Hao Wang

SKOLEM AND GÖDEL*

Many logicians would agree that Skolem (1887–1963) and Gödel (1906–1978) are the two greatest logicians of the century. Yet their styles, philosophies, and careers are strikingly different.

Gödel had already published some of his great works and had become world renowned by the time he was 25 years of age. Skolem began to publish his important papers only after he was 30, and his impact grew slowly over the years. Gödel was meticulous in writing for publication and published little after he reached 45. Skolem wrote informally, often even casually, continuing to publish into the last days of his life.

Gödel was a well-known absolutist and Platonist who had devoted much effort to studying and writing philosophy. Skolem was inclined to finitism and relativism, and rarely attempted to offer an articulate presentation of his coherent and fruitful philosophical viewpoint about the nature of mathematics and mathematical activity. Apart from mathematical logic, Gödel made contributions to the philosophy of mathematics and to fundamental physics. Skolem divided his work almost equally between logic and other parts of discrete mathematics, particularly algebra and number theory.

For many years I have been deeply involved with Gödel's work and his life. Even though I was for a long time intensely interested in Skolem's work in logic and made a careful study of it in the sixties, since then I have not followed carefully the important applications and developments of Skolem's ideas by many logicians. I know very little about his life and his work in fields other than logic. In my opinion, there is much room for interesting and instructive studies of Skolem's work and his life. One attraction for me in coming here to give the

*Delivered as the 1993 Skolem Lecture at the University of Oslo. Bibliography added by the editors.

Nordic Journal of Philosophical Logic, Vol. 1, No. 2, pp. 119–132.

Skolem Lecture is the opportunity to learn more about Skolem and about works devoted to the historical and conceptual study of his life and work, as well as the influences of his thoughts.

Recently, I came across Walter P. van Stigt's *Brouwer's Intuition* (1990), in which there is also a fairly extended account of Brouwer's life and general philosophy. I understand Dirk van Dalen is preparing a full biography of Brouwer. It seems to me that Skolem, in his own way, deserves to be studied in an analogous manner. In particular, Skolem's philosophy of mathematics and his implicit beliefs in the fruitful way to do mathematics very well represent a sound sense shared by many good mathematicians. It is challenging to bring out in an articulate manner what he believed, which he had only communicated informally and fragmentarily.

Last autumn William Boos sent me a typescript of his, written in July 1992, entitled *Thoralf Skolem, Hermann Weyl and "Das Gefühl der Welt als Begrenztes Ganzes"* (64 pages). Among other things, Boos sketched some of the historical and metaphysical implications of the 'Skolem functions' and tried to relate Skolem's philosophical views to two of his contemporaries: Weyl (1885–1955) and Wittgenstein (1889–1951). By the way, Bernays (1888-1977) was another contemporary. One surprising reference is to a review by Skolem (1910, signed 'Sk' only) of Weyl's 1910 essay on the definitions of fundamental mathematical concepts (reprinted in Weyl's collected works, 1968, Vol. 1, pp. 299–304) In this connection, Boos asks whether Skolem, as early as in 1910, embraced the first half but suspended judgment on the second half of Weyl's 'solution' to Richard's paradox in his essay: 'set theory has only to do with countably many relational concepts — not, however, with countably many things or sets.'

In order to facilitate references to the relevant writings by and on Skolem and Gödel, I quote primarily from Skolem's *Selected Works in Logic* (1970) and volume I of Gödel's *Collected Works* (1986). In particular, I generally adopt the abbreviations in the references (pp. 407–459) of the second book.

As already mentioned, Skolem's work in logic became known and appreciated only gradually. This was in part because logic was at first largely isolated from the principal interests of most mathematicians. In particular, this was, as reported by Fenstad (1970, p. 12), true of Skolem's own colleagues:

> Skolem's work in logic did not at the time create much interest among his Scandinavian colleagues, and later he indicated that he could not derive much inspiration because his papers remained unread. So from the beginning of the nineteen twenties he turned to more traditional and "respectable" fields — algebra and number theory.

