim 'No entity without identity', we should reject propositions altogether. Also, Davidson (e.g. in Davidson 1984b, pp. 126–127) objects that propositions do not serve any purpose in explaining how natural language works. Language is essentially public: it is a tool used for communication. Hence what we mean by what we say must be captured in terms of what is publicly accessible. But are propositions public in this way? When a speaker makes an utterance, how, exactly, is the proposition invoked (or referred to) by the utterance? Is it accessible to the hearer? It is not clear what the answers to these questions are. And if we are told that as hearers we do have available to us data that can help us grasp the proposition expressed by an utterance, why then cannot we account for meaning in terms of these data alone, and do away with propositions altogether?[3]

Suppose we accept these arguments and reject any appeal to propositions. In this case we need to come up with a new answer to the problem we started with, i.e., a new explication of the way Boolean algebra applies to 'and', 'or' and 'not'. What could such an alternative explication be? Well, Quine (one of the staunchest opponents of propositions) has a view of propositional calculus that avoids the introduction of propositions altogether. Let us consider this view and then see whether it extends to the discussion of Boolean algebra.

Quine's suggestion (e.g. in Quine 1953a, pp. 108–109) is that the axioms and theorems of classical propositional calculus are *schemas*. This means that these axioms and theorems are *not* open expressions, and that the x's and y's in them (or p's and q's) are *not* variables—they do not pick objects from some extra-linguistic domain. Instead, an axiom (or theorem) of the propositional calculus is just a *template for sentences*, it is a form common to sentences that we shall always be willing to agree to. For example, consider again the equation $x * y = y * x$. Its equivalent in propositional logic is $x \wedge y \Longleftrightarrow y \wedge x$ (i.e. $x \wedge y$ iff $y \wedge x$). According to Quine the latter formula is just a

[3] These remarks are not meant to do justice to the arguments against propositions. For a more elaborate presentation of such an argument see, e.g., Davidson 1984a, p. 99.

concise instruction manual, a schema that tells us how to create from any two sentences another sentence that has the special property of being universally affirmed. Applied to sentences (i) and (ii) above, this schema yields the following sentence:

(iii) John talks and Bill walks if and only if Bill walks and John talks.

Indeed, this sentence will never be objected to by a competent speaker of English. Thus we have an account of the way propositional identities are relevant to our understanding of the English logical connectives, an account that avoids the introduction of propositions.

We can try to extend Quine's explanation to the discussion of Boolean algebra, as follows.[4] The Boolean equations are notational variants of the theorems of (classical) propositional calculus: one could easily define a mapping between these two systems, a mapping in which (for example) the symbols '$\Longleftrightarrow$' and '$\wedge$' will be replaced with '$=$' and '$*$' respectively. Therefore it could be suggested that a Boolean equation too is a schema for natural language sentences, exactly like its propositional counterpart. *As templates for English sentences* there is no great difference between '$x \wedge y \Longleftrightarrow y \wedge x$' and '$x * y = y * x$'; each formula is as easy to use as the other.

However, I claim that this suggestion is counter-intuitive. Here is why. First (*premise 1*), Boolean equations are similar to ordinary numerical equations, such as $x + y = y + x$. Formulae of both kinds feature the identity sign (i.e., they are equations), and on both sides of this sign they have functional terms (i.e., terms that are (or at least look like) iterative applications of function symbols to variables and/or constants). Second (*premise 2*), the view that numerical equations are schemas is implausible, and has now very few, if any, supporters.[5] Therefore we get the *conclusion* that the schematic view of Boolean equations should be rejected too: if this view does not sit well with ordinary numerical equations (premise 2), and if numerical and Boolean

[4] It is certainly not my claim here that Quine would accept this extension of his view.