By the time Gödel began to study logic, Skolem had already published a number of important papers in logic. It is, therefore, not surprising that Gödel referred to some of these in his early work, although he was unable to read others which would have been more relevant (see below). On the other hand, even though Gödel had become the leading light in logic from 1930 on, Skolem rarely discussed or further developed Gödel's ideas. This was partly because Skolem was not interested in the internal development of set theory, partly because Gödel's work was often definitive with regard to the immediate problems under consideration, but Skolem tended to break new ground in his early work and to deal with simple loose ends in his later work.

Gödel made use of the Skolem normal form in his dissertation (see Gödel 1986, p. 77 and p. 109), referring to Skolem 1920. But for his purpose, Gödel had to retrace the steps of the reduction of every formula to this form to show that each step can be carried out in the initial formal system, to be proved to be complete. In his 1933i, Gödel further reduced the Skolem form to the special case with three initial universal quantifiers only (Gödel 1986, p. 323), again referring to Skolem 1920.

From 1932 to 1935, Gödel reviewed five papers by Skolem. Using the abbreviations in Gödel 1986 (pp. 421–422 and 451–452), these are:

Skolem	1931	1932	1933	1933a	1934
reviewed in Gödel	1932d	1932n	1934a	1934c	1935.

Of these papers and reviews the most interesting is Skolem 1933a, reviewed in Gödel 1934c: "On the impossibility of a complete characterization of the number sequence by means of a finite axiom system."

In addition, three papers by Skolem were discussed extensively by Gödel in the sixties in his correspondence with Jean van Heijenoort and with me: Skolem 1923a, 1928, and 1929. The reason is that these papers anticipated at least the mathematical part of Gödel's proof of the completeness of predicate logic, but Gödel had not seen them before he published his proof. There was therefore the problem of separating out Gödel's own advance beyond them.

It was probably in 1965 when Professor J.E. Fenstad wrote to invite me to write a survey of Skolem's work in logic, as an introduction to Skolem 1970. I accepted the invitation, not just because I valued Skolem's work, but also because I found his free and undogmatic spirit congenial: I felt strong sympathy with his status as a kind of *outsider* and his tendency to begin from scratch, to find the important in what was simple.

I worked on the project over an extended period, and in September 1967 I sent a draft to Bernays and Gödel for comment and criticism. As usual, Bernays sent me a number of helpful observations before long. On 7 December Gödel wrote me a long letter explaining both the relation between Skolem's and his own work on the completeness of predicate logic and his views on the relation between philosophy and the study of logic. — Eventually this correspondence led to a close association between Gödel and myself.

Since about 1950 I had been struck by the fact that all the pieces in Gödel's proof of the completeness of predicate logic had been available by 1929 in the work of Skolem (notably his 1923a), supplemented by a simple observation of Herbrand's (see the reference to his work under 1.2 on p. 24 of Wang 1970). In my draft I explained this fact and said that Gödel had discovered the theorem independently and given it an attractive treatment.

In his letter of 7 December 1967, Gödel said:

> Thank you very much for sending me your manuscript about Skolem's work. I am sorry for the long delay in my reply. It seems to me that, in some points, you don't represent matters quite correctly. So I wanted to consider carefully what I have to say. — You say, in effect, that the completeness theorem is attributed to me only because of my attractive treatment. Perhaps it looks this way, if the situation is viewed from the present state of logic by a superficial observer. The completeness theorem, mathematically, is indeed an almost trivial consequence of Skolem 1922.* However, the fact is that, at that time, nobody (including Skolem himself) drew this conclusion (neither from Skolem 1922 nor, as I did, from similar considerations of his own). — This blindness (or prejudice, or whatever you may call it) of logicians is indeed surprising. But I think the explanation is not hard to find. It lies in a widespread lack, at that time, of the required epistemological attitude toward metamathematics and toward nonfinitary reasoning.

Gödel had not seen Skolem 1923a before the publication of his own proof of the completeness of predicate logic. Indeed, it is now known that Gödel made several unsuccessful attempts to find a copy of Skolem 1923a in 1929 and 1930. If he had seen the paper before publishing his own, he would undoubtedly have cited it and shortened his own paper. It would have become clear that, mathematically, Gödel's proof did not add much to Skolem's work.

This example illustrates a rather general phenomenon with many of Skolem's writings which had often been published initially at places not easily accessible to those who worked on related problems. As a result, Skolem did not get the credit he deserved and others had to repeat his work. Another example is what is commonly known as the

*Ed. note: Skolem 1923a.

Zermelo-Fraenkel set theory. As I have elaborated in my survey of Skolem's work (Wang 1970, pp. 31–35), the axiom system should more appropriately be called the Zermelo-Skolem set theory.

There are various discussions of the relation of Gödel's completeness proof to the work of Skolem and Herbrand. On 14.8.64 Gödel wrote to Heijenoort (see his 1967, p. 510):

> As for Skolem, what he could justly claim, but apparently does not claim, is that, in his 1922 paper, he implicitly proved: 'Either A is provable or $\sim A$ is satisfiable' [in other words, if A is valid, then A is provable] ('provable' taken in an informal sense). However, since he did not clearly formulate this result (nor, apparently had made it clear to himself), it seems to have remained completely unknown, as follows from the fact that Hilbert and Ackermann in 1928 do not mention it in connection with their completeness problem.

In a paper written in 1955, Skolem explicitly discussed the difficulty involved in the notion of arbitrary domains (1955c; 1970, p. 582). He asked: 'But what does this really mean? What is the totality of all domains?' He then considered the possibility of using the Löwenheim theorem to simplify the definition of satisfiability by substituting the domain N of natural numbers for arbitrary domains. 'However, the formulation of Löwenheim's theorem requires either the notion "domain" in general ... or we must formulate the theorem by saying that if $\overline{F}$ is not provable, then F can be satisfied in N.'

Skolem went on to say that (1) without presupposing set theory, the second alternative is the only possible one, and (2) in that case validity can only mean provability in the pure predicate calculus. — It seems to follow that the completeness of the calculus is true by definition. Moreover, it is unclear how the reformulated Löwenheim theorem could be proved without first inferring from the nonprovability of not-F the existence of some model for F. — In short, Skolem appears to be saying in this connection that (a) without presupposing set theory, the completeness question disappears, and (b) even the Löwenheim theorem can only be restated, but not proved (without going through the notion of arbitrary domain or model). — In the concluding part of the paragraph, Skolem seems to suggest that different versions of the predicate calculus might make the notion of satisfiability relative to one kind of logic or another. One interpretation of this observation is that he wishes to suspend judgment also between different conceptions of predicate logic, such as the classical in contrast to the intuitionistic.

Apart from my discussion in Wang 1970, the relation between Skolem's work of the completeness proof is considered by Goldfarb, in his papers of 1971 and 1979, and, more recently, by Dreben and Heijenoort (Gödel 1986, pp. 50–56).

In his letter of 7.12.67 to me, Gödel explains at length how his 'objectivistic conception of mathematics and metamathematics' was fundamental to his work in logic (see Wang 1974, pp. 8–11), and then concludes by returning to Skolem:

> Skolem's epistemological views were, in some sense, diametrically opposed to my own. E.g., on p. 29 of his 1929 paper [Skolem 1929; 1970, p. 253], evidently because of the transfinite character of the completeness question, he tried to *eliminate* it, instead of answering it, using to this end a new definition of logical consequence, whose idea exactly was to *avoid* the concept of mathematical truth. Moreover, he was a firm believer in set theoretical relativism and in the sterility of transfinite reasoning for finitary questions (see p. 49 of his paper [1970, p. 273]).

There are two problems, on different levels, about Gödel's comments on the relation between Skolem's work and the completeness question. On the local level, Gödel said in his letter that the easy inference from Skolem 1923a to the completeness conclusion 'is definitely non-finitary, and so is any other completeness proof for the predicate calculus'. But Bernays had pointed out in his letter to me at the time that Skolem did not think of the theorems of elementary logic as given in a formal system and, therefore, that the question of full completeness had no meaning for Skolem. Moreover, Skolem did actually use nonfinitary reasoning in his early proof of Löwenheim's theorem. In any case, it is clear that Skolem had little interest in the formalization of logic, so that, as Gödel suggested, Skolem implicitly proved an informal version of the completeness theorem in Skolem 1923a: if A is valid, then A is provable (by familiar informal reasoning). — It is then easy to verify that actual steps in the informal proof can also be carried out in any familiar formal system for the predicate logic.