[5] The most notable exception is Hilbertian finitism, according to which universally quantified numerical statements *are* indeed schemas (or templates) for concrete numerical statements, which are finitistically verifiable (and therefore meaningful). Now I think that the analogy between Quine's schematic view of the propositional calculus and Hilbert's finitism is interesting, but as I do not subscribe to the Hilbertian view I do not see this analogy as supporting Quine's position. Of course, I do not intend to argue against finitism here, and therefore I shall take the rejection of finitism as a further (implicit) premise of my argument.

equations are similar in form (premise 1), then the schematic view should not be forced upon Boolean equations either.[6]

Now this objection is not claimed to be a knock down argument against the 'schematic' view of the way Boolean algebra applies to the English logical connectives (and certainly not against Quine's original suggestion as regards the propositional calculus). Similarly, the considerations presented above concerning the propositionalist view are not sufficient to reject that view either, and they are not presented here as such. It was not my objective to give an exhaustive discussion of the pros and cons of these two alternatives, and then to assess which is better. Rather, these two views are presented here because they mark the two sides of a dilemma facing anyone who wants to solve the problem we started with (i.e. anyone who wants to explain how the Boolean axioms apply to the English logical connectives). On the one hand, the Boolean identities are clearly equations, involving variables that pick their values from some domain. On the other hand, the only candidates for populating this domain seem to be propositions, which are highly problematic.

It might be objected now that in presenting this dilemma I have ignored an obvious way out of it, or around it. A well-known way to describe the Boolean operations is not through axioms, but through *truth tables*: the properties of the three Boolean operations are completely captured by specifying how they operate on the 2-element Boolean algebra that comprises of 0 and 1, or the True and the False. Construed this way, the Boolean operations can be viewed as being applied to variables *whose values are truth values of natural language sentences*; thus we apply Boolean algebra to language *as a system of equations*, which is what we wanted, and this without any involvement of propositions.

This suggestion is almost right. Its defect is that it conflates *sentences* and *utterances* (as is almost always done in formal logic): contrary to what is said in the previous paragraph, it is not natural language *sentences* that are true or false, but rather their utterances. In the previous paragraph the Boolean operations were shown to apply

[6]Moreover, I claim that there is a similar weakness in Quine's original suggestion, that the theorems of propositional calculus are schemas. Schemas are meta-linguistic mechanisms that are introduced in the discussion of a *concrete* object language. Is this true for the systems of propositional calculus? Should we view propositional theorems as schemas for English sentences when they appear in a paper in English, and for French sentences when they appear in a paper in French? Or maybe they are schemas for sentences of all languages, past and future? This seems implausible. Systems of propositional calculus are better viewed as mathematical constructions that apply to different cases, not as systems of schemas. For an elaboration of this approach see the treatment of Boolean algebra below.

as operations to values (i.e. truth values) of *utterances*, not sentences, while the English logical connectives apply to *sentences*, not utterances. (That is, a sentential connective such as 'and' connects sentences, not utterances or truth values.) If we distinguish between sentences and utterances, then we see that no direct account has been given of the connection between the Boolean axioms and natural language *sentences* after all.[7] However, as we see below we are on the right way.

II.

I claim now that a *measurement theoretic* approach to the problem we are facing offers us a way out of the dilemma stated towards the end of the previous section; it allows us to have the variables in the Boolean equations pick values from a domain of (abstract) objects, and this without a commitment to propositions. Summed up in one sentence, the solution is to see the variables in a Boolean identity as picking their values from a Boolean algebra (defined in pure mathematical terms), and then to view this Boolean algebra as being related to a system of natural language sentences in the same way that the (real) *numbers* are related to a system of concrete objects in *measurement*, e.g., of length or temperature. Let me elaborate and explain this point, appealing to the theoretical framework presented in Krantz et al. 1971.

The procedure of measurement is characterized in Krantz et al. (1971, p. 1) thus:

> When measuring some attribute of a class of objects or events, we associate numbers (or other familiar mathematical entities, such as vectors) with the objects in such a way that the properties of the attribute are faithfully represented as numerical properties.