The global problem is Gödel's belief that his 'objectivistic conception' is fruitful. This is true with regard to Gödel's own work, as explained in his letter to me. But it does not follow that Skolem's different, more or less finitary, conception of mathematics and metamathematics is not fruitful for obtaining (other) results. Indeed, Skolem's various important contributions to logic have demonstrated the fruitfulness of his conception, as much as Gödel's results have demonstrated that of his. — It seems to me that one can learn, by studying and reflecting on Skolem's work, one fruitful way of doing logic, which is different from Gödel's. That is also one reason why I believe it to be a valuable task to look for an articulate formulation of Skolem's philosophy of mathematics and mathematical activity.

Both Skolem and Gödel are well known for their results on the limitations of familiar methods in characterizing mathematical concepts such as sets and natural numbers. In 1922, Skolem developed what has since been known as 'Skolem's paradox', which shows that every thoroughgoing axiomatization of set theory, if consistent, has a countable model. Skolem speaks of 'a relativity of the set-theoretic notions' and goes on to say (1923a; 1970, p. 144):

In order to obtain something absolutely uncountable, we would have to have either an absolutely uncountably infinite number of axioms or an axiom that could yield an absolutely uncountable number of first-order propositions. But this would in all cases lead to a circular introduction of higher infinities; that is, *on an axiomatic basis higher infinities exist only in a relative sense.*

In 1929, stimulated by Brouwer's lecture in March, Gödel reflected on 'the inexhaustibility of mathematics' and told Carnap some of his ideas on 23.12.29, which can be compared with the above quotation. According to Carnap's diary, Gödel said on this occasion:

We admit as legitimate mathematics certain reflections on the grammar of a language that concerns the empirical. If one seeks to formalize such a mathematics, then with each formalization there are problems, which one can understand and express in ordinary language, but cannot express in the given formalized language. It follows (Brouwer) that mathematics is inexhaustible: one must always again draw afresh from the 'fountain of intuition.' There is, therefore, no characteristica universalis for the *whole* mathematics, and no decision procedure for the whole mathematics. In each and every closed language there are only countably many expressions. The *continuum* appears only in 'the whole of mathematics.' ... If we have *only one language*, and can only make 'elucidations' about it, then these elucidations are inexhaustible, they always require some new intuition again.

These observations by Gödel are both more general and less definite than Skolem's discussion. They seem to say that no language, being necessarily countable, could capture the continuum fully.

Gödel's further development of the idea in his famous result, obtained in 1930, shows that neither can the natural numbers be captured fully by any formal system, either directly or with the help of set theory.

In the last section (1970, pp. 269–272) of his 1929 paper, Skolem considers a fragment of number theory, and shows that it admits some simple nonstandard model by taking a suitable set of polynomials as the natural numbers. In a slightly earlier paper he says (1929a; 1970, p. 224):

A very probable consequence of this relativism is again that it cannot be possible to *completely* characterize the mathematical concepts; this already holds for the concept of the natural number. Thereby arises the question, whether the unicity or categoricity of mathematics might not be an illusion. Then it would not at all be strange if some problems were unsolvable; they would in fact not be decided by means of the principles which we are able to found them with, and it would not at all be necessary to resort to a new logic, as Brouwer does, in order to see this.

In 1933 Skolem published his famous result on the concept of natural number, which gives, for any 'axiom system' for the concept, a nonstandard model which has the same true (first-order) sentences as its standard model (Skolem 1933a and 1934, see also the 1954 lecture (1955d); compare, for Gödel's reviews, Gödel 1934c and 1935). — In the seventies, Gödel said to me that one should not construe this Skolem theory as establishing the impossibility of fully characterizing the concept of natural number by logic, because we can use the 'theory of concepts', which is also logic but goes beyond set theory in certain ways. — I do not fully understand Gödel's ideas, but I think it is interesting to consider the question whether or in what sense Skolem's theorem may be said to show the impossibility of capturing natural numbers by logic.

My main purpose in this lecture is to select a few quotations from Skolem and propose a few problems, which I find fascinating, for further study. One problem is the highly nonfinitary, nonconstructive character of 'Skolem functions', introduced in Skolem 1920. Another problem is the meaning of Skolem's 'relativism'. — Skolem's attitude toward set theory is worth considering in the context of the development of set theory and of Skolem's own contribution to it. Among other things, Skolem made several suggestive observations on the continuum hypothesis. — It may be argued that he was able to make his particular contributions to the foundations of set theory precisely because he was skeptical toward Cantorian set theory.

As Boos has pointed out in the typescript mentioned above, the Skolem functions are closely related to the τ-symbol, introduced by Hilbert in 1923 to formulate the laws governing quantifiers. For example, in 1929 Skolem quoted with approval Weyl's observation that Hilbert's τ-symbol is a contrived 'divine automat': 'If we had access to such an automat, we would be relieved of all pains; but the belief in its existence is of course the purest nonsense' (1929a; 1970, p. 220).

The difference between Skolem's and Hilbert's uses of such an 'automat' is, I believe, the fact that the Skolem functions are employed only hypothetically, to select objects from a domain assumed to exist and to have certain properties — whereas Hilbert uses his 'automat' to represent the very essence of our nonconstructive reasoning over infinite ranges, in the sense of the 'actual' infinite.

Skolem's observations on the continuum hypothesis may be viewed as an early conjecture that it is not decidable by the familiar axioms of set theory:

1923a (1970, p. 149, note 2). — Since Zermelo's axioms do not determine the domain B [the model for them], it is very improbable that all cardinality problems are decidable by means of these axioms. For example, it is quite probable that what is called the continuum problem is not solvable at all on this basis; nothing need be decided about it.

1929a (1970, p. 222) [comment on Hilbert's attempt to prove the continuum hypothesis]. — It seems in fact that Hilbert wants to uphold the Cantorian views in their old absolutist sense, which seems to me very strange; it is revealing that he has never found it necessary to deal with the relativism, which I proved for every finitistically formulated axiomatization of set theory.

Skolem's skepticism toward Cantorian set theory continued over the years, despite the clarifications in the work of Zermelo, Gödel, and others in the thirties and the forties. For instance, the following two observations indicate that his views did not change between 1915 and 1955:

1923a (1970, p. 152) [conclusion of the paper]. — The most important result above is that set-theoretic notions are relative. I had already communicated it to F. Bernstein in Göttingen in the winter of 1915–16. ... I believed that it was so clear that axiomatization in terms of sets was not a satisfactory ultimate foundation of mathematics that mathematicians would, for the most part, not be very much concerned with it. But in recent times I have seen to my surprise that so many mathematicians think that these axioms of set theory provide the ideal foundation for mathematics; therefore it seemed to me that the time had come to publish a critique.

1955c (1970, p. 583). — Is Cantor's set theory still going strong? ... Sometimes I have had the task to write reviews of articles, where set theoretic notions and theorems are used without any explanation of what kind of set theory is meant. It is a disagreeable job to write reviews in such cases. One does not know what the author really means.

Apart from incidental philosophical observations in the middle of technical articles, Skolem also wrote a few essays of a philosophical character, usually on the occasion of giving a lecture. Among them are his 1938 lecture in Zürich (1941a), his 1950 lecture in Cambridge, Mass. (1950d), and his 1958 lecture in Paris (1958a).

The record of the first lecture includes discussions with Bernays. — Bernays observed that things like the 'Skolem paradox' were to be understood as a limitation of the use of formal systems to capture our intuitive mathematical concepts of set and number. Skolem's reply is thus reported in the record: 'Mr. Skolem thinks that one does not have to view the situation from such an angle. In his view, the best way is to refer in each domain of research to an appropriate formalism. This manner of proceeding does not imply any restriction on the possibilities of reasoning, for one has always the liberty of passing to a more extended formalism' (Skolem 1970, p. 480).