In order to see how this characterization is made formally precise, let us consider the measurement of length as an example. Given a class of physical objects D, we have a basic procedure through which to compare these objects: for each pair of objects we can check which is longer. This basic procedure imposes length relations among the elements of D (and their possible concatenations), thus giving rise to the following *empirical relational structure*[8] $\langle D^{\circ}, \geq, \circ \rangle$:

[7]Note that there is no problem in viewing *truth tables* as applying to *sentences*, albeit indirectly, through the mediation of sentence utterances. For example, the truth table of negation can be taken as saying this: given a sentence p, the sentence $\sim p$ is such that for any true utterance of p a concomitant utterance of $\sim p$ would be false, and vice versa. However, construed this way truth tables do not involve the Boolean operations *as operations*; they are much more like Quine's schemas. Therefore they too do not count as solutions to our problem.

[8]Krantz et al. 1971, p. 8.

(1) D° is the closure of D under concatenation,

(2) $\geq$ is the binary relation 'at least as long as' on D°,

(3) $\circ$ is the (ternary) concatenation relation (that obtains between any two objects and their concatenation).

The procedure of length-measurement can be defined now as follows: it consists in our establishing a *homomorphism*—a structure-preserving mapping—from the empirical structure $\langle D^\circ, \geq, \circ \rangle$ into $\langle \mathrm{Re}^+, \geq, + \rangle$—the positive real numbers with 'less than' and plus defined on them. The existence of such a homomorphism is exactly what enables us represent the length properties of an object through the association of a number to it (as its length).[9]

Let us return now to our semantic discussion. Consider a class of utterances of English sentences, and suppose that we are interested in the structure applicable to this class of utterances due to appearances of the words 'and', 'or', and 'not' as sentential connectives. The 'raw' data (or the basic facts) that we have available to us consist in connections between appearances of the above words in sentences and truth/falsity of utterances of sentences. For example, we could find that whenever (an utterance of) the conjunction of two sentences is true then each of the conjuncts is true, that an utterance of sentence (i) above is true iff a concomitant utterance of sentence (ii) is also true (i.e., that conjunction is commutative), and so forth. We could go on and collect a large body of data of this kind, which gives rise to the following empirical relational structure $\langle K, \Rightarrow, \circ_{\mathrm{and}}, \circ_{\mathrm{or}}, \circ_{\mathrm{not}}, \rangle$:

(1) K is the set of sentences in the language,

(2) $\Rightarrow$ is a binary relation on K, obtaining between two sentences s and t if and only if whenever an utterance of s is true a concomitant utterance of t would be true as well,

(3) $\circ_{\mathrm{and}}$ is a ternary relation on K, obtaining among s, t and 's and t' (for all s, t in K). The ternary relation $\circ_{\mathrm{or}}$ and the binary relation $\circ_{\mathrm{not}}$ are defined in a similar fashion.

Now as in the previous example, we can represent the structure of this empirical relational structure by establishing the existence of a *homomorphism* between this structure and an adequate mathematical structure. Such an adequate mathematical structure will not be the real numbers this time, though. Rather, it is easy to see that (i) the

[9]For more details, see Krantz et al. 1971, pp. 71–77.

required structure will be a sufficiently rich Boolean algebra, and that (ii) the required homomorphism will be constructed thus:

– the set of propositionally atomic sentences of the language will be mapped onto a set of mutually independent generators of the Boolean algebra

– if an element x is assigned to sentence S1 and an element y is assigned to sentence S2, then the element $x * y$ should be assigned to the conjunction of S1 and S2 (i.e. to the sentence created from S1, 'and' and S2 in the standard way); similarly for the other connectives.[10]

The claims made in the previous paragraph are easy to verify and will not be proved here. These claims consist in a measurement theoretic *representation theorem*: they establish the fact that the relevant truth functional properties of the sentences within the system are faithfully represented by the algebraic properties of the algebra elements assigned to them by the homomorphism.

The measurement theoretic approach yields the following solution to the problem we started with, then: we should view the Boolean axioms as indeed obtaining in pure mathematical structures, and then construe such structures as representing (some of) the semantic properties of natural language sentences in the same way that numbers represent properties of physical objects in numeric measurement.[11] The expression 'in the same way' in the previous sentence does not mean, of course, that there is a quantitative aspect to the application of Boolean algebras to natural language; the numeric aspect of measurement is not preserved when we move to the semantics case. Rather, the essential (and characteristic) feature of measurement that is preserved is the representation of (some of the properties of) an empirical relational structure by homomorphically embedding it into an adequate pure mathematical one.