In the Cambridge lecture, Skolem recommended further developments of predicative set theory (pp. 525–526) and concluded with what he took to be the most important observation. — He pointed out that the use of quantifiers is the most difficult thing and proposed to develop mathematics without using them (pp. 526–527):

> I think that the fear that mathematics will be crippled by the restriction to the use of only free logical variables is exaggerated. I am aware that it may look different to mathematicians accustomed to analysis — to the theory of functions say — and those only working in the theory of numbers, but there are certainly many more ways of treating mathematics than we know today.
>
> Now I will not be misunderstood. I am no fanatic, and it is not my intention to condemn the nonfinitistic ideas and methods. But I should like to emphasize that the finitistic development as far as it may be carried out has a very great advantage with regard to clearness and security. Further there may be good reason to conjecture that it can be carried out very far, if one would make serious attempts in that direction.

In his Paris lecture, Skolem said that we can prove the consistency of formal systems which use only free variables by our intuition of mathematical induction, and that 'in the other cases we can adopt the opportunistic standpoint (see my Cambridge lecture, p. 700). — My point of view is then that we use formal systems for the development of mathematical ideas' (p. 634).

This observation seems to involve a change of emphasis from the Cambridge lecture, where he said that 'this standpoint has the unpleasant feature that we can never know when we have finished the foundation of mathematics', after defining the opportunistic standpoint as follows (p. 524):

> One desires only to have a foundation which makes it possible to develop present-day mathematics, and which is consistent so far as is known yet. Should any contradiction occur, we may try to make such restrictions in the underlying postulates that the deduction of the contradiction proves impossible. This may perhaps be called the opportunistic standpoint. It is a very practical one.

If we put together these observations from 1950 and 1958 by Skolem, we see three components or levels in his position on the study of the foundations of mathematics. First, whenever possible, avoid the use of quantifiers and adhere to the finitistic standpoint. Second, it is desirable to develop further predicative set theory, which, though not as clear as finitary mathematics, is comparatively transparent. Third, adopt the opportunistic standpoint in areas where the first two approaches are insufficient (at present).

Expressed in this way, Skolem's views are not as different from Gödel's views, in the form they emerged from his conversations with me, as commonly believed. For example, both Gödel and I and indeed most logicians agree that the degree of 'clearness and security' decreases as we move from finitistic mathematics to predicative set theory to classical analysis and then to Cantorian set theory. Moreover, Gödel said to me, and I agree with him: which of these theories to prefer depends on how much clarity and certainty one desires. This position seems to agree with the spirit of Skolem's observation in 1950: 'Which one of the different theories shall we prefer? That depends on the desires we have in the foundation of mathematics' (p. 524).

In my 1958 essay "Eighty years of foundational studies", I proposed a scheme of replacing the conflict of the different schools on the foundations with a sequential demarcation of things like finitistic, intuitionistic, predicative, and classical mathematics, and suggested that the more instructive study is to clarify the interconnections between these areas: especially a clarification of what natural steps are involved in going from a more transparent domain to a less transparent one. For instance, Gödel's interpretation of intuitionistic arithmetic by a slight extension of finitistic mathematics and his translation of the classical arithmetic into the intuitionistic are striking examples of this enterprise. — I have no doubt that Skolem would also find such a program congenial.

I had only occasional personal contacts with Skolem. — In 1954, both he and I took part in a symposium at the International Congress of Mathematicians in Amsterdam. (The proceedings were published in 1955 as a volume entitled *Mathematical Interpretation of Formal Systems*.) In autumn 1957 I went to the University of Notre Dame to give a lecture; Skolem came to it and asked a question. In autumn 1961 I sent to many logicians, for verification and criticism, copies of a manuscript, which reduces predicate logic, by using the intuitive tool of certain tiling problems, to formulas of the AEA form. Skolem was one of only two recipients who responded to the request for comments — Dana Scott being the other. — My impression from these casual encounters does not contradict Professor Fenstad's evaluation that 'Skolem was very modest and retiring'.

It is quite possible that one would not have learnt any more mathematics and philosophy from Skolem through personal contacts than from just studying his writings. However that may be, after spending so many hours of my life with his work, it is a special experience to set foot in his home country for the first time and to honor him at his home institution.