A natural question (or objection) that could arise here is this. It is a well known fact that the sentences of a given formal language can be partitioned into classes of propositionally equivalent sentences, and that the equivalence classes of such a partition (together with 'and'

[10]Notice that any instance of a propositional logical truth would be assigned 1 by a homomorphism of this kind, and any absurdity would be assigned 0. (1 and 0 are the distinguished elements of a Boolean algebra, not integers.)

[11]The measurement theoretic approach presented here is new, but it arises in a natural way from a holistic conception of meaning that is well established in contemporary philosophy of language; for details, see Dresner 1998, pp. 1–37.

and ‘not’) consist in a Boolean algebra—the (propositional) Tarski-Lindenbaum algebra of the language. In what way is the viewpoint suggested here different, then, from what is already implicit in this well-known construction?

The answer to this question is as follows. It is certainly correct that the viewpoint suggested here does not go *formally* beyond the Tarski-Lindenbaum construction. Indeed, the canonical way to construct a Boolean algebra into which the above-defined linguistic empirical structure can be homomorphically embedded is through this construction. However, I claim that *philosophically* the measurement theoretic viewpoint is of significance: it provides us with the right framework through which to describe the connection between the Boolean algebra resulting from the Tarski-Lindenbaum construction and natural language sentences. Quite often this connection is made (albeit implicitly) through an appeal to *propositions*: the elements of the Tarski-Lindenbaum structure are thought of as being (or standing for) the propositions expressed by sentences that take part in the construction. However, we saw in section (I) that this option is problematic in several ways. The conceptual framework of measurement allows for a superior alternative to the propositionalist conception of the Tarski-Lindenbaum algebra: in this framework the construction of such an algebra is viewed as a part of a proof of a *representation theorem*, i.e. a proof of the existence of a homomorphism between a linguistic empirical relational structure and a pure mathematical entity. In fact, in the measurement theoretic framework the formation of (abstract) equivalence classes of concrete entities is a *typical* move towards proving a representation theorem; for an easy example, see the proof that the numbers can represent any empirically occurring linear order.[12]

The introduction of the measurement theoretic outlook into the context of the Tarski-Lindenbaum construction is therefore natural, and allows us to have the connection between language and the algebra without any heavy metaphysical baggage. Furthermore, as I show in Dresner 1998 and 2000, the measurement theoretic outlook opens the door for several new applications of algebraic conceptualization to problems in formal semantics and the theory of meaning. These applications can be developed in a natural way when the connection between language and the algebraic constructions is construed measurement theoretically, as elaborated here.

Let me address another pair of related objections to the analogy made above between the elements of a Boolean algebra and numbers.

[12] Krantz et al. 1971, pp. 14–17.

It might be claimed that our acquaintance with the two types of mathematical entities is different (and that therefore their use cannot be the same). First, the numbers are just one structure, while there are many Boolean algebras; how can we tell which Boolean algebra to apply in each particular case? And second, we have names for the numbers, we know each of them well; the elements of an arbitrary Boolean algebra, on the other hand, are unnamed and unfamiliar.

These objections can be answered as follows. First, it is not the case that there is only one system of numbers; there are many of them (such as the integers, the rationals, and the reals, to name just a few of the more 'pedestrian' systems). When we use a system of numbers we just have to make sure that it is adequate for our needs; if there is more than one such system (and we do not specify which is used), then what we say might be harmlessly ambiguous. For example, suppose you say that this table is two and a half feet high. Is it important (or even meaningful) to ask you whether you have in mind the system of the rational numbers, or the reals? Of course not. What is important to know is only that you implicitly appeal to a system of numbers that can capture the relevant data about the table: if the rationals are sufficient for your needs, then 2.5 could be thought of as an element of the rationals, and otherwise it could be thought of as a real.

Similarly, when we use a Boolean algebra to 'measure' a class of sentences K it is implicitly assumed that we use an algebra that meets our needs—no further specification is required. (For example, the Boolean algebra of four elements (there is only one such algebra) will not be applicable to a class of utterances that includes utterances of the sentences "John walks" , "Bill talks" and "John walks and Bill talks" because it is not sufficiently rich, and hence a larger one will have to be used.)

The second objection (i.e. that we do not have names for elements of Boolean algebras) can be answered in two ways. One is that in fact we do *not* have names for all numbers either, e.g. for most of the real numbers, but this does not make these numbers less useful in any way. Therefore the same situation with the elements of Boolean algebras should not worry us: although we do not have a name (or even a description) for every element of every Boolean algebra, still these algebras are well studied and well understood, and are therefore useful in 'semantic measurement.' And second, there is actually a very good reason why everyday language does not provide us with means to refer to so-called 'Boolean numbers.' The assignment of these (so-called) numbers to sentences was shown here to represent structure arising only from appearances of 'and', 'not' and 'or' in sentences (as sentential connectives). However, in everyday life we are never interested in the significance that utterances have due to appearances of these words alone; rather, we are interested in the significance utterances have due to all of their syntactical structure. The full syntactical structure of sentences imposes on a class of utterances of these sentences a structure which is much richer than what a mere Boolean algebra can capture, and this structure is what we (as speakers of a natural language) are interested in. I discuss algebraic structures that are richer than Boolean algebras, and that can therefore capture more of the structure of natural language, in Dresner 1998.

We proceed now to verify that this new solution to our initial problem has indeed the advantages it was claimed to have; that is, it is shown that this new solution avoids the dilemma mentioned several paragraphs above.

First, consider the difference between what is said here and the Quinean solution to the problem. The measurement-theoretic solution allows us to maintain that the Boolean identities apply to language as equations. These equations, exactly like their more familiar numerical counterparts, involve variables that pick their values from a domain of abstract (mathematical) objects. Thus the measurement-theoretic solution manifests one of the two desirable features referred to above: it does not force us to view the Boolean formulae in an unnatural way (i.e. as schemas).

Moving on to the second horn of the dilemma, the propositional horn, it might seem as if the measurement-theoretic approach is of no great help. That is, it could be objected that in the measurement-theoretic solution propositions masquerade as the elements of a mathematical structure, but they are still propositions; they are still these inscrutable abstract entities that are somehow relevant to the meaning of sentences. The suggested measurement-theoretic approach tries to give propositions some much needed mathematical respectability, the objection goes on to say, but substantially it achieves nothing.

This objection could not be further from the truth. To see this, let us recall the problematic features of propositions, as discussed above, and verify that the solution suggested here is free of these features.

First, the relation between an utterance (of a sentence) and the proposition it is supposed to express is obscure: it is implausible to suggest that utterances of sentences *refer* to propositions (because sentences are not names), and no articulated alternative to this suggestion is ever given. Well, does the measurement-theoretic solution feature this troublesome characteristic? Of course not. The abstract elements of a Boolean algebra are assigned to sentences in the same familiar fashion in which numbers are assigned to physical objects, such as tables and chairs. A chair does not *refer* to the number that gives its height (say in inches); a chair does not refer to anything at all, because it is a chair, not a name. Similarly, an utterance does not *refer* to the 'Boolean number' that helps represent the connections (of a very special kind) this utterance has to other utterances. Rather, the utterance is related to the abstract 'Boolean number' in an alternative way, a way which is common to many applications of mathematics to the (non-mathematical) world. This is not to say, of course, that the status of abstract mathematical objects is not philosophically problematic, or that the way these objects apply to the world is in no need of explanation. Rather, my claim is only that through the solution given here we dispose of a seemingly intractable problem special to semantics (i.e., the question how utterances invoke propositions) and we replace it with a problem that is much more general and widely addressed (i.e., the question what numbers are and how they apply to things). This seems to me to be a good philosophical deal.[13]

A second cluster of problems that propositions have is related to their role in explaining how communication through language actually works. How can a hearer 'grasp' an abstract entity (i.e., a proposition) which is the meaning of an utterance made by a speaker? What access does the hearer have to this abstract entity? And if the hearer does have some data that should help him interpret (i.e., understand) an utterance, how is the meaning of the utterance (as an abstract object) related to these data? Isn't it redundant? These are some of the questions that were posed above to the friend of propositions.