References

Boos, William. Jul. 1992. Thoralf Skolem, Hermann Weyl and "Das Gefühl der Welt als Begrenztes Ganzes". Typescript.

Gödel, Kurt. 1932d. Besprechung von "Über einige Satzfunktionen in der Arithmetik". *Zentralblatt für Mathematik und ihre Grenzgebiete*, 2, p. 3. Reprinted in Gödel 1986, pp. 240–242.

Gödel, Kurt. 1932n. Besprechung von "Über die symmetrisch allgemeinen Lösungen im identischen Kalkul". *Zentralblatt für Mathematik und ihre Grenzgebiete*, 4, p. 385. Reprinted in Gödel 1986, p. 262.

Gödel, Kurt. 1933i. Zum Entscheidungsproblem des logischen Funktionenkalküls. *Monatshefte für Mathematik und Physik*, 40, pp. 433–443. Reprinted in Gödel 1986, pp. 306–326.

Gödel, Kurt. 1934a. Besprechung von "Ein kombinatorischer Satz mit Anwendung auf ein logisches Entscheidungsproblem". *Zentralblatt für Mathematik und ihre Grenzgebiete*, 7, pp. 97–98. Reprinted in Gödel 1986, p. 372.

Gödel, Kurt. 1934c. Besprechung von "Über die Unmöglichkeit einer vollständigen Charakterisierung der Zahlenreihe mittels eines endlichen Axiomensystems". *Zentralblatt für Mathematik und ihre Grenzgebiete*, 2, p. 3. Reprinted in Gödel 1986, pp. 378–380.

Gödel, Kurt. 1935. Besprechung von "Über die Nichtcharakterisierbarkeit der Zahlenreihe mittels endlich oder abzählbar unendlich vieler Aussagen mit ausschließlich Zahlenvariablen". *Zentralblatt für Mathematik und ihre Grenzgebiete*, 10, p. 49. Reprinted in Gödel 1986, p. 384.

Gödel, Kurt. 1986. *Collected Works,* edited by Solomon Feferman, vol. 1. Oxford University Press, New York.

Goldfarb, Warren D. 1971. Review of Skolem 1970. *Journal of Philosophy*, 68, pp. 520–530.

Goldfarb, Warren D. 1979. Logic in the twenties: The nature of the quantifier. *Journal of Symbolic Logic*, 44, pp. 351–368.

van Heijenoort, Jean (ed.). 1967. *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931.* Harvard University Press, Cambridge, Mass.

Skolem, Thoralf. 1910. H. Weyl. Über die Definitionen der mathematischen Grundbegriffe. *Jahrbuch über die Fortschritte der Mathematik*, 41, pp. 89–90. Review of Weyl 1910.

Skolem, Thoralf. 1920. Logisch-kombinatorische Untersuchungen über die Erfüllbarkeit und Beweisbarkeit mathematischen Sätze nebst einem Theoreme über dichte Mengen. *Skrifter utgit av Videnskabsselskapet i Kristiania, I. Matematisk-naturvidenskabelig klasse*, 4, pp. 1–36. Reprinted as 1920a in Skolem 1970, pp. 103–136.

Skolem, Thoralf. 1923a. Einige Bemerkungen zur axiomatischen Begründung der Mengenlehre. In *Matematikerkongressen i Helsingfors den 4–7 juli 1922, Den femte skandinaviska matematikerkongressen, Redogörelse*, pp. 217–232, Helsinki. Akademiska Bokhandeln. Reprinted as 1922b in Skolem 1970, pp. 137–152.

Skolem, Thoralf. 1928. Über die mathematische Logik. *Norsk matematisk tidsskrift*, 10, pp. 125–142. Reprinted as 1928a in Skolem 1970, pp. 189–206.

Skolem, Thoralf. 1929. Über einige Grundlagenfragen der Mathematik. *Skrifter utgit av Det Norske Videnskaps-Akademi i Oslo I. Matematisk-naturvidenskabelig klasse*, 4, pp. 1–49. Reprinted as 1929b in Skolem 1970, pp. 227–273.

Skolem, Thoralf. 1929a. Über die Grundlagendiskussionen in der Mathematik. In *Proceedings of the 7th Scand. Math. Congress*, pp. 3–21. Reprinted in Skolem 1970, pp. 207–225.

Skolem, Thoralf. 1931. Über einige Satzfunktionen in der Arithmetik. *Skrifter utgitt av Det Norske Videnskaps-Akademi i Oslo, I. Matematisk naturvidenskapelig klasse*, 7, pp. 1–28. Reprinted as 1930c in Skolem 1970, pp. 281–306.

Skolem, Thoralf. 1932. Über die symmetrisch allgemeinen Lösungen im identischen Kalkul. *Skrifter utgitt av Det Norske Videnskaps-Akademi i Oslo, I. Matematisk naturvidenskapelig klasse*, 6, pp. 1–32. Reprinted as 1931d in Skolem 1970, pp. 307–336.

Skolem, Thoralf. 1933. Ein kombinatorischer Satz mit Anwendung auf ein logisches Entscheidungsproblem. *Fundamenta mathematicae*, 20, pp. 254–261. Reprinted as 1933c in Skolem 1970, pp. 337–344.

Skolem, Thoralf. 1933a. Über die Unmöglichkeit einer Charakterisierung der Zahlenreihe mittels eines endlichen Axiomensystems. *Norsk Matematisk Forening, Skrifter*, pp. 73–82. Reprinted as 1933d in Skolem 1970, pp. 345–354.

Skolem, Thoralf. 1934. Über die Nicht-charakterisierbarkeit der Zahlenreihe mittels endlich oder abzählbar unendlich vieler Aussagen mit ausschließlich Zahlenvariablen. *Fundamenta mathematicae*, 23, pp. 150–161. Reprinted as 1934b in Skolem 1970, pp. 355–366.

Skolem, Thoralf. 1941a. Sur la Porté du Théorème de Löwenheim-Skolem. In *Les Entretiens de Zürich, 6–9 Dec. 1938*, pp. 25–47. Reprinted in Skolem 1970, pp. 455–482.

Skolem, Thoralf. 1950d. Some remarks on the foundation of set theory. In *Proceedings of International Congress of Mathematics, Massachusetts 1950*, pp. 695–704. Reprinted in Skolem 1970, pp. 519–528.

Skolem, Thoralf. 1955c. A critical remark on foundational research. *Kongelig Norske Videnskabsselskabs Forhandlinger, Trondheim*, XXVIII, no. 20, pp. 100–105. Reprinted in Skolem 1970, pp. 581–586.

Skolem, Thoralf. 1955d. Peano's axioms and models of arithmetic. *Studies in Logic and the Foundation of Mathematics*, pp. 1–14. Reprinted in Skolem 1970, pp. 587–600.

Skolem, Thoralf. 1958a. Une relativisation des notions mathématiques fondamentales. In *Colloques internationaux des Centre National de la Recherche Scientifique, Paris 1958*, pp. 13–18. Reprinted in Skolem 1970, pp. 633–638.

Skolem, Thoralf. 1970. *Selected Works in Logic,* edited by Jens E. Fenstad. Universitetsforlaget, Oslo.

van Stigt, Walter P.˙1990. *Brouwer's Intuitionism.* No. 2 in Studies in the History and Philosophy of Mathematics. North–Holland, Amsterdam.

Wang, Hao. 1958. Eighty years of foundational studies. *Dialectica*, 12.

Wang, Hao. 1970. A survey of Skolem's work in logic. In Skolem 1970, pp. 17–52.

Wang, Hao. 1974. *From Mathematics to Philosophy.* Humanities Press, New York.

Weyl, Hermann. 1910. Über die Definitionen der mathematischen Grundbegriffe. *Mathematisch-naturwissenschaftliche Blätter*, 7, pp. 89–90 and 109–113. Reprinted in Weyl 1968, pp. 298–304.

Weyl, Hermann. 1968. *Gesammelte Abhandlungen,* edited by K. Chandrasekaran, vol. 1. Springer Verlag, Berlin.