Now the measurement-theoretic account given here is free of these problems altogether.[14] First and foremost, note that an element of a Boolean algebra that is assigned here to a sentence *is not* the utterance's meaning (and this is reason enough to avoid calling abstract Boolean entities 'propositions'). It is easy to see why. A number that expresses the height of a chair represents the relations the chair has to other objects in one specific respect; it says nothing more about the chair. Similarly, when a Boolean number is assigned to an utterance of

[13]Notice that the account I give here is not committed to any particular position concerning the existence of mathematical entities, be they real numbers or the elements of a Boolean algebra. I certainly talk here of mathematical objects as abstract entities, as we usually do; however, this does not rule out attempts to do away with such talk. Any account of numerical statements that avoids the postulation of numbers as abstract objects will clearly apply to the discussion given here as well.

[14]As said above, it is not my aim to demonstrate that the friend of propositions cannot solve these problems. Such a demonstration is not necessary for an argument in favor of my account.

a sentence in the way described above it represents only a very specific type of data about the utterance, i.e. the truth-functional relations the utterance has to other utterances *due to appearances of 'and', 'or' and 'not' in sentences* (as sentential connectives). Therefore the Boolean number cannot say anything else about the sentence: it is applied to the sentence in order to capture the above data alone. Thus in this case there is no entity which is the meaning of a sentence; there are just semantically relevant properties that a sentence has within a class of sentences, properties that can be represented through an assignment of elements of an algebraic structure to them.

Of course, the above-described assignment of Boolean algebra elements to the sentences in a given class tells us only little about the meaning of these sentences: sentences have significance due to syntactical features other than appearances of the logical connectives. However, this is not to say that in order to account for this further significance of sentences we need to introduce entities that are meanings; I argue in Dresner 1998 that an account similar to the one given here can be sufficient.

The solution I present here, then, does not stipulate the existence of entities that are the meanings of utterances, and therefore it avoids all the problems that arise from such a stipulation. Nevertheless, the measurement-theoretic solution does involve abstract entities (assigned to utterances), so it could still be asked whether these entities are relevant to what we learn when we come to understand someone's utterances. (Recall that this question was raised as regards propositions.) Well, according to the measurement-theoretic approach the assignment of an algebra to a class of sentences K represents the structure that K has due to certain basic data (i.e., truth of utterances). It follows that if these data are such that an interpreter could come to know them, then the algebraic structure (that represents these data) is relevant to what interpreters can learn. If, on the other hand, the data are out of public reach, then so is their systematic representation.[15] Either way, the connection (or lack thereof) between the abstract entities assigned to utterances and what interpreters can learn about these utterances is clear.[16]

[15]This second option is much less plausible, for reasons that were considered above.

[16]It should be noted, though, that the measurement-theoretic framework is *not* claimed here to capture *how* we humans store and make use of semantic information; it is not presented here as a part of a theory in psychology or in cognitive science. Rather, like any other theory in formal semantics the aim of this theory is to provide an adequate formal representation of semantic facts, and this in a way which fits best (and helps bring out) our understanding of what language is.

We see that the measurement-theoretic account of the way Boolean identities apply to the logical sentential connectives avoids major problems that face other views. Also, it has the advantage of answering a question in semantics through the use of ideas and concepts that are not limited to this field (i.e. the account appeals to the general notion of a mathematical construction applying to the non-mathematical world). For these reasons this account should be preferred over its competitors.[17]

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[17]In Dresner 1998 I show that the measurement-theoretic approach can be extended to first-order logic (through the use of cylindric algebras) and beyond, and that it yields further results that are beyond the scope of this paper.